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Výzkumný úkol

Studium měření doby letu atomů antivodíku v experimentu AEGIS

Praha 2010 **Michal Špaček**

Abstrakt

Název práce:
Studium měření doby letu atomů antivodíku v experimentu AEGIS

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Abstrakt: Tato práce se zabývá pohybem atomů antivodíku v obecných elektrických a magnetických polích a v gravitačním poli Země. V úvodu autor objasňuje, které části experimentu AEGIS se práce dotýká a jaký význam pro experiment má rozřešení této problematiky. Podrobně jsou odvozeny kinematika nábojové výměny a kinematika deexcitace urychleného antivodíku - výsledky jsou ověřeny nezávislými postupy a demonstrovány na jednoduchých příkladech. V rámci kvaziklasického modelu, který je postupně budován a analyzován, je nalezena pohybová rovnice antivodíku ve vnějším poli a jsou rozebrána její omezení. Průběžně jsou zdůrazňovány důležité vlastnosti antivodíku.

Klíčová slova: AEGIS, antivodík, positronium, nábojová výměna, deexcitace, vlastní elektrický dipól, indukovaný elektrický dipól, vlastní magnetický dipól, indukovaný magnetický dipól, pohybová rovnice dipólu, elektrické pole, magnetické pole, gravitačni pole, starkovské urychlení rydbergovských atomů.

Abstract

Title:
Study of the antihydrogen atoms' time of flight in the AEGIS experiment

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Branch of study: Nuclear engineering

Type of work: Research project

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Abstract: This work deals with the motion of antihydrogen atoms in general electric and magnetic fields and in the Earth's gravitational field. The Author first clarifies which part of the AEGIS experiment is concerned and what is the importance of solving the problematics. Charge exchange kinematics and antihydrogen deexcitation kinematics are derived in detail - the outcomes are verified by independent derivation methods and illustrated with simple examples. Within a quasiclassical model which is constructed and analysed progressively, the antihydrogen's equation of motion in an external field is found and its limits are analysed. The important qualities of the antihydrogen are stressed continuously.

Key words: AEGIS, antihydrogen, positronium, charge exchange, deexcitation, intrinsic electric dipole, induced electric dipole, intrinsic magnetic dipole, induced magnetic dipole, dipole equation of motion, electric field, magnetic field, gravitational field, Stark acceleration of Rydberg atoms.

Obsah

1 Introduction

This text's aim is to provide an analysis how to deal with the antihydrogen kinematics and dynamics in the AEGIS experiment at CERN.

The first scientific goal of AEGIS (Antimatter Experiment: Gravity, Interferometry, Spectroscopy) is to study the gravitational interaction of antimatter, for the first time in history. For this purpose, a device has been designed for antihydrogen production and experimental use [1].

Although antimatter has been known since 1930's, the scientists haven't got the opportunity to test its gravity properties experimentally yet. There are many difficulties: The antimatter itself, whether natural or artificially prouduced, is extremelly rare; much of antimatter are charged particles (positrons, antiprotons, ...) therefore a presence of any weak electric or magnetic field makes gravity studies unfeasible; and much of antimatter propagates almost at the speed of light (antineutrinos).

Gravity is the weakest of all the fundamental interactions. If its repulsion magnitude is compared to Coulombic repulsion of two protons at Bohr radius, the ratio gives 10^{-36}. Gravity prevails if the distances are enormous or if the two interacting bodies are huge. However, at AEGIS, any long distance measurements can not be afforded and even though one of the interacting body (the Earth) is huge enough, the second (an antimatter particle) is always microscopic.

Fortunately, there is no need the gravity prevails in this sense - the interaction's another characteristic is that it can not be "cancelled"[1]. If an area is prepared with both electric and magnetic fields screened out ("cancelled"), the gravity would remain weak but also the only effective interaction - this is the general way of gravity measurements on antimatter at AEGIS.

There are three major theories of gravity interaction of antimatter [2]. The first theory predicts for antimatter exactly the same behaviour as it is known for a classical matter; the second theory proposes analogy to Coulombic force in which antimatter particle would fall up in the Earth's gravitational field; and the third theory suggests existence of two more exotic potentials so antimatter would fall down but a bit faster thana classical matter.

No matter which of these theories (or another one) is valid, due to small dimensions of the AEGIS experiment device it is clearly reasonable to treat the gravitational acceleration (positive or negative) as constant in the device's volume (that is the gravitational field would be uniform).

Because of the gravity's relative weakness, for studying gravity behaviour on charged antimatter particles one would need both electric and magnetic fields to be screened out with a precision 10^{-36} or better. This is completely impossible so the only promising way now is to work with electrically neutral antimatter particles. Another problem resulting from the gravity's weakness is that for particles with large initial velocities almost nothing could be judged by their trajectories.

Antimatter is not only so rare on Earth, it also can not survive a touch with a classical matter - this always leads to a complete annihilation of all concerned particles. On account of that, one has to keep antimatter in a space of very hight vacuum and slow the antiparticles down so that they stay in their preparation volume until the experimental device is ready for measurements.

For AEGIS, antihydrogen was found to be the only possible particle for gravity studies and measurements. It is (when not ionised) electrically neutral[2] and it can be handled at velocities small enough to make gravity measurements possible.

[1] According to one of the theories of antimatter's gravitational interaction, the gravity could be "cancelled"in the same way, as the electric interaction of the Earth in space is - the both charge types are of the same number and space distribution. However, antimatter amount is still easily overwhelmed by the mass of the Earth. So the statement about gravity's uncancellation remains valid.

[2] Even though a particle is electrically neutral as a whole, it can still exhibit electrical qualities, especially if it has a non-uniform charge distribution in space. Then electric and magnetic fields play roles and may affect the gravity measurements. On the other hand, the fields' outscreening does not have to be as much as 10^{-36}. This will be discussed later.

An antihydrogen atom is a compound state of one antiproton (negative charge) and one positron (positive charge) in a general quantum state $\Psi_{n,l,m}$. Its mass is equal to the hydrogen atom in the same state. Antihydrogen is likely not to exist in nature close to Earth at all so one has to produce it artifically through so called charge exchange reaction.

In charge exchange, a positronium (a compound state of one positron and one electron) hits an antiproton which displaces the electron in the positronium. One antihydrogen and one electron are obtained as products. Note that the quantum state of the gained antihydrogen depends on that of the positronium.

In AEGIS, it is impossible to keep antihydrogen in the vacuum volume at small velocities for a long time and the experimental device can not work continuously. However, one is able to handle its element - antiproton - very well with a system of electrods (Malmberg-Penning trap) inside a strong magnetic field; the antiprotons follow closed curves at given small average velocities.

The philosophy of the experiment is following: Antiprotons are being trapped untill their number is large enough. Then (still inside the trap) they undergo the charge exchange reactions which result in antihydrogen production. Since all the antihydrogen rise almost simultaneously, the trap electrodes change their mode in this moment and become an accelerator (so called Stark accelerator of Rydberg atoms) suddenly; the magnetic field is not switched off - it is time-independent. The electric field of the accelerator makes a gradient which forces the antiatoms to move in the field even though they are electrically neutral[3]. The accelerator gives each suitable antihydrogen an impulse and the antiatoms begin to fly just about horizontally. The electrodes are turned off after the antiatoms leave their space and no significant electric field is in the presence for the time. Now it is necessary that the antiatoms reach a space where the gravitational field would be the only effective one (see above). Accordingly, the antiatoms have to fly through a zone of a magnetic field gradient from a strong field area, where they rose, to almost zero field volume. There (as other fields are succesfully screened out) the antiatoms follow a parabola typical for a uniform gravitational field and some of them hit a position sensitive detector. Unfortunately, this coordinate itslef can not provide any information about the antimatter's gravitational acceleration. To fit a parabola, one need together with the final point coordinates another information - starting point coordinates and starting velocity vector[4].

It is impossible to measure these starting quantities directly. Instead, one knows the coordinates of the antihydrogen production zone, the time of starting and switching the accelerator off, the time of antihydrogen hitting the position sensitive detector and the electric and magnetic field properties. The crucial then is to know how much of the total time of flight is spent in the final (gravity-only) area and whether the antihydrogen keeps to fly horizontally or is elevated or dropped a bit by the electric and magnetic fields.

This text tries to find the dynamics of antihydrogen moving in combined electric, magnetic and gravitational field to answer the questions in the previous paragraph. Much of the brief analysis in this introduction is discussed in detail further as well as all the important characteristics of antihydrogen.

[3] In an electric field with a gradient, the antihydrogen gains induced electric dipole moment which is sensitive to that gradient retroactively and forces the antihydrogen to accelerate.

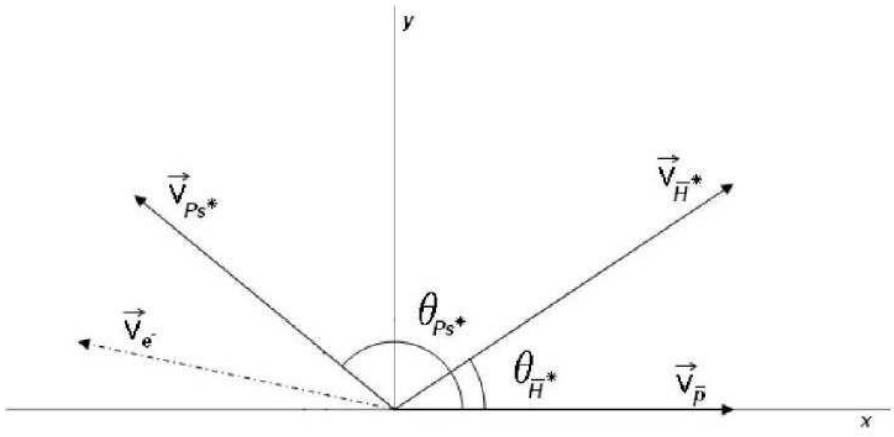
[4] These starting quantities may be given at any point of the antihydrogen's trajectory which is in the area with electric and magnetic field screened out.

