Czech Technical University
Faculty of Nuclear Sciences and Physical Engineering
Department of Physics

Bachelor‘s degree project

Libor Novák

Praha–2008

Czech Technical University
Faculty of Nuclear Sciences and Physical Engineering
Department of Physics

Evaluation of Nuclear Processes in High-parameter Plasmas

Bachelor‘s degree project

Author: **Libor Novák**
Advisor: **Prof. Ing. Ladislav Drška, CSc.**
School year: **2007/2008**

I declare that I worked on my bachelor‘s degree project alone and used source materials mentioned on the enclosed list only.

Prohlašuji, že jsem svou bakalářskou práci vypracoval samostatně, a že jsem použil pouze podklady uvedené v přiloženém seznamu.

V Praze 25. 7. 2008

Acknowledgement

I would like to thank to my advisor, Prof. Ing. Ladislav Drška, CSc., for his help with this bachelor‘s degree project, his patience and willingness.

Název práce:
Výpočet jaderných procesů ve vysokoparametrovém plazmatu

Autor: Libor Novák

Obor: Fyzikální inženýrství

Druh práce: Bakalářská práce

Vedoucí práce: Prof. Ing. Ladislav Drška, Csc. Katedra fyzikální elektroniky, Fakulta jaderná a fyzikálně inženýrská, České vysoké učení technické v Praze

Abstrakt: Slučováním lehkých atomových jader se uvolňuje velké množství energie. Jádra jsou však kladně nabita, a proto je pro jejich sloučení nutné překonat odpudivou coulombickou sílu. Částice se přitom dostanou do stavu s vysokou energií, který můžeme označit jako plazma. V této práci jsou počítány výtěžky jaderných reakcí $D-D$, $D-T$ a $D-He^3$, které jsou získány jako numerické řešení systému obyčejných diferenciálních rovnic v programu Maple 11. Pro jednoduchost se předpokládá, že reagující částice mají rovnovážné, tedy Maxwellovo rozdělení. Výpočty jsou realizovány pro energie 20 a $100keV$.

Klíčová slova: termonukleární fúze, účinný průřez, Maxwellovo rozdělení, výtěžek reakce.

Title: **Evaluation of Nuclear Processes in High-parameter Plasmas**

Author: Libor Novák

Abstract: By fusion of light atomic nuclei together, a large amount of energy is released. However, the nuclei are positively charged; therefore in order to fuse, they must overcome the repulsive Coulomb force. In so doing, the particles acquire a high-energy level, which can be denoted as plasma. In this work, the respective reaction yields of $D-D$, $D-T$ and $D-He^3$ are evaluated. These yields are obtained as a numerical solution of ordinary differential equations system in program Maple 11. A Maxwellian distribution of reactive particles is assumed for simplicity. Evaluations are realized for energies 20 and $100keV$.

Key words: thermonuclear fusion, cross section, Maxwellian distribution, reaction yield.

Contents

Declaration . . . 3
Acknowledgement . . . 4
Abstract . . . 9
Contents . . . 9

1 Energy sources **13**
1.1 World Energy Situation . . . 13
1.2 Fusion Energy . . . 14

2 Fusion Reactions **15**
2.1 Binding Energy Release . . . 15
2.2 Reactions . . . 16

3 Reaction Yield **17**
3.1 Distribution Function . . . 17
3.2 Coulomb Barrier . . . 17
3.3 Cross Section . . . 20
3.4 Reaction Rate . . . 22
3.4.1 Two Colliding Beams . . . 22
3.4.2 Target with Maxwellian Distribution . . . 22
3.4.3 Two Maxwellian Distributions . . . 23
3.5 Reaction Rate Parameter . . . 24

4 Temporal Progress of Fusion Reactions **27**

5 Advanced Fuels **33**
Conclusion . . . 34
Bibliography . . . 35

Chapter 1

Energy sources

1.1 World Energy Situation

Today's total amount of energy consumed in the whole world is around $12TWyr$ [2]. Predictions say that energy consumption should increase and in the next 20-30 years could reach $16-20TWyr$ [2]. There are two main reasons: 1. By 2025 the world population is predicted to reach 8 billion [1]. 2. Average power consumption per capita will rise from $2.1kW$ to about $3kW$ [2].

At present, about 90% of the energy is produced by burning fossil fuels [2], but this could impose several huge problems in the next years:

- decrease of the fossil fuels may lead to political instabilities in the world
- fossil resources are important for chemical and pharmaceutical industry as base material
- substances released during the burning can change our environment

It means that human society should restrict using the fossil resources as a fuel, and find alternative energy sources. Renewable sources such as biogas, biomass, water, solar or wind power are not available everywhere and have only a limited potential, e.g. $1GW$ power plant could be replaced by about $100km^2$ photovoltaic panels or by $30000km^2$of wood [2].