2 Charge exchange kinematics

In charge exchange reaction, an antiproton replaces an electron in a positronium and as a result a neutral antihydrogen is obtained. It is the only mechanism of antihydrogen production in AEGIS. In this chapter, the quantities linked to an antiproton and a positronium are denoted as initial, whereas those concerning antihydrogen or electron as final. Only the velocity vectors infinitesimally before (respectivelly after) the reaction are considered. Symbolically:

$$Ps + \bar{p} \rightarrow \bar{H} + e^{-} \tag{2.1}$$

In this chapter, the focus is on the charge exchange kinematics. The main goal is to derive an identity putting all the proper quantities together, namely: initial velocities of an antiproton and of a positronium ($v_{\bar{p}}$ and v_{Ps}), the antihydrogen's final velocity ($v_{\bar{H}}$), angles between these velocity vectors (θ_{Ps} a $\theta_{\bar{H}}$) and the difference in the binding energy of the initial positronium and the final antihydrogen (Q). The derivation will be performed in a more general relativistic way.



Obrázek 2.1: The setting of the charge exchange reaction. In the picture, the θ_e angle is missing, however, it is well-defined in the text and its placing is intuitive. The excitations of the positronium and the antihydrogen are reminded by asterisk signs but not used in the text anymore. Taken from [1].

It is necessary to define the setting closer. The fact that only the instantenous initial velocities matter[1] supports an inertial frame of reference implementation. The frame of reference is attached to the laboratory, does not rotate and a classical Cartesian coordinate system is used. The initial velocity vectors of an antiproton and a positronium unambiguously form a plane in which the whole reaction takes place. The x-axis is chosen in that way, the vector of an antiproton points in the axis' positive direction; the y-axis is perpendicular to x-axis and lays fully in the plane given by the initial vectors; and the z-axis is perpendicual to both previous axes and plays no role in the kinematics; the origin is put in the point where the charge exchange takes place. These assumption make it clear that for every single charge exchange

[1]The kinematics does not depend on the choice of the coordinate system's origin and also not on a position itself. If the antiproton undergoes nonzero acceleration in the moment of the charge exchange reaction, it is caused by the present external force field (generally the combined electric, magnetic and gravitational field). Since the incipient antihydrogen differs in proper dynamical qualities from both positronium and antiproton, the antihydrogen gains acceleration from the field "suddenly"(so called jump change). Therefore only the instantaneous velocities of initial and final particles matter. The antihydrogens' position (known at least approximately from the experimental geometry) and instantaneous velocity (given by the kinematic identity) serve as initial conditions for accelerated motion in the external fields.

reaction event, a different coordinate system is separately materialized. However, since the kinematic identity will be formulated only in the terms of scalars, there is no loss of generality.

Every velocity vector is given by three independent real numbers. In the described coordinate system, the z coordinate is zero for every vector of the charge exchange reaction so two coordinates are non-trivial. These could be the two remaining Cartesian coordinates as well as two polar coordinates - the magnitude of the vector (never negative) and its polar angle. Let this polar angle be measured contra-clockwise from the positive x half-axis.

Following from how the coordinate system was defined, the angle is always zero for the antiproton. Therefore only its velocity magnitude is non-trivial and is denoted as $v_{\bar{p}}$. The three remaining velocity vectors need two coordinates to be fully described - a pair of a velocity magnitude and a polar angle. For the positronium these are v_{Ps} and θ_{Ps}, for the antihydrogen $v_{\bar{H}}$ and $\theta_{\bar{H}}$ and for the electron v_e and θ_e. The meaning of the three angle quantities are according to the coordinate system definition their orientation in reference to the velocity vector of the initial antiproton.

The last which is necessary to define is the Q quantity. It is the difference between the binding energies of the positronium and the antihydrogen (both can be in a general quantum state). There are two possible ways of definition:

$$Q = \left((m_{\bar{p}} + m_{Ps}) - (m_{\bar{H}} + m_e) \right) c^2 \tag{2.2a}$$

$$Q = R_{\bar{H}} \left(\frac{1}{n_{\bar{H}}^2} - \frac{1}{2n_{Ps}^2} \right) \tag{2.2b}$$

The R quantity is the Rydberg constant of antihydrogen [3]:

$$R = \frac{q_{\bar{e}}^4}{8h^2\epsilon_0^2} \cdot \frac{m_{\bar{p}} m_{\bar{e}}}{m_{\bar{p}} + m_{\bar{e}}} \tag{2.3}$$

If the mass of the antiproton in the second fraction is replaced by the mass of electron, the Rydberg constant of positronium is obtained.

The energy of a relativistic body (an indice $\alpha = \bar{p}, Ps, \bar{H}, e$ denotes the particular particle) and its Taylor power series expansion to the fourth order in v give:

$$E_\alpha = \frac{m_\alpha c^2}{\sqrt{1 - \frac{v_\alpha^2}{c^2}}} \approx m_\alpha c^2 + \frac{1}{2} m_\alpha v_\alpha^2 + \frac{3}{8} m_\alpha \frac{v_\alpha^4}{c^4} \tag{2.4}$$

Similarly for the particle's linear momentum:

$$p_\alpha = \frac{m_\alpha v_\alpha}{\sqrt{1 - \frac{v_\alpha^2}{c^2}}} \approx m_\alpha v_\alpha + \frac{1}{2} m_\alpha \frac{v_\alpha^3}{c^2} \tag{2.5}$$

Besides kinematic derivation further, this formula is important for one more reason: It shows that linear momentum of a body and its velocity are always parallel. Therefore all the reasoning about a frame of reference and a coordinate system in the beginning of this chapter applies to linear momentums as well as to velocities[2].

With no doubt, the energy conservation holds in charge exchange:

$$E_{\bar{p}} + E_{Ps} = E_{\bar{H}} + E_e \tag{2.6}$$

[2]The velocity model for the definition of the frame of reference and of the coordinate system was introduced because it illustrates the motivation of the kinematics. However, the linear momentum is much more suitable as the starting point of identity's derivation since it is very often conserved in a reaction in contrast to velocity.

Because of the outlined relativistic approximations and the way the Q quantity has been defined, the previous equation could be written as:

$$\frac{1}{2}m_e v_e^2 + \frac{3}{8}m_e\frac{v_e^4}{c^4} = \frac{1}{2}m_{\bar{p}}v_{\bar{p}}^2 + \frac{3}{8}m_{\bar{p}}\frac{v_{\bar{p}}^4}{c^4} + \frac{1}{2}m_{Ps}v_{Ps}^2 + \frac{3}{8}m_{Ps}\frac{v_{Ps}^4}{c^4} - \frac{1}{2}m_{\bar{H}}v_{\bar{H}}^2 - \frac{3}{8}m_{\bar{H}}\frac{v_{\bar{H}}^4}{c^4} + Q \quad (2.7)$$

If both terms containing the electron velocity are exclusively moved on the left-hand side of the equation and the whole right-side is denoted as P_1, one gets a biquadratic equation for the electron velocity v_e:

$$v_e^4 + \left(\frac{4}{3}c^2\right)v_e^2 - \left(\frac{8c^2P_1}{3m_e}\right) = 0 \quad (2.8)$$

There is only one physical solution (that is a real one) to the equation:

$$v_{e,1}^2 = \frac{2}{3}c^2\left(\sqrt{1 + \frac{6P_1}{m_e c^2}} - 1\right) \quad (2.9)$$

Now one can apply the same procedure to the linear momentum conservation law which is of course also valid. However, this conservation law is 3-dimensional. From how the coordinate system was defined, two equations are sufficient since the third (linked to the z-axis) says trivially $0 = 0$. The pair is:

$$p_{\bar{p}} + p_{Ps}\cos\theta_{Ps} = p_{\bar{H}}\cos\theta_{\bar{H}} + p_e\cos\theta_e \quad (2.10a)$$

$$p_{Ps}\sin\theta_{Ps} = p_{\bar{H}}\sin\theta_{\bar{H}} + p_e\sin\theta_e \quad (2.10b)$$

The terms including $p_{\bar{H}}$ (and $\theta_{\bar{H}}$ as well) can be easily transfered onto the other sides in each of the equations. Then, by squaring the both equations, summing them and by using the Pythagorean identity, one gets a single equation instead of two, with the θ_e angle completely eliminated.

$$p_{\bar{p}}^2 + p_{Ps}^2 + p_{\bar{H}}^2 - 2p_{\bar{p}}p_{\bar{H}}\cos\theta_{\bar{H}} + 2p_{\bar{p}}p_{Ps}\cos\theta_{Ps} - 2p_{Ps}p_{\bar{H}}\cos(\theta_{Ps} - \theta_{\bar{H}}) = p_e^2 \quad (2.11)$$

Although the proper Taylor series expansion of linear momentum has been already introduced, a bit more is necessary to know in this phase of derivation: expansion of a product of two momentums. Using the equation (2.5) and keeping the terms to fourth order in v, this is obtained:

$$p_\alpha p_\beta \approx m_\alpha m_\beta v_\alpha v_\beta + \frac{1}{2}m_\alpha m_\beta\frac{v_\alpha v_\beta}{c^2}\left(v_\alpha^2 + v_\beta^2\right) \quad (2.12)$$

For $\alpha = \beta$ simply:

$$p_\alpha^2 \approx m_\alpha^2 v_\alpha^2 + \frac{m_\alpha^2 v_\alpha^4}{c^2} \quad (2.13)$$

Plugging these into the (2.11), another biquadratic equation for v_e is obtained, this time within the linear momentum conservation law.

$$v_e^4 + \left(c^2\right)v_e^2 - \left(\frac{c^2P_2}{m_e^2}\right) = 0 \quad (2.14)$$

P_2 refers to:

$$P_2 = P_{2a} + P_{2b} + P_{2c} + P_{2d} \tag{2.15}$$

$$P_{2a} = m_{\bar{p}}^2 v_{\bar{p}}^2 + m_{\bar{p}}^2 \frac{v_{\bar{p}}^4}{c^2} + m_{Ps}^2 v_{Ps}^2 + m_{Ps}^2 \frac{v_{Ps}^4}{c^2} + m_{\bar{H}}^2 v_{\bar{H}}^2 + m_{\bar{H}}^2 \frac{v_{\bar{H}}^4}{c^2} \tag{2.15a}$$

$$P_{2b} = -2 m_{\bar{p}} m_{\bar{H}} v_{\bar{p}} v_{\bar{H}} \left(1 + \frac{v_{\bar{p}}^2 + v_{\bar{p}}^2}{2c^2}\right) \cos\theta_{\bar{H}} \tag{2.15b}$$

$$P_{2c} = 2 m_{\bar{p}} m_{Ps} v_{\bar{p}} v_{Ps} \left(1 + \frac{v_{\bar{p}}^2 + v_{Ps}^2}{2c^2}\right) \cos\theta_{Ps} \tag{2.15c}$$

$$P_{2d} = -2 m_{\bar{H}} m_{Ps} v_{\bar{H}} v_{Ps} \left(1 + \frac{v_{\bar{H}}^2 + v_{Ps}^2}{2c^2}\right) \cos(\theta_{Ps} - \theta_{\bar{H}}) \tag{2.15d}$$

Like the first one (following from the energy conservation), this biquadratic equation has only one physical solution, too:

$$v_{e,2}^2 = \frac{1}{2} c^2 \left(\sqrt{1 + \frac{4P_2}{m_e^2 c^2}} - 1\right) \tag{2.16}$$

Setting the two solutions $v_{e,1}$ and $v_{e,2}$ equal, the whole kinematics is expressed by a single identity. However, any information about the electron (that is v_e and θ_e) has been lost. On the other hand, mathematically, one can get it by return if necessary and the little mass of the electron compared to the antihydrogen suggests its negligable role in the reaction kinematics. The identity in the terms of P_1 and P_2 is:

$$\frac{64P_1^2}{m_e^2 c^4} + \frac{9P_2^2}{m_e^4 c^4} + \frac{8P_1}{m_e c^2} - \frac{48 P_1 P_2}{m_e^3 c^4} - \frac{4P_2}{m_e^2 c^2} = 0 \tag{2.17}$$

Both P_1 and P_2 have been defined and the last step of the derivation is to plug the two definitions into (2.17) but now keeping only the terms to second order in velocities[3]. The algebraic process is very long and error prone but it leads to the final result:

$$A_1 v_{\bar{H}}^2 + A_2 v_{\bar{p}}^2 + A_3 v_{Ps}^2 + A_4 v_{\bar{p}} v_{\bar{H}} \cos\theta_{\bar{H}} + A_5 v_{\bar{p}} v_{Ps} \cos\theta_{Ps} + A_6 v_{Ps} v_{\bar{H}} + A_7 Q = 0 \tag{2.18}$$

[3]The fourth order terms served in the biquadratic equation finding and are no more required.

The A_1, A_2, A_3, A_4, A_5, A_6, A_7 symbols' meanings are:

$$A_1 = 1 + \frac{m_e}{m_{\bar{H}}} + 16\frac{Q}{m_{\bar{H}}c^2} + 12\frac{Q}{m_e c^2} \tag{2.18a}$$

$$A_2 = \left(1 - \frac{m_e}{m_{\bar{p}}} - 16\frac{Q}{m_{\bar{p}}c^2} + 12\frac{Q}{m_e c^2}\right)\left(\frac{m_{\bar{p}}}{m_{\bar{H}}}\right)^2 \tag{2.18b}$$

$$A_3 = \left(1 - \frac{m_e}{m_{Ps}} - 16\frac{Q}{m_{Ps}c^2} + 12\frac{Q}{m_e c^2}\right)\left(\frac{m_{Ps}}{m_{\bar{H}}}\right)^2 \tag{2.18c}$$

$$A_4 = -2\left(1 + 12\frac{Q}{m_e c^2}\right)\left(\frac{m_{\bar{p}}}{m_{\bar{H}}}\right) \tag{2.18d}$$

$$A_5 = 2\left(1 + 12\frac{Q}{m_e c^2}\right)\left(\frac{m_{\bar{p}} m_{Ps}}{m_{\bar{H}}^2}\right) \tag{2.18e}$$

$$A_6 = -2\left(1 + 12\frac{Q}{m_e c^2}\right)\left(\frac{m_{Ps}}{m_{\bar{H}}}\right) \tag{2.18f}$$

$$A_7 = -2\left(1 + 8\frac{Q}{m_e c^2}\right)\left(\frac{m_e}{m_{\bar{H}}^2}\right) \tag{2.18g}$$

It is reasonable now to use these approximations:

$$\frac{Q}{m_{\bar{H}}c^2}, \frac{Q}{m_{\bar{p}}c^2}, \frac{Q}{m_{Ps}c^2}, \frac{Q}{m_e c^2}, \frac{m_e}{m_{\bar{H}}}, \frac{m_e}{m_{\bar{p}}} \ll 1 \tag{2.19a}$$

$$m_{Ps} \simeq 2m_e, m_{\bar{H}} \simeq m_{\bar{p}} \tag{2.19b}$$

The last two approximations do not apply to the Q quantity. After implementation of all these approximations, the final identity can be written and understood in the form of a quadratic equation for the antihydrogen velocity:

$$v_{\bar{H}}^2 + B_1 v_{\bar{H}} + B_2 = 0 \tag{2.20}$$

$$B_1 = -2v_{\bar{p}}\cos\theta_{\bar{H}} - 4\left(\frac{m_e}{m_{\bar{p}}}\right) v_{Ps}\cos(\theta_{Ps} - \theta_{\bar{H}}) \tag{2.20a}$$

$$B_2 = v_{\bar{H}}^2 + 2\left(\frac{m_e}{m_{\bar{p}}}\right)^2 v_{Ps}^2 + 4\left(\frac{m_e}{m_{\bar{p}}}\right) v_{\bar{p}} v_{Ps}\cos\theta_{Ps} - \frac{2m_e Q}{m_{\bar{p}}^2} \tag{2.20b}$$

The formal solution is:

$$v_{\bar{H}} = \frac{1}{2}\left(-B_1 \pm \sqrt{B_1^2 - 4B_2}\right) \tag{2.21}$$

Exactly the same formula is the result of a fully non-relativistic approach, too. The conservation laws are:

$$\frac{1}{2}m_{Ps}v_{Ps}^2 + \frac{1}{2}m_{\bar{p}}v_{\bar{p}}^2 = \frac{1}{2}m_e v_e^2 + \frac{1}{2}m_{\bar{H}}v_{\bar{H}}^2 - Q \tag{2.22a}$$

$$m_{\bar{p}}v_{\bar{p}} + m_{Ps}v_{Ps}\cos\theta_{Ps} = m_e v_e \cos\theta_e + m_{\bar{H}}v_{\bar{H}}\cos\theta_{\bar{H}} \tag{2.22b}$$

$$m_{Ps}v_{Ps}\sin\theta_{Ps} = m_e v_e \sin\theta_e + m_{\bar{H}}v_{\bar{H}}\sin\theta_{\bar{H}} \tag{2.22c}$$

The derivation is very similar, only the solutions of two quadratic equations are set equal instead of two biquadratic equations. The result is the same. This could serve as a control.

Note that the relation in the AEGIS Proposal [1] is different from the one just derived, namely both B_1 and B_2 are different (if the former identity takes the same form of quadratic equation) - they are denoted now as $\tilde{B}_1$ and $\tilde{B}_2$:

$$\tilde{B}_1 = -4(\frac{m_e}{m_{\bar{p}}})v_{Ps}\cos(\theta_{Ps} - \theta_{\bar{H}}) \tag{2.23a}$$

$$\tilde{B}_2 = v_{\bar{p}}^2 + 2(\frac{m_e}{m_{\bar{p}}})v_{Ps}^2 - 2v_{\bar{p}}v_{Ps}\cos\theta_{Ps} - \frac{2m_e Q}{m_{\bar{p}}^2} \tag{2.23b}$$

In a special case when the velocities of the entering particles (antiproton and positronium) are zero, the relations are the same and give the same results. Otherwise, the difference is easy to see. The identity is usefull for instance if one need to know the maximum velocity of an antihydrogen on the exit of the reaction.

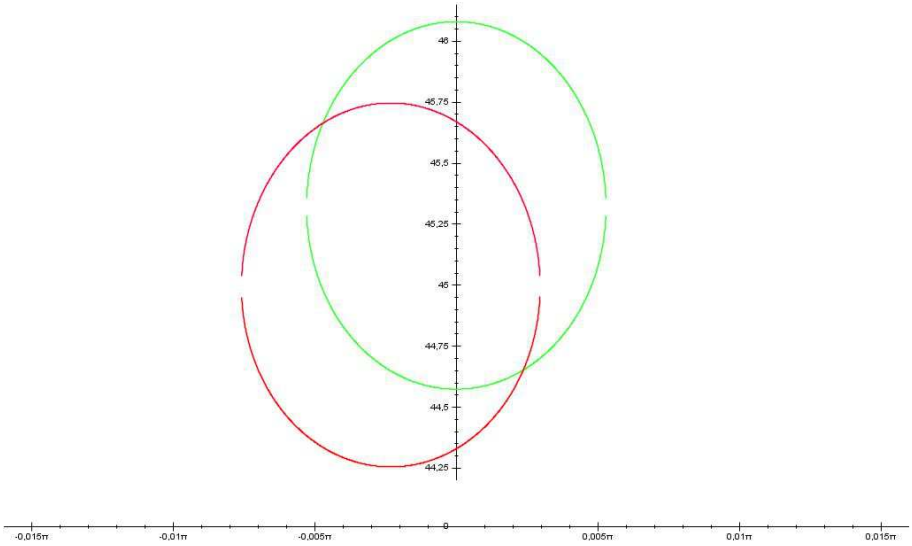
To demonstrate the difference of the former and the new identity on an example, let the antiproton and the positronium fly on the same line (the faster positronium following the antiproton) with sample velocites and principal quantum numbers[4]:

$$v_{\bar{p}} = 45\ \mathrm{m/s}, v_{Ps} = 300\ \mathrm{m/s}, \theta_{Ps} = 0, \theta_{\bar{H}} = 0, n_{Ps} = 29, n_{\bar{H}} = 41 \tag{2.24}$$

Then, according to the new formula, the antihydrogen velocity just after the reaction is $45.3 \pm 0.2\ \mathrm{m/s}$, whereas the former identity gives $158.4\ \mathrm{m/s}$.