Present nuclear power plants use fission of the uranium or plutonium as the energy source. This plants produce highly radioactive waste, however there are no problems with its disposal. Fission energy is the most ecological energy source, which human society can now use on a large scale. However the world uranium reserves can suffice for only several hundred years [4] at the present ratio of contribution to primary energy production, which makes only 6%[2]. Still, it is the best solution of energy problems for the next decades.

1.2 Fusion Energy

At this time, we know only one energy source, which could fully substitute the fossil fuels energy for manny years to the future. If we could succeed to create a workable thermonuclear reactor, we would have an energy source for thousands of years, because we do not need uranium - as a fuel. For the first generation of these reactors instead, deuterium and tritium could be used. The former can be obtained from water, the latter can be created from lithium, which occurs naturally. There are large reserves of this element, for example in abandoned uranium mines in the Czech republic. This reactor would require only a small quantity of fuel to work, so considering the water and lithium reserves on our planet, it would be practically inexhaustible source of energy[3]. Moreover, it holds the promise of being a safe and clean energy production method.

For the first time on our planet, fusion energy was released by explosion of a bomb as well as fission energy. More than 40 years after the hydrogen bomb we still do not have available fusion power plants, while the first fission power plants were developed within just severel years after the fission bomb. There are many reasons for this, but the main one is, that it is more complicated to build fusion reactor than the fission one. Fission is started by slow neutrons, which are electrically neutral, so they can brake the positively charged heavy nuclei. But fusion is different.

Against the fission reactions there are reactions between positively charged light nuclei, so in order to fuse, they must overcome the repulsive Coulomb force. We know only one way, how to do this. We have to heat the fuel to very high temperatures. The reaction between deuterium and tritium requires over 100 million centigrade degrees (it responds to the kinetic energy approximately $10keV$)! Moreover there are no material vessels to confine such a matter. Fortunately, the fact that under this conditions particles are electrically charged (atoms divide into electrons and nuclei) makes it possible to confine them by electric or magnetic fields.

The research of nuclear fusion is now at a stage, that scientists have a fundamental understanding of its function. But it will be one of the most difficult technical problems ever solved, to build economically feasible thermonuclear reactor. Presently, the most developed experimental fusion facility which would be able to produce electrical energy calls tokamak, which works on the principle of magnetic confinement. In 2007 began building of the biggest tokamak in the world, called ITER. It is an international project and will be built at Cadarache in France. Its purpose is to show, if tokamaks could be used as power plants. The ITER‘s construction is expected to cost around 5 billion USD[5]. It is a huge amount, but compared to building of other scientific or energy projects as LHC (2,5 billion USD)[6], ISS (100 billion USD!)[7] or fission power station in Temelin (5 billion USD)[8] it is not so much money.

Chapter 2

Fusion Reactions

2.1 Binding Energy Release

Nuclear energy can be released in two ways. The binding energy per nucleon of the different elements has a maximum for iron, as shown in Figure 2.1. It means that we can obtain energy either by fusion of light elements together or by fission of heavy nuclei. In both cases the total mass of the final products is smaller than that of the reacting atoms. The excess mass Δm is converted according to Einstein's equation into kinetic energy of reaction products and is obtained experimentally.

$$E = \Delta m c^2 \tag{2.1}$$

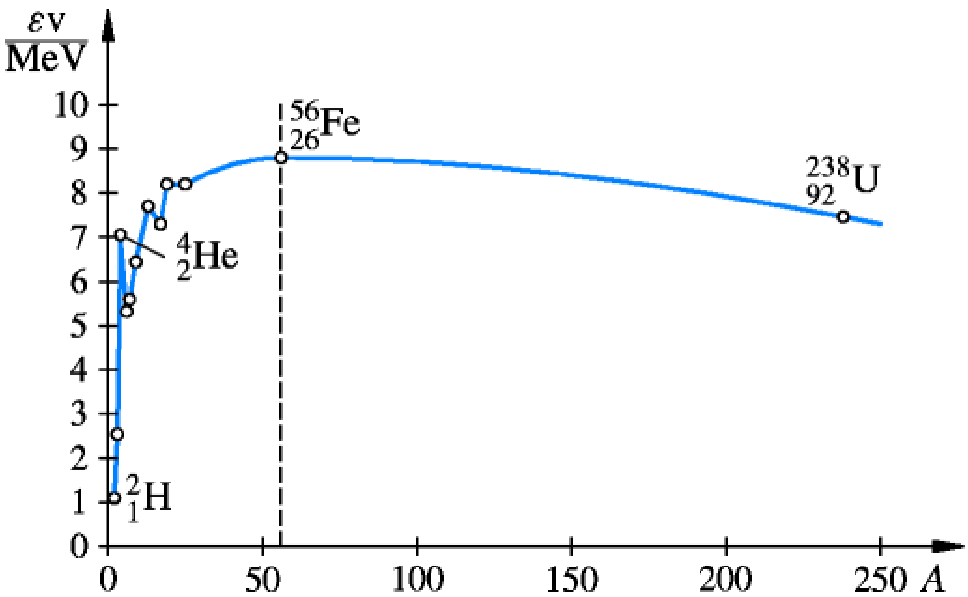

Figure 2.1: Average binding energy per nucleon

2.2 Reactions

In this text, we will consider these 4 reactions[13]:

$$
\begin{aligned}
D + T &\rightarrow \alpha(14.1MeV) + n(3.5MeV) && (2.2)\\
D + He^3 &\rightarrow \alpha(3.6MeV) + p(14.7MeV) && (2.3)\\
D + D &\rightarrow He^3(0.82MeV) + n(2.45MeV) && (2.4)\\
D + D &\rightarrow T(1.01MeV) + p(3.02MeV) && (2.5)
\end{aligned}
$$

The reaction (2.2) between hydrogen isotopes deuterium and tritium is most important for controlled fusion research, because of its huge cross section, which reaches its maximum at the relatively modest energy of $64keV$ (see Fig. 3.3)[12]. Due to $Z = 1$, this reaction has a relatively small value of ϵ_G (see section 3.2) and hence a relatively large tunnel penetrability. Energy released during this reaction ($17.6MeV$) is quite large, e.g. compared to the $D - D$ reactions. Disadvantage of this reaction are the fast neutrons, which carry most of the energy released and can not be controlled by electromagnetic field and the tritium, which is radioactive and almost does not appear in nature. It will be probably generated during the reaction from lithium.

The reactions (2.4) and (2.5) between 2 deuterium particles are nearly equiprobable[12] and have much smaller cross section than the $D-T$ reaction. In the $10-100keV$ energy interval, the cross sections for each of them are about 100 times smaller than for DT. Tritium, which is generated in the reaction (2.5) and helium from the reaction (2.4) can both react with deuterium. Temporal progress of these situations is discussed in chapter 4.

The reaction (2.3) between deuterium and helium can be partially classified as advanced fuel reaction, because it produces only charged particles. However we have to consider also reactions between deuterium particles, which can produce neutrons. For this reaction isotope He^3 is needed, but it does not occur naturaly. Energy yield from this reaction is comparable to the $D-T$ reaction. Though the cross section is bigger than for the $D - D$, it is still much smaller opposite to the $D - T$ (see Fig. 3.3).

Chapter 3

Reaction Yield

3.1 Distribution Function

Let $f(x_1, x_2, x_3, v_1, v_2, v_3, t) = f(\vec{x}, \vec{v}, t)$ be the distribution function. It means that number of particles in a volume V can be found by integrating f over all particles velocities and the volume[11]:

$$N(t) = \int_{-\infty}^{\infty} \mathrm{d}\vec{v} \int_{V} \mathrm{d}\vec{x} f(\vec{x}, \vec{v}, t) \tag{3.1}$$

The particle density is an integral of f over all velocities[11]:

$$n(\vec{x}, t) = \int_{-\infty}^{\infty} \mathrm{d}\vec{v} f(\vec{x}, \vec{v}, t) \tag{3.2}$$

The average value of any function $F(s)$ is defined as[11]:

$$< F(s) >= \frac{\int F(s) f(s)\, \mathrm{d}s}{\int f(s)\, \mathrm{d}s} \tag{3.3}$$

At equilibrium, particles have a Maxwellian distribution function[11]:

$$f_M(\vec{x}, \vec{v}, t) = n(\vec{x}, t) \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left(-\frac{mv^2}{2kT}\right) \tag{3.4}$$

A plasma confined by magnetic fields is never in a state of thermodynamic equilibrium[10]. However we can think of the Maxwellian as the first approximation of the real distribution.

3.2 Coulomb Barrier

Nuclear fusion probability is limited by the repulsive Coulomb barrier. The Coulomb potential which the particles have to overcome in order to fuse is given by[12]:

$$V(r) = \frac{Z_1 Z_2 e^2}{r} \tag{3.5}$$

where Z_1 and Z_2 are the atomic numbers, e the electron charge and r the distance between the nuclei. This relation can be applied at distances greater than:

$$r_n \approx 1.44 \times 10^{-13}(A_1^{1/3} + A_2^{1/3})cm \tag{3.6}$$

where A_1 and A_2 are the mass numbers of the interacting nuclei. At smaller distances the Coulomb force is exceeded by the attractive nuclear force. At present, nobody knows the exact form of the nuclear potential, but experimentally it was discovered, that the potential well of depth U_0 is about $30 - 40MeV$.