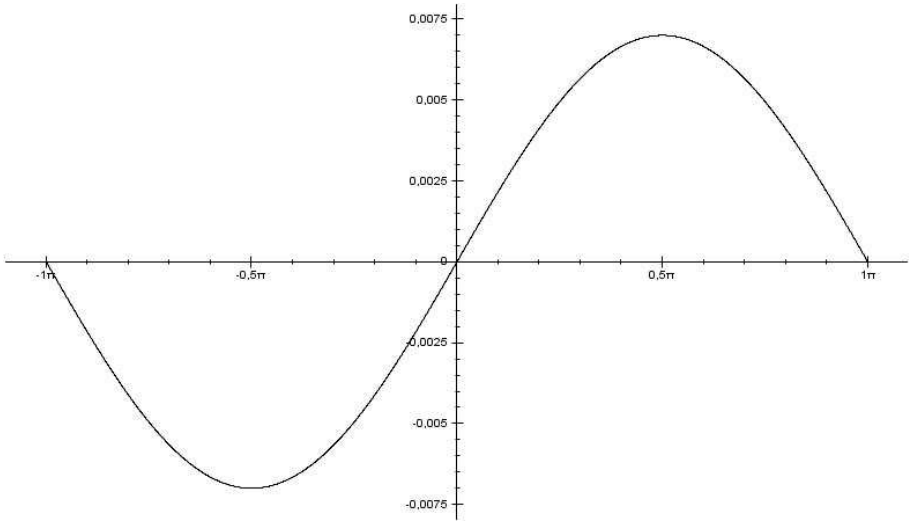
[4]For the chosen pair $n_{Ps} = 29$ and $n_{\bar{H}} = 41$, the absolute value of Q quantity is the smallest up to $n_{Ps} = 43$ and $n_{\bar{H}} = 60$. Therefore the difference in the two indentities should be most noticeable.

Furthermore, the vector of the final antihydrogen can not be known precisely since all the information about the electron has been eliminated - there is a class of possible pairs $v_{\bar{H}}$ - $\theta_{\bar{H}}$ solving the identity. However, the effect of the electron is very small. In the picture below, it is shown (still within the previous values) how the class of solutions looks like (the green curve) and how it looks like if the angle of the positronium is changed to perpendicular $\theta_{Ps} = \frac{\pi}{2}$ (the red curve):



Obrázek 2.2: Two classes of solutions of the identity - the green curve for $\theta_{Ps} = 0$ and the red curve for $\theta_{Ps} = \frac{\pi}{2}$. The antihydrigen velocity magnitude is on the vertical axis and the angle of the antihydrogen on the horizontal axis.

The maximum relative difference is $1.3\ \%$. For the same values, if one is interested what is the functional dependence of the deviation of the curve centres (from the previous picture) on the θ_{Ps} angle, this is obtained:



Obrázek 2.3: The dependence of the deviation of the antihydrogen velocity solutions class centre on the angle of the positronium. The horizontal axis refers to that angle and the vertical means the velocity deviation.

The deviation (still the effect of the elimination of the electron’s information) is negligable.

3 Antihydrogen deexcitation kinematics

In this chapter, the focus is on this reaction:

$$\bar{H}^*_n \rightarrow \bar{H}^*_{n'} + \gamma \tag{3.1}$$

$$1 \leq n' < n \tag{3.1a}$$

$$n, n' \in N \tag{3.1b}$$

In words: An antihydrogen atom in an excited state n switches to a lower energy level n' (”deexcites”) which leads to a single photon radiation.

It is the antihydrogen’s nature that it prefers to get rid of any extra internal energy and for the AEGIS experiment it has to be taken into account.

All the possible energy states are discrete and they are all unambigiously labeled with a natural number (n or n') called the principal quantum number. The final state is not necessary the ground state.

In this chapter, the term deexcitation photon will be used to refer to the photon in (3.1) and in accordance with this notation, the antihydrogen on the left-hand side is called the initial antihydrogen and the one on the right-hand side is called the final antihydrogen[1]. The quantities linked to the final antihydrogen have usualy a prime aside.

The rest energy of an antihydrogen is given by the summ of the rest energies of both its components (an antiproton and a positron) reduced by an amount of the system binding energy:

$$E(n) = m_{\bar{p}}c^2 + m_{\bar{e}}c^2 - \frac{R}{n^2} \tag{3.2a}$$

$$E(n') = m_{\bar{p}}c^2 + m_{\bar{e}}c^2 - \frac{R}{n'^2} \tag{3.2b}$$

In this sense, the rest masses of antihydrogen are simply:

$$m_0 = m_{\bar{p}} + m_{\bar{e}} - \frac{R}{n^2c^2} \tag{3.3a}$$

$$m'_0 = m_{\bar{p}} + m_{\bar{e}} - \frac{R}{n'^2c^2} \tag{3.3b}$$

The total antihydrogen energy in a frame of reference is then given by a product of its rest energy and its Lorentz factor (”gamma-factor”):

$$E = E(v, n) = E(n)\gamma(v) = \frac{m_{\bar{p}}c^2 + m_{\bar{e}}c^2 - \frac{R}{n^2}}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{m_0c^2}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{3.4a}$$

$$E' = E(v', n') = E(n')\gamma(v') = \frac{m_{\bar{p}}c^2 + m_{\bar{e}}c^2 - \frac{R}{n'^2}}{\sqrt{1 - \frac{v'^2}{c^2}}} = \frac{m'_0c^2}{\sqrt{1 - \frac{v'^2}{c^2}}} \tag{3.4b}$$

For sure, there is a frame of reference in which the initial antihydrogen stays at rest.

[1] In the chapter of Charge exchange kinematics, the antihydrogen was always refered to be final because its definition was different even though it was not necessary to attribute anything to the antihydrogen. Here, the term initial is used as well and the two ensure the definiteness.

One can write the conservation laws of the deexcitation in this frame of reference quite easily; in fact, the conservation equations take their simpliest forms there.

$$m_0c^2 = E' + E_\gamma \tag{3.5}$$

$$p' \cos\theta_{\bar{H}'} = p_\gamma \cos\theta_\gamma \tag{3.6a}$$

$$p' \sin\theta_{\bar{H}'} = p_\gamma \sin\theta_\gamma \tag{3.6b}$$

The meanings of the symbols in the first equation (energy conservation) was shown in the beginning of this chapter. Like in any other double-particle decays, the three linear momentum vectors form a plane so only two equations for linear momentum conservation are necessary. If the polar coordinates are chosen, the p' and p_γ quantities refer to linear momentum magnitudes of the final antihydrogen and the deexcitation photon respectively (both never negative). The $\theta_{\bar{H}'}$ quantity is the polar angle of the final antihydrogen in the polar coordinates system and θ_γ is defined for the deexcitation photon in the same way. The inertial frame of reference, where the initial antihydrogen stays at rest and where the polar coordinates are defined, will be used for most of this chapter.

Anyway, from that the chosen frame of reference is the centre of mass one, then it should be clear that $p' = p_\gamma$: Since the linear momentum of the initial antihydrogen is always zero, the linear momentums of the two final particles have to be equal in magnitudes (and opposite in directions). The proof is that the $p' = p_\gamma$ information follows from the $(3.6\mathrm{a})$ and $(3.6\mathrm{b})$ laws - squaring them, summing and using of Pythagorean identity results in the same equation for linear momentum conservation.

Any photons' energy and linear momentum satisfy:

$$p_\gamma = \frac{E_\gamma}{c} \tag{3.7}$$

Setting this and the previous momentum information equal, one gets:

$$\frac{E_\gamma}{c} = p_\gamma = p' \tag{3.8}$$

So the energy of the deexcitation photon can be expressed as a multiple of the final antihydrogen linear momentum magnitude:

$$E_\gamma = p'c \tag{3.9}$$

This can be plugged into the energy conservation equation:

$$m_0c^2 = E' + p'c \tag{3.10}$$

Now, the final antihydrogen's energy is tranfered onte the opposite side of the equation:

$$p'c = m_0c^2 - E' \tag{3.11}$$

Squaring of this formula leads to:

$$\left(p'c\right)^2 = \left(m_0c^2 - E'\right)^2 \tag{3.12}$$

Now the $(p'c)^2$ is expressed. Furthermore, there is one more general way of how the quantity can be found - the Lorentz invariant of the final antihydrogen. It says:

$$\left(p'c\right)^2 = E'^2 - \left(m_0c^2\right)^2 \tag{3.13}$$

Setting the two expressions equal:

$$m_0^2c^4 - 2m_0c^2E' + E'^2 = E'^2 - m'^2_0c^4 \tag{3.14}$$

Eliminating the E'^2 terms:

$$2m_0c^2E' = m'^2_0c^4 + m_0^2c^4 \tag{3.15}$$

Now it is easy to find E'. Moreover, an independent definition of E' is what the $(3.4\mathrm{b})$ says. Now these two expressions of the E' quantity can be set equal:

$$\frac{m'^2_0c^4 + m_0^2c^4}{2m_0c^2} = E' = \frac{m'_0c^2}{\sqrt{1 - \frac{v'^2}{c^2}}} \tag{3.16}$$

If both sides are multiplied with $\frac{1}{m'_0c^2}$, the Lorentz factor of the final antihydrogen stands on its side of the equation alone:

$$\frac{1}{\sqrt{1 - \frac{v'^2}{c^2}}} = \frac{m'^2_0c^4 + m_0^2c^4}{2m_0m'_0c^4} \tag{3.17}$$

Setting the both fractions reciprocal:

$$\sqrt{1 - \frac{v'^2}{c^2}} = \frac{2m_0m'_0c^4}{m'^2_0c^4 + m_0^2c^4} \tag{3.18}$$

Squaring:

$$1 - \frac{v'^2}{c^2} = \left(\frac{2m_0m'_0c^4}{m'^2_0c^4 + m_0^2c^4}\right)^2 \tag{3.19}$$

Now the goal is to find the value of v' or $\frac{v'}{c}$.

$$\frac{v'^2}{c^2} = 1 - \frac{2m_0m'_0c^4}{m'^2_0c^4 + m_0^2c^4}^2 = \frac{m'^4_0c^8 + 2m_0^2m'^2_0c^8 + m_0^4c^8 - 4m_0^2m'^2_0c^8}{\left(m'^2_0c^4 + m_0^2c^4\right)^2} =$$

$$= \frac{m'^4_0c^8 - 2m_0^2m'^2_0c^8 + m_0^4c^8}{\left(m'^2_0c^4 + m_0^2c^4\right)^2} = \left(\frac{m_0^2c^4 - m'^2_0c^4}{m'^2_0c^4 + m_0^2c^4}\right)^2 \tag{3.20}$$

So the result is:

$$v' = \frac{m_0^2c^4 - m'^2_0c^4}{m'^2_0c^4 + m_0^2c^4}c \tag{3.21}$$

The just derived velocity of the final antihydrogen is linked to the final antihydrogen's linear momentum and possesses analogical qualities - the velocity, like the momentum, is one of the polar coordinates and therefore is never negative.