From equations (3.5) and (3.6) we can evaluate the height of the Coulomb barrier V_c:

$$V_b \approx V(r_n) = \frac{Z_1 Z_2}{A_1^{1/3} + A_2^{1/3}} MeV \tag{3.7}$$

It means that nuclei would have to gain a huge kinetic energy in order to get over this barrier. Two particles with relative energy $\epsilon < V_b$ can only approach each other up to the classical turning point:

$$r_{tp} = \frac{Z_1 Z_2 e^2}{\epsilon} MeV \tag{3.8}$$

Fortunatelly, the quantum mechanics allows for tunnelling a potential barrier of finite extension, thus making fusion reactions between nuclei with energy smaller than the height of the barrier possible.

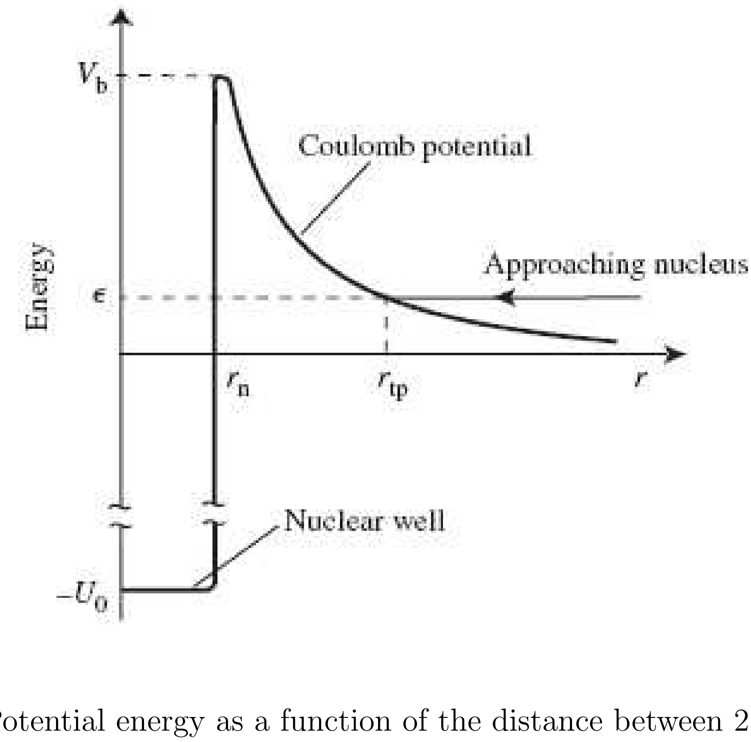

Figure 3.1: Potential energy as a function of the distance between 2 charged nuclei

A widely used parametrization of fusion reaction cross-section is:

$$\sigma \approx \sigma_{geom} \times T \times R, \tag{3.9}$$

where σ_{geom} is a geometrical cross-section, T is the barrier transparency, and R is the probability that nuclei come into contact fuse. The σ_{geom} can be expressed by the de-Broglie wavelength λ:

$$\sigma_{geom} \approx \lambda^2 = \left(\frac{\hbar}{m_r v}\right)^2 \propto \frac{1}{\epsilon}, \tag{3.10}$$

where $\hbar$ is the reduced Planck constant, v the relative velocity of the reacting nuclei, m_r their reduced mass and ϵ center-of-mass energy.

$$v = |\vec{v_1} - \vec{v_2}| \tag{3.11}$$

$$m_r = \frac{m_1 m_2}{m_1 + m_2} \tag{3.12}$$

$$\epsilon = \frac{1}{2} m_r v^2 \tag{3.13}$$

The barrier transparency T shall be written as:

$$T \approx T_G = \exp(-\sqrt{\epsilon_G}/\epsilon) \tag{3.14}$$

T_G is known as Gamow factor and ϵ_G is the Gamow energy.

$$\epsilon_G = (\pi \alpha_f Z_1 Z_2)^2 2 m_r c^2 = 986.1 Z_1^2 Z_2^2 A_r keV \tag{3.15}$$

$$\alpha_f = e^2/\hbar c = 1/137.04 \tag{3.16}$$

$$A_r = m_r/m_p \tag{3.17}$$

where α_f is the fine-structure constant commonly used in quantum mechanics.

Equation (3.14) holds as far as $\epsilon \ll \epsilon_G$ and implies that the chance of tunneling decreases rapidly with the atomic number and mass, thus providing a first simple explanation for the fact that fusion reactions of interest for energy production on earth only involve the lightest nuclei.

The cross section is often expressed as:

$$\sigma(\epsilon) = \frac{S(\epsilon)}{\epsilon} \exp(-\sqrt{\epsilon_G}/\epsilon), \tag{3.18}$$

where the function $S(\epsilon)$ is called the astrophysical S factor, which for many important reactions is a slightly varying function of the energy.

3.3 Cross Section

Cross section σ is defined to be a proportionality constant between a fractional attenuation of particle beam in distance $\mathrm{d}s$ and target particles per unit area in distance $\mathrm{d}s$. This definition is symmetric in the two types of particles, since the relative velocity is the same viewed from either particle. If we would direct a monoenergetic particle beam on a stationary target, the number of collisions among the particles on a small distance $\mathrm{d}s$ will be proportional to the uncollided beam particles density n_1 and to the target particle density n_2[9]:

$$\frac{\mathrm{d}n_1}{\mathrm{d}s} = -\sigma n_1 n_2 \tag{3.19}$$

where the minus sign indicates that the density of uncollided beam particles is decreasing as a result of collisions. Thence it follows:

$$\sigma = -\frac{\mathrm{d}n_1}{n_1}\frac{1}{n_2\mathrm{d}s} \tag{3.20}$$

Cross section is obtained experimentally. It can be established by a theory, which uses a tunnel efect to evaluate σ like in[12], but this theory only transfers the problem to the astrophysical S factor, which for many important reactions is a weakly varying function of the energy and must be obtained experimentally, too.

For reactions (2.2)-(2.4), the total cross section in barns ($1barn = 10^{-28}m^2$) as a function of E, the energy in keV of the incident particle, assuming the target ion at rest, can be fitted by[13]:

$$\sigma(E) = \frac{A_5 + A_2[(A_4 - A_3E)^2 + 1]^{-1}}{E[\exp(A_1E^{-1/2}) - 1]} \tag{3.21}$$

	$D-T$	$D-He^3$	$D-D$
A_1	45.95	89.27	47.88
A_2	50200	25900	482
A_3	1.368×10^{-2}	3.98×10^{-3}	3.08×10^{-4}
A_4	1.076	1.297	1.177
A_5	409	647	0

Table 3.1: Coefficients for equation (3.11)

In the last column of Table 3.1 are coefficients for reaction (2.4), but the reaction (2.5) has nearly the same cross section as (2.4). In Figure 3.2 these three cross sections are plotted for energies $1 - 1000keV$. Because of large differences among these functions, a logarithmic graph of them is shown in Figure 3.3.