The fraction is appropriate to abbreviate even though the reason is just aesthetic.

$$v' = \frac{m_0^2 - m'^2_0}{m'^2_0 + m_0^2}c \tag{3.22}$$

The rest masses of the initial and final antihydrogen atoms has been already introduced and now they will be replaced with their definitions in the expression for v':

$$v' = \frac{\left(m_{\bar{p}} + m_{\bar{e}} - \frac{R}{n^2c^2}\right)^2 - \left(m_{\bar{p}} + m_{\bar{e}} - \frac{R}{n'^2c^2}\right)^2}{\left(m_{\bar{p}} + m_{\bar{e}} - \frac{R}{n^2c^2}\right)^2 + \left(m_{\bar{p}} + m_{\bar{e}} - \frac{R}{n'^2c^2}\right)^2}c =$$

$$= \frac{(m_{\bar{p}} + m_{\bar{e}})^2 - 2\,(m_{\bar{p}} + m_{\bar{e}})\,\frac{R}{n^2c^2} + \frac{R^2}{n^4c^4} - (m_{\bar{p}} + m_{\bar{e}})^2 + 2\,(m_{\bar{p}} + m_{\bar{e}})\,\frac{R}{n'^2c^2} - \frac{R^2}{n'^4c^4}}{(m_{\bar{p}} + m_{\bar{e}})^2 - 2\,(m_{\bar{p}} + m_{\bar{e}})\,\frac{R}{n^2c^2} + \frac{R^2}{n^4c^4} + (m_{\bar{p}} + m_{\bar{e}})^2 - 2\,(m_{\bar{p}} + m_{\bar{e}})\,\frac{R}{n'^2c^2} + \frac{R^2}{n'^4c^4}}c =$$

$$= \frac{-2\,(m_{\bar{p}} + m_{\bar{e}})\,\frac{R}{c^2}\left(\frac{1}{n^2} - \frac{1}{n'^2}\right) + \frac{R^2}{c^4}\left(\frac{1}{n^4} - \frac{1}{n'^4}\right)}{2\,(m_{\bar{p}} + m_{\bar{e}})^2 - 2\,(m_{\bar{p}} + m_{\bar{e}})\,\frac{R}{c^2}\left(\frac{1}{n^2} + \frac{1}{n'^2}\right) + \frac{R^2}{c^4}\left(\frac{1}{n^4} + \frac{1}{n'^4}\right)}c \tag{3.23}$$

It follows that the velocity of the final antihydrogen depends on the initial and final states' principal numbers only - the rest in (3.23) are constants, namely: the rest mass of an antiproton, the rest mass of a positron, the Rydberg constant of an antihydrogen and the speed of light in vacuum. They are all known with a very high accurancy. A brief analysis of the (3.23) equation indicates that the fraction is dimensionless since the dimension of the rightmost speed of light corresponds with the final antihydrogen velocity's dimension on the left-hand side of the equation. Therefore both the numerator and the denominator are of the same dimension and for the purpose of a further analysis, no matter whether SI units or atomic units are chosen.

For instance, if one prefers to work in the atomic units, then these values will be substituted for the symbols of the constants [3]:

$$m_{\bar{p}} = 938272013 \text{ eV}$$

$$m_{\bar{e}} = 510999 \text{ eV}$$

$$R = 13.6 \text{ eV}$$

$$c = 1$$

Only the principal numbers and the rightmost c constant will be untouched:

$$v = \frac{-2.553 \cdot 10^{10}\left(\frac{1}{n^2} - \frac{1}{n'^2}\right) + 1.85 \cdot 10^2\left(\frac{1}{n^4} - \frac{1}{n'^4}\right)}{1.763 \cdot 10^{18} - 2.553 \cdot 10^{10}\left(\frac{1}{n^2} + \frac{1}{n'^2}\right) + 1.85 \cdot 10^2\left(\frac{1}{n^4} + \frac{1}{n'^4}\right)}c \tag{3.24}$$

This should help to simplify the expression of the final velocity a bit.

In the denominator, both the second and the third term are limited because $(\frac{1}{n^2} + \frac{1}{n'^2}) \leq 2$ and $(\frac{1}{n^4} + \frac{1}{n'^4}) \leq 2$ for all possible pairs of natural numbers n and n'. So there is no need to discuss the particular values of the second and the third term, since they form less then one nth of the denominator.

In the numerator, one can see that $0 \leq \frac{1}{n^2} \leq 1$, $\leq \frac{1}{n'^2} \leq 1$, $\leq \frac{1}{n^4} \leq 1$ and $\leq \frac{1}{n'^4} \leq 1$ which leads to $-1 \leq \frac{1}{n^2} - \frac{1}{n'^2} \leq 1$ and $-1 \leq \frac{1}{n^4} - \frac{1}{n'^4} \leq 1$. Furthermore, $\frac{1}{n^4} - \frac{1}{n'^4} = \frac{n'^4 - n^4}{n^4n'^4} = \frac{(n'^2+n^2)}{n^2n'^2}\frac{(n'^2-n^2)}{n^2n'^2} = (\frac{1}{n^2} + \frac{1}{n'^2})(\frac{1}{n^2} - \frac{1}{n'^2})$. It was mentioned in the analysis of the denominator that $(\frac{1}{n^2} + \frac{1}{n'^2}) \leq 2$ and now it follows that the term $\frac{1}{n^4} - \frac{1}{n'^4}$ can not be more than twice as higher than $\frac{1}{n^2} - \frac{1}{n'^2}$. Therefore the second term in the numerator can be neglected, too.

These sensible approximations simplify the formula for the final antihydrogen velocity:

$$v' = \frac{-2\left(m_{\bar{p}} + m_{\bar{e}}\right)\frac{R}{c^2}\left(\frac{1}{n^2} - \frac{1}{n'^2}\right)}{2\left(m_{\bar{p}} + m_{\bar{e}}\right)^2}c = \frac{R}{m_{\bar{p}}c^2 + m_{\bar{e}}c^2}\left(\frac{1}{n'^2} - \frac{1}{n^2}\right)c \tag{3.25}$$

The fully non-relativistis approach can serve as a control. The proper energy and linear momentum conservation laws are:

$$-\frac{R}{2n^2} = -\frac{R}{2n'^2} + \frac{1}{2}\left(m_{\bar{p}} + m_{\bar{e}}\right)^2 v'^2 + E_\gamma \tag{3.26}$$

$$0 = \left(m_{\bar{p}} + m_{\bar{e}}\right)v' - \frac{E_\gamma}{c} \rightarrow E_\gamma = \left(m_{\bar{p}} + m_{\bar{e}}\right)cv' \tag{3.27}$$

Plugging the second equation into the first one, the result is a quadratic equation for the final antihydrogen velocity.

$$v'^2 + 2cv' - \frac{2R}{m_{\bar{p}} + m_{\bar{e}}}\left(\frac{1}{n'^2} - \frac{1}{n^2}\right) = 0 \tag{3.28}$$

Its positive solution is:

$$v' = \frac{-2c + \sqrt{4c^2 + 4c^2\frac{2R}{m_{\bar{p}}c^2 + m_{\bar{e}}c^2}\left(\frac{1}{n'^2} - \frac{1}{n^2}\right)}}{2} \tag{3.29}$$

The Taylor series of the to the first order:

$$v' = \left(-1 + 1 + \frac{1}{2}\cdot 2\cdot\frac{\frac{R}{c^2}}{m_{\bar{p}}c^2 + m_{\bar{e}}}\left(\frac{1}{n'^2} - \frac{1}{n^2}\right)\right)c = \frac{R}{m_{\bar{p}}c^2 + m_{\bar{e}}c^2}\left(\frac{1}{n'^2} - \frac{1}{n^2}\right)c \tag{3.30}$$

The results of two different approaches are the same.

Some of the characteristics of the result are simple to find. For $1 \leq n' \leq n$, the v' value is positive, naturally, and for $n' = n$ (nothing happens to the antihydrogen), the velocity is zero, as expected - note the frame of reference where the initial antihydrogen stays at rest has not been abandoned.

The result depends on four constants: R, $m_{\bar{p}}$, $m_{\bar{e}}$ and c. If the values are used instead of symbols, this is obtained:

$$v' = 4.343046\left(\frac{1}{n'^2} - \frac{1}{n^2}\right)\left[\frac{m}{s}\right] \tag{3.31}$$

The number 4.343046 (with its proper SI unit metre per second) also means the theoretically maximum velocity which the final antihydrogen can gain in the frame of reference where the initial antihydrogen stays at rest.

If one would like now to work with quantities in the laboratory frame of reference, they had to transfer the v' result into that frame of reference. According to the fact that the values of the initial antihydrogen velocity in the laboratory frame of reference[2] would not be greater than 10000 metres per secend, the classical Galilean velocity transformation can be used - the possible difference to the "slow Lorentz transformation" [4] would be approximately 10^{-11}.

If the Cartesian coordinates of the initial antihydrogen velocity in the laboratory frame of reference are denoted as v_x, v_y and v_z and the Cartesian coordinates of the final antihydrogen velocity in the

[2]This is the same as the relative velocity of the two frames of reference - the laboratory one and the one in which the initial antihydrogen stays at rest.

laboratory frame of reference ${v_x}'$, ${v_y}'$ and ${v_z}'$, then (using polar coordinates for the final antihydrogen velocity) this follows:

$${v_x}' = v_x + \frac{R}{m_{\bar{p}}c^2 + m_{\bar{e}}c^2}\left(\frac{1}{n'^2} - \frac{1}{n^2}\right) c\cos\phi\cos\theta \tag{3.32a}$$

$${v_y}' = v_y + \frac{R}{m_{\bar{p}}c^2 + m_{\bar{e}}c^2}\left(\frac{1}{n'^2} - \frac{1}{n^2}\right) c\sin\phi\cos\theta \tag{3.32b}$$

$${v_z}' = v_z + \frac{R}{m_{\bar{p}}c^2 + m_{\bar{e}}c^2}\left(\frac{1}{n'^2} - \frac{1}{n^2}\right) c\sin\theta \tag{3.32c}$$

Finally, one can evaluate v' for chosen pairs of n and n':

$$n = 35, n' = 34 \rightarrow v' = 2.116 \cdot 10^{-4}\left[\frac{\mathrm{m}}{\mathrm{s}}\right] \tag{3.33a}$$

$$n = 35, n' = 1 \rightarrow v' = 4.340\left[\frac{\mathrm{m}}{\mathrm{s}}\right] \tag{3.33b}$$

Evidently, the effect of the deexcitation can reach the order of metres per second and the smaller the deexcitaion difference in principal numbers is, the smaller is the velocity change effect.