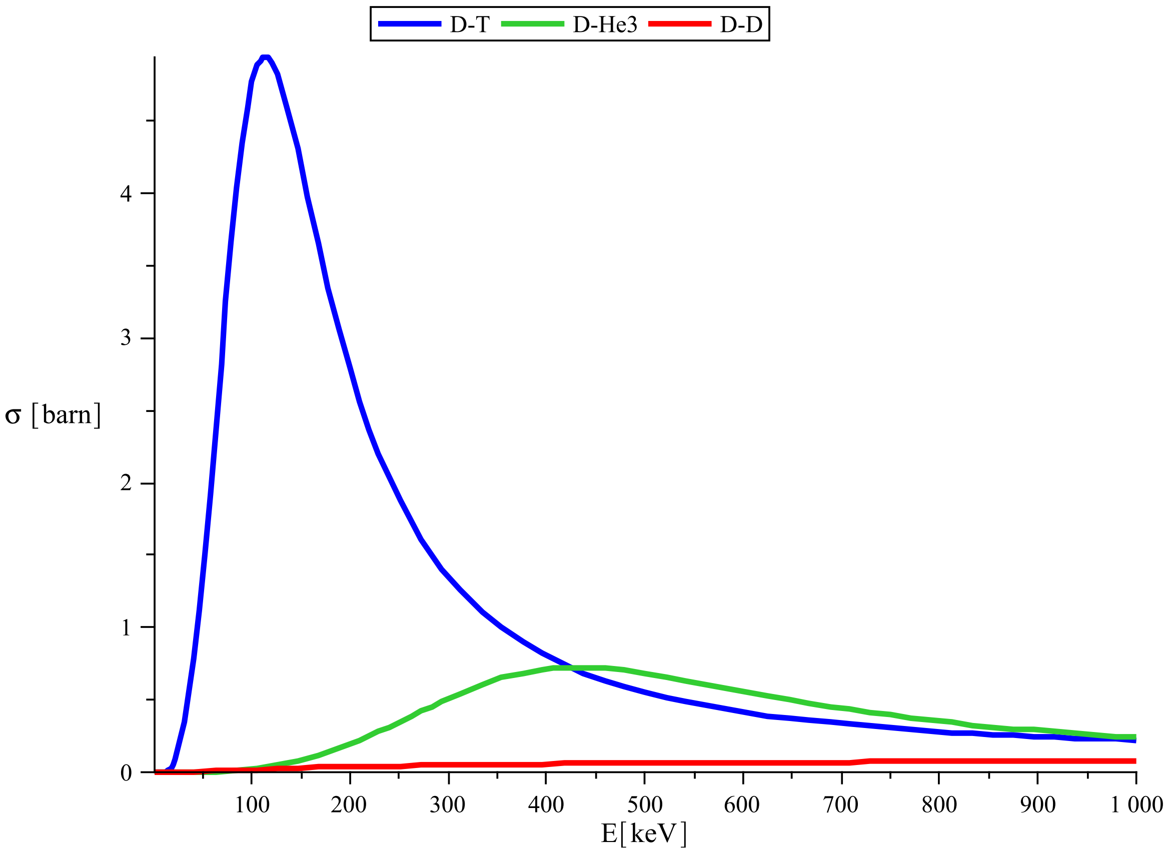

Figure 3.2: Cross section

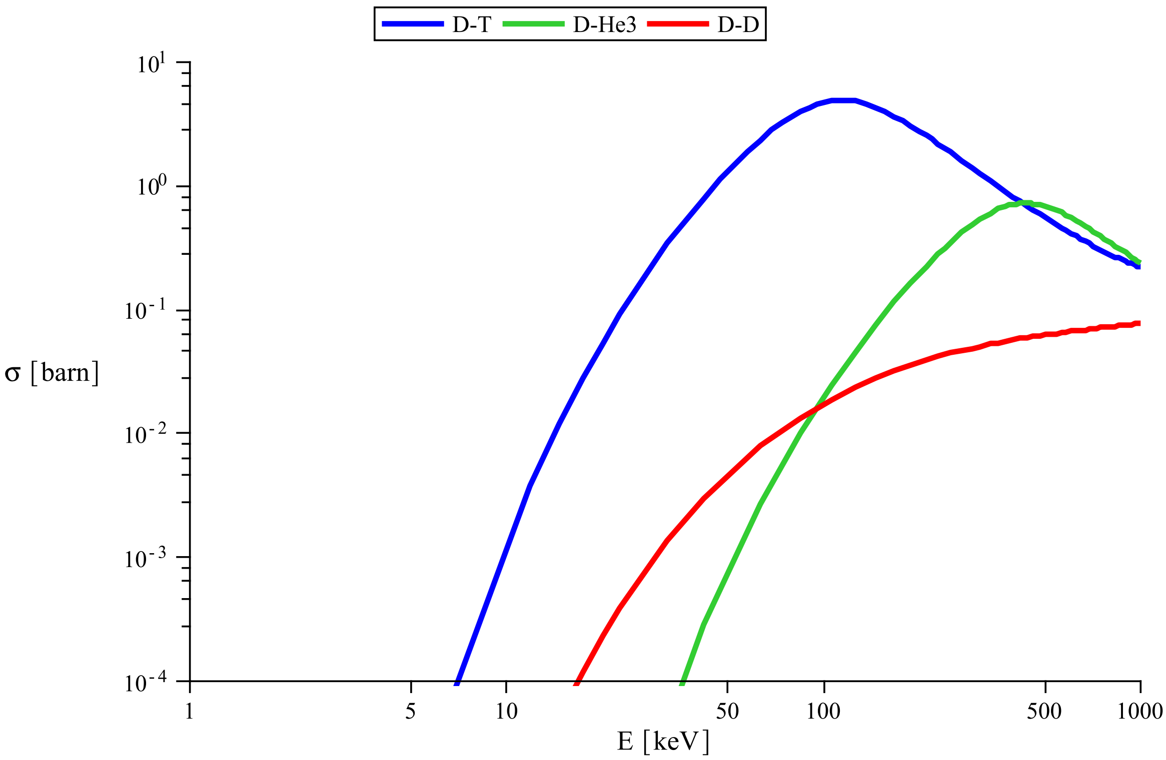

Figure 3.3: Cross section on log scale

3.4 Reaction Rate

When the target particles are at rest and the beam particles move with a constant velocity, the reaction rate per unit volume is defined to be[9]:

$$r(\vec{x},t) = -\frac{\mathrm{d}n_1(\vec{x},t)}{\mathrm{d}t} = n_1(\vec{r},t)n_2\sigma\frac{\mathrm{d}s}{\mathrm{d}t} = n_1(\vec{x},t)n_2\sigma v \tag{3.22}$$

If the target particles move with velocity $\vec{v_2}$, we have to replace the speed v by the magnitude of relative velocity $v = |\vec{v_1} - \vec{v_2}|$.

Using the equation (3.2) we can express the reaction rate generally as:

$$r(\vec{x},t) = \int \mathrm{d}\vec{v_1} \int \mathrm{d}\vec{v_2} f(\vec{x},\vec{v_1},t) f(\vec{x},\vec{v_2},t)\sigma(v)v \tag{3.23}$$

3.4.1 Two Colliding Beams

In case of two colliding beams with constant velocities $\vec{v_1}$ and $\vec{v_2}$ the product $\sigma(v)v$ is independent of the integrating variables. Then the reaction rate may be expressed as:

$$r(\vec{x},t) = n_1(\vec{x},t)n_2(\vec{x},t)\sigma(v)v \tag{3.24}$$

3.4.2 Target with Maxwellian Distribution

When a beam with a constant velocity $\vec{v_1}$ collides with a plasma (particles with different velocities), the reaction rate must be rearranged as follows:

$$r(\vec{x},t) = n_1(\vec{x},t) \int \mathrm{d}\vec{v_2} f(\vec{x},\vec{v_2},t)\sigma(v)v \tag{3.25}$$

For target with Maxwellian distribution (3.4), this becomes:

$$r(\vec{x},t) = n_1(\vec{x},t)n_2(\vec{x},t)\left(\frac{m_2}{2\pi kT_2}\right)^{3/2} \int \mathrm{d}\vec{v_2} \exp\left(-\frac{m_2 v_2^2}{2kT_2}\right)\sigma(v)v \tag{3.26}$$

If we express the average value of $\sigma(v)v$ over a Maxwellian distribution using Eq. (3.2)-(3.4), then we can write:

$$\langle\sigma(v)v\rangle = \left(\frac{m_2}{2\pi kT_2}\right)^{3/2} \int \mathrm{d}\vec{v_2} \exp\left(-\frac{m_2 v_2^2}{2kT_2}\right)\sigma(v)v \tag{3.27}$$

Now we can obtain the reaction rate as:

$$r(\vec{x},t) = n_1(\vec{x},t)n_2(\vec{x},t)\langle\sigma(v)v\rangle \tag{3.28}$$

3.4.3 Two Maxwellian Distributions

In case of reaction between two species of particles, each having Maxwellian distributions, charakterized by m_1, T_1 and m_2,T_2, Eq. (3.13) becomes:

$$r(\vec{x},t) = n_1(\vec{x},t)n_2(\vec{x},t)\left(\frac{m_1}{2\pi kT_1}\right)^{3/2}\left(\frac{m_2}{2\pi kT_2}\right)^{3/2}\int \mathrm{d}\vec{v_1}\mathrm{d}\vec{v_2}\exp\left(-\frac{m_1v_1^2}{2kT_1}-\frac{m_2v_2^2}{2kT_2}\right)\sigma(v)v \tag{3.29}$$

Let $\beta_1 = m_1/2kT_1$,$\beta_2 = m_2/2kT_2$ and $\vec{v} = \vec{v_1} - \vec{v_2}$. Then

$$\exp\left(-\frac{m_1v_1^2}{2kT_1}-\frac{m_2v_2^2}{2kT_2}\right) = \exp\left[-\beta_1(\vec{v_2}+\vec{v})^2 - \beta_2 v_2^2\right] \tag{3.30}$$

$$= \exp\left[-\frac{\beta_1\beta_2 v^2}{\beta_1+\beta_2} - (\beta_1+\beta_2)\left(\vec{v_2}+\frac{\beta_1\vec{v}}{\beta_1+\beta_2}\right)^2\right] \tag{3.31}$$

Using equation $\mathrm{d}\vec{v_1} = \mathrm{d}\vec{v}$(holding $\vec{v_1}$ constant during this integration) and Eq.(3.20), it is possible to convert Eq.(3.19) to the form:

$$r(\vec{x},t) = n_1n_2\left(\frac{\beta_1\beta_2}{\pi^2}\right)^{3/2}\int \mathrm{d}\vec{v}\exp\left(-\frac{\beta_1\beta_2v^2}{\beta_1+\beta_2}\right)\sigma(v)v\int \mathrm{d}\vec{v_2}\exp\left[-(\beta_1+\beta_2)\left(\vec{v_2}+\frac{\beta_1\vec{v}}{\beta_1+\beta_2}\right)^2\right] \tag{3.32}$$

To solve the integral over $\vec{v_2}$ we will use the substitution: $\vec{u} = \vec{v_2} + \frac{\beta_1\vec{v}}{\beta_1+\beta_2}$. Then $\mathrm{d}\vec{v_2} = \mathrm{d}\vec{u}$(holding $\vec{v}$ constant during integration). We will denote $\beta \equiv \frac{\beta_1\beta_2}{\beta_1+\beta_2}$. Now we can write:

$$r(\vec{x},t) = n_1n_2\left(\frac{\beta_1\beta_2}{\pi^2}\right)^{3/2}\int \mathrm{d}\vec{v}\exp(-\beta v^2)\sigma(v)v\int \mathrm{d}\vec{u}\exp\left[-(\beta_1+\beta_2)u^2\right] \tag{3.33}$$

$$= n_1n_2\left(\frac{\beta_1\beta_2}{\pi^2}\right)^{3/2}\int \mathrm{d}\vec{v}\exp(-\beta v^2)\sigma(v)v\left(\frac{\pi}{\beta_1+\beta_2}\right)^{3/2} \tag{3.34}$$

$$= n_1n_2\left(\frac{\beta}{\pi}\right)^{3/2}\int \mathrm{d}\vec{v}\exp(-\beta v^2)\sigma(v)v \tag{3.35}$$