4 Antihydrogen dynamics

This chapter presents the derivation of a simple antihydrogen dynamics in electric ($\vec{E}(\vec{R},t)$), magnetic ($\vec{B}(\vec{R},t)$) and gravitational field $\vec{g}$. The limits of the approach will be stressed continuously. The main phenomenon which affects the dynamics is the deexcitation of the antihydrogen - the possible change of kinematic qualities in such an event was discussed in the chapter Antihydrogen deexcitation kinematics. The outcomes of the chapter Charge exchange kinematics serve as the initial conditions for an equation of motion which this chapter aims to find.

Before starting, it is useful to remind the dynamics of a classical charged particle in an electromagnetic field. The equation of motion is:

$$\ddot{\vec{P}}(\vec{R}) = \frac{M\ddot{\vec{R}}}{\sqrt{1-\frac{\dot{\vec{R}}^2}{c^2}}} = q\left(\vec{E}(\vec{R},t) + \dot{\vec{R}} \times \vec{B}(\vec{R},t)\right) \tag{4.1}$$

where P means the linear momentum of the charged particle, M its mass and $\vec{R}$ the current position of the particle (then $\dot{\vec{R}}$ refers to the velocity); c is the speed of light and $\vec{g}$ the gravitational acceleration, as usual. The "dot"sign always refers to the first total time derivative (like $\dot{\vec{R}} = \frac{d}{dt}\vec{R}$)

In this chapter, a general charge q (in the Lorentz force, for example) will be distinguished from a particular charge Q (the charge of an antiproton, for instance).

This equation includes the so called Lorentz force which is nonzero if the particle posses an intrinsic electric charge - this charge has to be constant and globally nonzero. However, this does not imply that a particle with a zero charge is insensive to electromagnetic field and experiences only the gravity. An antihydrogen atom is such a particle. Even though its global charge is zero (if not ionized), the space distribution of the charge need not be uniform. This kind of space distribution is possible to represent in so called multiple expansion whose first order term is the electric dipole.

In a given frame of reference, this is the definition of the electric dipole $\vec{p}$ located at $\vec{R}$:

$$\vec{p}(\vec{R}) = \int_V \rho(\vec{r})\left(\vec{r} - \vec{R}\right) d^3\vec{r} \tag{4.2a}$$

for the continuous charge distribution $\rho(\vec{R})$; and for a set of N point charges (q_j):

$$\vec{p}(\vec{R}) = \sum_{j=1}^{N} q_j\left(\vec{r}_j - \vec{R}\right) \tag{4.2b}$$

There are two different types of an electric dipoles - an intrinsic electric dipole and an induced electric dipole. The intrinsic dipole moment is possessed by a molecule of water, for instance, its absolute value is constant and only the space orientation of the vector is influenced by the field. Although this is not what an antihydrogen atom represents, the derivation of such dipole's dynamics in an electric field will help to find the more complicated dynamics of the induced dipole in electromagnetic field.

Electric charge is positive or negative. Assume the two kinds are possible to treat separately (possible to summarize them and to find the centres of their distributions - $q_+ = q(\vec{r}_+)$ and $q_- = q(\vec{r}_-)$). In addition, if these charges are equal in the absolute value ($|q_+| = |q_-| = Q$), the definition of the electric dipole becomes:

$$\vec{p}(\vec{R}) = Q\vec{l} \tag{4.3}$$

where the $\vec{l}$ quantity stands for the relative position of the two charges:

$$\vec{l} = \vec{r}_+ - \vec{r}_- \tag{4.4}$$

A charged particle in an electric field experiences a force:

$$\vec{F}(\vec{r}) = q\vec{E}(\vec{r}) \tag{4.5}$$

For the two charges forming the intrinsic dipole it follows:

$$\vec{F}_+ = \vec{F}(\vec{r}_+) = q_+\vec{E}(\vec{r}_+) = Q\vec{E}(\vec{r}_+) \tag{4.6a}$$

$$\vec{F}_- = \vec{F}(\vec{r}_-) = q_-\vec{E}(\vec{r}_-) = -Q\vec{E}(\vec{r}_-) \tag{4.6b}$$

It is possible to expand one of the previous equation (say the first one) in Taylor series to the first order in l:

$$\vec{F}_+ = Q\vec{E}(\vec{r}_+) = Q\vec{E}(\vec{r}_- + \vec{l}) \approx Q\left(\vec{E}(\vec{r}_-) + \left(\vec{l}\nabla_{r_-}\right)\vec{E}(\vec{r}_-)\right) \tag{4.7}$$

The definition of a special operator was used:

$$\vec{a}\nabla_b = \sum_{j=1}^{3} a_j \frac{\partial}{\partial b_j} \tag{4.8}$$

The j indices just the Cartesian coordiante. The meaning of the Taylor expansion is that the distance l between the two charges is very small with respect to the molecule's displacement as it moves and the change of the electric field intensity along l the first order effect.

The total force on the intrinsic dipole is given by the sum of the two forces:

$$\vec{F} = \vec{F}_+ + \vec{F}_- \approx Q\left(\vec{E}(\vec{r}_-) + \left(\vec{l}\nabla_{r_-}\right)\vec{E}(\vec{r}_-)\right) - Q\vec{E}(\vec{r}_-) = \left(\vec{l}\nabla_{r_-}\right)\vec{E}(\vec{r}_-) \tag{4.9}$$

There is no need to keep the subscribe of the negative charge anymore - for an obsrever, the difference between $\vec{r}_+$ and $\vec{r}_-$ is undetectable ($\vec{r}_+, \vec{r}_- \gg \vec{l}$) and $\vec{r} \approx \vec{r}_-$ can be used instead:

$$\vec{F} = \left(\vec{l}\nabla_r\right)\vec{E}(\vec{r}) \tag{4.10}$$

In (4.10), the definition of the operator provides:

$$\vec{l}\nabla_r = \sum_{j=1}^{3} l_j \frac{\partial}{\partial x_i} \tag{4.11}$$

where x_i are the Cartesian coordinates of $\vec{r}$.

The equation (4.10), however, does not provide the complete information about the dynamics since the two coupled charges have more than three degrees of freedom (namely five). The constraint here is the constant distance between the two charges and as a whole, the molecule can be treated as a rigid body. The corresponding theory suggests the second (independent) equation of motion which is based on torque [5] and [4]. Its derivation follows from the $(4.6\mathrm{a})$ and $(4.6\mathrm{b})$ like the force equation - it is their another "combination". In [4], this is proposed:

$$\vec{M} = (\vec{r}_+ - \vec{r}_-) \times \vec{F}_+ = Q(\vec{r}_+ - \vec{r}_-) \times \vec{E}(\vec{r}_+) = Q\vec{l} \times \vec{E}(\vec{r_+}) = \vec{p} \times \vec{E}(\vec{r}_+) \tag{4.12}$$

Like before, the $\vec{r}_+ \approx \vec{r}$ approximation can be implemented and the torque equation becomes:

$$\vec{M} = \vec{p} \times \vec{E}(\vec{r}) \tag{4.13}$$

The configuration of the intrinsic dipole is now described by three Cartesian coordinates $\vec{r}$ and by two independent coordinates of its orientation $\vec{p}$. Therefore the intrinsic dipole is said to be point-like in this model.

To sum up, the intrinsic electric dipole has five degrees of freedom and just a one equation of motion would be not sufficient to know its dynamics completely. Beside the force equation, the sedond torque equation is introduced. The pair of equation of its dynamics is:

$$\vec{F} = \left(\vec{l}\nabla_r\right)\vec{E}(\vec{r}) \tag{4.14a}$$

$$\vec{M} = \vec{p} \times \vec{E}(\vec{r}) \tag{4.14b}$$

This outcome is very often found in textbooks, however, the antihydrogen atom can not be counted as the intrinsic electric dipole. It is reasonable now to remind some of its qualities. The antihydrogen is a coupled state of two charges: the positive one ($q_{\bar{e}} = Q$) is linked to the positron (its mass is $m_{\bar{e}}$) and the negative one ($q_{\bar{p}} = -Q$) is linked to the antiproton (whose mass is $m_{\bar{p}}$). If the antihydrogen is set free (outside any electric or magnetic field), the centres of the two charges are situated in the same place - the antihydrogen possess zero global charge (anytime if not ionized) and, moreover, there is no prefered direction of its charge distribution orientation. In other words, its electric dipole is zero so, in the first order, one can neglect its electric qualities. On the other hand, if the same antihydrogen atom is put into nonzero electric field, both of the charges will tend to move in opposite directions - even in a homogeneous electric field, the antihydrogen stays at rest, however, it gains the induced electric dipole.

Evidently, the induced electric dipole is very different from the intrinsic one: outside the field the induced electric dipole was zero, whereas inside a field, the electric dipole was gained by the particle. Therefore the condition that the intrinsic dipole has to be constant in magnitude does not hold. The constant magnitude of a dipole is equivalent to the presence of a holonomic constraint. In the case of antihydrogen, just a different holonomic constraint applies, thus in this sense the two kinds of dipoles are not extremely distinct.

The mechanism is that the antihydrogen gains an induced electric dipole in an electric field and the dipole is sensitive to that field retroactively.

If the positions of the centres of the two charges are denoted as $r_{\bar{p}}$ and $r_{\bar{e}}$, the variability of the distance between them has to be taken into account:

$$\vec{l} = \vec{l}(\vec{R}) = \vec{r_{\bar{e}}} - \vec{r_{\bar{p}}} \tag{4.15}$$

The $\vec{R}$ quantity is the position of centre of the antihydrogen mass:

$$\vec{R} = \frac{m_{\bar{p}}\vec{r_{\bar{p}}} + m_{\bar{e}}\vec{r_{\bar{e}}}}{m_{\bar{p}} + m_{\bar{e}}} \tag{4.16}$$

In this model, the induced dipole is treated as point-like (in the sense introduced above), too. The $\vec{R}$ quantity will refer to the position of the whole antihydrogen.

The $\vec{l}$ quantity's dependence on $\vec{R}$ implicitly says that the distance between the centres of the two charges is given by the electric intensity at $\vec{R}$: $\vec{l} = \vec{l}(\vec{E}) = \vec{l}(\vec{E}(\vec{R})) = \vec{l}(\vec{R})$. Note that the intrisic dipole's definition formula $\vec{p}(\vec{R}) = Q\vec{l} = Q\vec{l}(\vec{R})$ remains valid for the induced dipole, therefore $\vec{p} = \vec{p}(\vec{E}(\vec{R})) = \vec{p}(\vec{R})$. According to the source [6], the dependance is linear:

$$\vec{p}(\vec{R}) = \tilde{\alpha}\vec{E}(\vec{R}) \tag{4.17}$$

The constant of proportionality $\tilde{\alpha}$ will be discussed later.

The equations (4.15) and (4.16) together form simultaneous equations for $r_{\bar{p}}$ and $r_{\bar{e}}$. The solution is simple to find:

$$\vec{r}_{\bar{p}} = \vec{R} - \frac{m_{\bar{e}}}{m_{\bar{p}} + m_{\bar{e}}}\vec{l}(\vec{R}) \tag{4.18a}$$

$$\vec{r}_{\bar{e}} = \vec{R} + \frac{m_{\bar{p}}}{m_{\bar{p}} + m_{\bar{e}}}\vec{l}(\vec{R}) \tag{4.18b}$$

The two charges experience the Lorentz force introduced in the beginning of this chapter. First, the same kind of expansions like in the intrinsic dipole's section will be implemented for the electric forces:

$$\vec{F}_{E\bar{p}} = \vec{F}_E(\vec{r}_{\bar{p}}) = -Q\vec{E}(\vec{r}_{\bar{p}}) = -Q\vec{E}(\vec{R} - \frac{m_{\bar{e}}}{m_{\bar{p}} + m_{\bar{e}}}\vec{l}(\vec{R})) \approx$$

$$\approx -Q\vec{E}(\vec{R}) + \frac{m_{\bar{e}}}{m_{\bar{p}} + m_{\bar{e}}}Q\left(\vec{l}(\vec{R})\nabla_R\right)\vec{E}(\vec{R}) \tag{4.19a}$$

$$\vec{F}_{E\bar{e}} = \vec{F}_E(\vec{r}_{\bar{e}}) = +Q\vec{E}(\vec{r}_{\bar{e}}) = +Q\vec{E}(\vec{R} + \frac{m_{\bar{p}}}{m_{\bar{p}} + m_{\bar{e}}}\vec{l}(\vec{R})) \approx$$

$$\approx +Q\vec{E}(\vec{R}) + \frac{m_{\bar{p}}}{m_{\bar{p}} + m_{\bar{e}}}Q\left(\vec{l}(\vec{R})\nabla_R\right)\vec{E}(\vec{R}) \tag{4.19b}$$

The total electric force on the induced electric dipole is the sum of the previous two:

$$\vec{F}_E = \vec{F}_{E\bar{p}} + \vec{F}_{E\bar{e}} = \left(\vec{d}(\vec{R})\nabla_R\right)\vec{E}(\vec{R}) \tag{4.20}$$

Now, the same expansion for the magnetic forces on the charges:

$$\vec{F}_{B\bar{p}} = \vec{F}_B(\vec{r}_{\bar{p}}) = -Q\dot{\vec{r}}_{\bar{p}} \times \vec{B}(\vec{r}_{\bar{p}}) =$$

$$= -Q\left(\dot{\vec{R}} - \frac{m_{\bar{e}}}{m_{\bar{p}} + m_{\bar{e}}}\dot{\vec{l}}(\vec{R})\right) \times \vec{B}(\vec{R} - \frac{m_{\bar{e}}}{m_{\bar{p}} + m_{\bar{e}}}\vec{l}(\vec{R})) \approx$$

$$\approx -Q\left(\dot{\vec{R}} - \frac{m_{\bar{e}}}{m_{\bar{p}} + m_{\bar{e}}}\dot{\vec{l}}(\vec{R})\right) \times \left(\vec{B}(\vec{R}) - \frac{m_{\bar{e}}}{m_{\bar{p}} + m_{\bar{e}}}\left(\vec{l}(\vec{R})\nabla_R\right)\vec{B}(\vec{R})\right) =$$

$$= -Q\dot{\vec{R}} \times \vec{B}(\vec{R}) + \frac{m_{\bar{e}}}{m_{\bar{p}} + m_{\bar{e}}}Q\dot{\vec{R}} \times \left(\left(\vec{l}(\vec{R})\nabla_R\right)\vec{B}(\vec{R})\right) +$$

$$+\frac{m_{\bar{e}}}{m_{\bar{p}} + m_{\bar{e}}}Q\dot{\vec{l}}(\vec{R}) \times \vec{B}(\vec{R}) - \frac{m_{\bar{e}}^2}{(m_{\bar{p}} + m_{\bar{e}})^2}Q\dot{\vec{l}}(\vec{R}) \times \left(\left(\vec{l}(\vec{R})\nabla_R\right)\vec{B}(\vec{R})\right) \tag{4.21a}$$

$$\vec{F}_{B\bar{e}} = \vec{F}_B(\vec{r}_{\bar{e}}) = +Q\dot{\vec{r}}_{\bar{e}} \times \vec{B}(\vec{r}_{\bar{e}}) =$$

$$= +Q\left(\dot{\vec{R}} + \frac{m_{\bar{p}}}{m_{\bar{p}} + m_{\bar{e}}}\dot{\vec{l}}(\vec{R})\right) \times \vec{B}(\vec{R} + \frac{m_{\bar{p}}}{m_{\bar{p}} + m_{\bar{e}}}\vec{l}(\vec{R})) \approx$$

$$\approx +Q\left(\dot{\vec{R}} + \frac{m_{\bar{p}}}{m_{\bar{p}} + m_{\bar{e}}}\dot{\vec{l}}(\vec{R})\right) \times \left(\vec{B}(\vec{R}) + \frac{m_{\bar{p}}}{m_{\bar{p}} + m_{\bar{e}}}\left(\vec{l}(\vec{R})\nabla_R\right)\vec{B}(\vec{R})\right) =$$

$$= +Q\dot{\vec{R}} \times \vec{B}(\vec{R}) + \frac{m_{\bar{p}}}{m_{\bar{p}} + m_{\bar{e}}}Q\dot{\vec{R}} \times \left(\left(\vec{l}(\vec{R})\nabla_R\right)\vec{B}(\vec{R})\right) +$$

$$+\frac{m_{\bar{p}}}{m_{\bar{p}} + m_{\bar{e}}}Q\dot{\vec{l}}(\vec{R}) \times \vec{B}(\vec{R}) + \frac{m_{\bar{p}}^2}{(m_{\bar{p}} + m_{\bar{e}})^2}Q\dot{\vec{l}}(\vec{R}) \times \left(\left(\vec{l}(\vec{R})\nabla_R\right)\vec{B}(\vec{R})\right) \tag{4.21b}$$

The sum of the previous two forces is the total magnetic force on the induced electric dipole:

$$\vec{F}_B = \vec{F}_{B\bar{p}} + \vec{F}_{B\bar{e}} =$$

$$= Q\dot{\vec{R}} \times \left(\left(\vec{l}(\vec{R})\nabla_R\right)\vec{B}(\vec{R})\right) + Q\dot{\vec{l}}(\vec{R}) \times \vec{B}(\vec{R}) +$$

$$+\frac{(m_{\bar{p}} + m_{\bar{e}})(m_{\bar{p}} - m_{\bar{e}})}{(m_{\bar{p}} + m_{\bar{e}})^2} Q\dot{\vec{l}}(\vec{R}) \times \left(\left(\vec{l}(\vec{R})\nabla_R\right)\vec{B}(\vec{R})\right) =$$

$$= \dot{\vec{d}}(\vec{R}) \times \vec{B}(\vec{R}) + \left(\dot{\vec{R}} + \frac{1}{q}\frac{m_{\bar{p}} - m_{\bar{e}}}{m_{\bar{p}} + m_{\bar{e}}}\dot{\vec{d}}(\vec{R})\right) \times \left(\left(\vec{d}(\vec{R})\nabla_R\right)\vec{B}(\vec{R})\right) \tag{4.22}$$

Summing the forces $\vec{F}_E$ and $\vec{F}_B$, one gets the total force which the antihydrogen would experience if there were no more antihydrogen dynamic qualities (like magnetic dipole, for instance). The force would be:

$$\vec{F}_{EB} = \vec{F}_E + \vec{F}_B =$$

$$= \left(\vec{d}(\vec{R})\nabla_R\right)\vec{E}(\vec{R}) + \dot{\vec{d}}(\vec{R}) \times \vec{B}(\vec{R}) + \left(\dot{\vec{R}} + \frac{1}{q}\frac{m_{\bar{p}} - m_{\bar{e}}}{m_{\bar{p}} + m_{\bar{e}}}\dot{\vec{d}}(\vec{R})\right) \times \left(\left(\vec{d}(\vec{R})\nabla_R\right)\vec{B}(\vec{R})\right) \tag{4.23}$$

The already discussed retroactivity of the electric field to the induced electric dipole will be now fully implemented into the force from the information $\vec{p}(\vec{R}) = \tilde{\alpha}\vec{E}(\vec{R})$:

$$\vec{F}_{EB} = \tilde{\alpha}\left(\vec{E}(\vec{R})\nabla_R\right)\vec{E}(\vec{R}) + \tilde{\alpha}\dot{\vec{E}}(\vec{R}) \times \vec{B}(\vec{R}) +$$

$$+\tilde{\alpha}\left(\dot{\vec{R}} + \frac{1}{q}\frac{m_{\bar{p}} - m_{\bar{e}}}{m_{\bar{p}} + m_{\bar{e}}}\tilde{\alpha}\dot{\vec{E}}(\vec{R})\right) \times \left(\left(\vec{E}(\vec{R})\nabla_R\right)\vec{B}(\vec{R})\right) \tag{4.24}$$

Following the derivation of intrinsic dipole's dynamics, one would try to find the torque equation. However, the introduced formula $\vec{p}(\vec{R}) = \tilde{\alpha}\vec{E}(\vec{R})$ makes the situation easier. If the torque on the induced electric dipole is similar to that of the intrinsic electric dipole, the cross product of two identical vectors gives zero:

$$\vec{M} = \vec{p}(\vec{R}) \times \vec{E}(\vec{R}) = \tilde{\alpha}\vec{E}(\vec{R}) \times \vec{E}(\vec{R}) = 0 \tag{4.25}$$

A different proof supports this estimation: The constraint $\vec{p}(\vec{R}) = \tilde{\alpha}\vec{E}(\vec{R})$ actually not only asserts the magnitude of the induced electric dipole[1] but also its orientation. While the constraint of the constant magnitude reduced the number of degrees of freedom of the intrinsic electric dipole from six to five, induced dipole's constraint reduces the number of degrees of freedom from six to three. Therefore the force equation (with just the electric dipole implemented so far) fully describes the dynamics of the antihydrogen.

However, this is not everything because the antihydrogen has a magnetic dipole, too: in fact, two kinds which are both sensitive to the magnetic field. The first kind is the induced magnetic dipole which is very similar to the induced electric dipole. For any magnetic dipole $\vec{m}$, the source [4] states the force equation:

$$\vec{F}_M(\vec{R}) = \left(\vec{m}(\vec{R})\nabla_R\right)\vec{B}(\vec{R}) \tag{4.26}$$

[1] The magnitude of the intrinsic electric dipole is explicitely given, too, albeit trivially.

And the same source provides the dependence of $\vec{m}(\vec{R})$ on $\vec{B}(\vec{R})$:

$$\vec{m}(\vec{R}) = \tilde{\beta}\vec{B}(\vec{R}) \tag{4.27}$$

Because of this similarity, the induced magnetic dipole's contribution to the total force of antihydrogen will be:

$$\vec{F}_{M_{ind}} = \tilde{\beta}\left(\vec{B}(\vec{R})\nabla_R\right)\vec{B}(\vec{R}) \tag{4.28}$$

The two quantities $\tilde{\alpha}$ and $\tilde{\beta}$ are independent of the fields, however, they depend on the particular inner state of the antihydrogen. The source [6] provides estimations for both of them (in the case of a hydrogen atom):

$$\tilde{\alpha} = f(n,l,m)4\pi\epsilon_0 r_B^3 \tag{4.29a}$$

$$\tilde{\beta} = -\frac{q_{\bar{e}}^2 < r_0^2 >}{6m_{\bar{e}}} \tag{4.29b}$$

The senses of symbols are: ϵ_0 is the vacuum permitivity, r_B is the Bohr radius; r_0 is the root mean square of the electron radius and has to be treated by quantum mechanics tools as well as $f(n,l,m)$ where the three parameters are the principal, azimuthal and magnetic quantum numbers. Fortunately, the source [6] states that it should $f(1,0,0) = \frac{9}{2}$ - this perfectly agrees with something solved in [7]:

$$f(n,l,m) = \frac{1}{8}\left(17n^2 - 9m^2 + 19\right) \tag{4.30}$$

There is no torque equation for the introduced induced magnetic dipole, too. The two induced dipoles point exactly in the same direction like the proper fields' lines. In other words, both of them answer to the change of the field immediately. The causality is not the problem since the reaction takes place in the same place as the action - the (4.17) and (4.27) could be both just "adiabatic"approximations. However, in this model the both equations are treated as valid.

The intrinsic magnetic dipole $\vec{\mu}$ experience torque but to keep within the simple model, its orientation is prescribed at every instant:

$$\vec{\mu}(\vec{R}) = \frac{\left|\vec{\mu}(\vec{R})\right|}{\left|\vec{B}(\vec{R})\right|}\vec{B}(\vec{R}) \tag{4.31}$$

The intrinsic magnetic dipole experiences the force:

$$\vec{F}_{M_{int}} = \frac{\left|\vec{\mu}(\vec{R})\right|}{\left|\vec{B}(\vec{R})\right|}\left(\vec{B}(\vec{R})\nabla_R\right)\vec{B}(\vec{R}) \tag{4.32}$$

Finally, the ordinary gravitational force is implemented:

$$\vec{F}_G = \left(m_{\bar{p}} + m_{\bar{e}}\right)\vec{g} \tag{4.33}$$

If all the intermediate outcomes are summarized, the total force on the antihydrogen atom in the combined fields within the model is:

$$\vec{F} = \tilde{\alpha}\left(\vec{E}(\vec{R})\nabla_R\right)\vec{E}(\vec{R}) + \tilde{\alpha}\dot{\vec{E}}(\vec{R}) \times \vec{B}(\vec{R}) +$$

$$+\tilde{\alpha}\left(\dot{\vec{R}} + \frac{1}{q}\frac{m_{\bar{p}} - m_{\bar{e}}}{m_{\bar{p}} + m_{\bar{e}}}\tilde{\alpha}\dot{\vec{E}}(\vec{R})\right) \times \left(\left(\vec{E}(\vec{R})\nabla_R\right)\vec{B}(\vec{R})\right) +$$

$$+\left(\frac{|\vec{\mu}|}{\left|\vec{B}(\vec{R})\right|} + \tilde{\beta}\right)\left(\vec{B}(\vec{R})\nabla_R\right)\vec{B}(\vec{R}) + (m_{\bar{p}} + m_{\bar{e}})\,\vec{g} \tag{4.34}$$

And the equation of motion is:

$$(m_{\bar{p}} + m_{\bar{e}})\,\ddot{\vec{R}} = \tilde{\alpha}\left(\vec{E}(\vec{R})\nabla_R\right)\vec{E}(\vec{R}) + \tilde{\alpha}\dot{\vec{E}}(\vec{R}) \times \vec{B}(\vec{R}) +$$

$$+\tilde{\alpha}\left(\dot{\vec{R}} + \frac{1}{q}\frac{m_{\bar{p}} - m_{\bar{e}}}{m_{\bar{p}} + m_{\bar{e}}}\tilde{\alpha}\dot{\vec{E}}(\vec{R})\right) \times \left(\left(\vec{E}(\vec{R})\nabla_R\right)\vec{B}(\vec{R})\right) +$$

$$\left(\frac{|\vec{\mu}|}{\left|\vec{B}(\vec{R})\right|} + \tilde{\beta}\right)\left(\vec{B}(\vec{R})\nabla_R\right)\vec{B}(\vec{R}) + (m_{\bar{p}} + m_{\bar{e}})\,\vec{g} \tag{4.35}$$

The Newtonian formulation of mechanics is chosen since the number of the degrees of freedom is three and since for all the forces, a simple expression in Cartesian coordinate system is known. The non-relativistic approach agrees with the fact that the velocities of antihydrogen will be definitely smaller than a few tenths kilometres per second. The result and its derivation are "quasi-classical"- they treat the antihydrogen as a classical point-like particle and its qualities like electric dipole and magnetic dipole are prescribed from the outcomes of quantum mechanics or experiments.

The initial values are $\vec{R}(t_0)$ and $\dot{\vec{R}}(t_0)$ where t_0 refers to the time of the antihydrogen creation.

The final equation of motion can be written with the Cartesian indices rigorously emphasized. Let
$\vec{E}(\vec{R},t) = (E_1(\vec{R},t), E_2(\vec{R},t), E_3(\vec{R},t)) = (E_1, E_2, E_3)$,
$\vec{B}(\vec{R},t) = (B_1(\vec{R},t), B_2(\vec{R},t), B_3(\vec{R},t)) = (B_1, B_2, B_3)$,
$\vec{g} = (g_1, g_2, g_3)$, $\vec{R} = (X_1, X_2, X_3)$ and $\dot{\vec{R}} = (\dot{X}_1, \dot{X}_2, \dot{X}_3)$; ϵ_{ijk} is the Levi-Civita symbol. The definition of the total time derivative is used. In this form, there are three equations of motion ($i = 1, 2, 3$):

$$(m_{\bar{p}} + m_{\bar{e}})\,\ddot{X}_i = \tilde{\alpha}\left(\sum_{l=1}^{3} E_l\frac{\partial E_i}{\partial X_l}\right) + \epsilon_{ijk}\tilde{\alpha}\left(\frac{\partial E_j}{\partial t} + \sum_{l=1}^{3}\frac{\partial E_j}{\partial X_l}\dot{X}_l\right)B_k +$$

$$+\epsilon_{ijk}\tilde{\alpha}\left(\dot{X}_j + \frac{1}{q}\frac{m_{\bar{p}} - m_{\bar{e}}}{m_{\bar{p}} + m_{\bar{e}}}\tilde{\alpha}\left(\frac{\partial E_j}{\partial t} + \sum_{l=1}^{3}\frac{\partial E_j}{\partial X_l}\dot{X}_l\right)\right)\left(\sum_{l=1}^{3} E_l\frac{\partial B_k}{\partial X_l}\right) +$$

$$+\left(\sqrt{\frac{\sum_{l=1}^{3}\mu_l^2}{\sum_{l=1}^{3}B_l^3}} + \tilde{\beta}\right)\left(\sum_{l=1}^{3} B_l\frac{\partial B_i}{\partial X_l}\right) + (m_{\bar{p}} + m_{\bar{e}})\,g_i \tag{4.36}$$

If the number of degrees of freedom were higher than three and therefore the force equation were not enough, to find the equations of motion, the Lagrange formulation of mechanics would be easier to implement (any problems concerning number of degrees of freedom, constraints or correct identification of the forces are solved automatically). If $\vec{F}_l$ denotes to coordiantes of all the Cartesian forces experienced by the system or by its components, if parametres q_i (generalized coordinates) are used instead of the

Cartesian coordinates x_l and if $T = T(q_1, ...)$ stands for the kinetic energy, then the equations of motion for all possible i are [8]:

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_i} + \frac{\partial T}{\partial q_j} = \sum_l F_l \frac{\partial x_l}{\partial q_i} \tag{4.37}$$

Then the particular form for antihydrogen depends on how the equations (4.17) and (4.27) are modified.

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