Similarly as in the section (3.4.2), we can express the average value of $\sigma(v)v$ over a Maxwellian distribution charakterized by the parameter β as:

$$\langle\sigma(v)v\rangle = \left(\frac{\beta}{\pi}\right)^{3/2}\int \mathrm{d}\vec{v}\exp(-\beta v^2)\sigma(v)v \tag{3.36}$$

The quantity $\langle\sigma(v)v\rangle$ is called the reaction rate parameter.

The reaction rate can be expressed now as:

$$r(\vec{x},t) = n_1(\vec{x},t)n_2(\vec{x},t)\langle\sigma(v)v\rangle \tag{3.37}$$

3.5 Reaction Rate Parameter

As shown in the previous section, for evaluation of reaction rate we need to know the reaction rate parameter. There are manny publications, which contain values of this parameter for various reactions and temperatures. However a lot of modern computer simulations of fusion reaction rates utilize fitting functions based on data that were published almost thirty years ago.

On this account, for evaluation of reaction rate parameter we will use Bosch and Hale fusion reactivity model[15], which is based on R-matrix theory in conjunction with more recent experimental cross section data. Moreover, Bosch and Hale give energies ranges over which the model is valid (for reaction (2.3) it is $0.5 - 190keV$, for the other reactions $0.2 - 100keV$). Equations (3.28) through (3.31) are the result of the R-matrix fit to the experimental data. Table 3.2 shows the coefficients used in these equations. Equation (3.31) determines reaction rate parameter in cm^3s^{-1} as a function of thermal energy in keV.

$$B_G = \frac{1}{137}\pi Z_1 Z_2 \sqrt{2m_r c^2} \tag{3.38}$$

$$\theta = E/\left[1 - \frac{E(C_2 + E(C_4 + C_6 E))}{1 + E(C_3 + E(C_5 + C_7 E))}\right] \tag{3.39}$$

$$\zeta = \left(\frac{B_G^2}{4\theta}\right)^{1/3} \tag{3.40}$$

$$\langle\sigma(v)v\rangle = C_1 \theta \exp(-3\zeta)\sqrt{\zeta/m_r c^2 E^3} \tag{3.41}$$

	$D-T$	$D-He^3$	$D-D_{He^3}$	$D-D_T$
C_1	1.17×10^{-9}	5.51×10^{-10}	5.43×10^{-12}	5.66×10^{-12}
C_2	1.51×10^{-2}	6.42×10^{-3}	5.86×10^{-3}	3.41×10^{-3}
C_3	7.52×10^{-2}	-2.03×10^{-3}	7.68×10^{-3}	1.99×10^{-3}
C_4	4.61×10^{-3}	-1.91×10^{-5}	0	0
C_5	1.35×10^{-2}	1.36×10^{-4}	-2.96×10^{-6}	1.05×10^{-5}
C_6	-1.07×10^{-4}	0	0	0
C_7	1.37×10^{-5}	0	0	0
$m_r c^2$ (keV)	1124656	1124572	937814	937814

Table 3.2: Coefficients for equations (3.27)-(3.30)

BUCKY is a one-dimensional hydrodynamics code developed by the University of Wisconsin that models high energy density fusion plasma[15]. It was used to generate reaction rate parameter as a function of plasma thermal energy (3.32). Table 3.3 shows the coefficients used in this equation. Its advantage is the simplicity, however there are no ranges, over which this formula is valid. Moreover, there are no coefficients for reactions (2.4) and (2.5) separately, which we will need for the next evaluations.

$$\langle\sigma(v)v\rangle = \exp(\frac{A_1}{E^r} + A_2 + A_3E + A_4E^2 + A_5E^3 + A_6E^4) \quad (3.42)$$

In [13] there is not mention of any fitting function of the reaction rate parameter as in the case of cross section. There are only several values written for each reaction.

	$D-T$	$D-He^3$	$D-D$
A_1	-21.377692	-27.764468	-15.511891
A_2	-25.20405	-31.023898	-35.318711
A_3	-7.1013427×10^{-2}	2.7889999×10^{-2}	1.2904737×10^{-2}
A_4	1.937545×10^{-4}	-5.5321633×10^{-4}	$2.6797766\times10{-}4$
A_5	$4.9246592\times10{-}6$	$3.0293927\times10{-}6$	-2.9198658×10^{-6}
A_6	-3.9836572×10^{-8}	-2.5233325×10^{-9}	1.2748415×10^{-8}
r	0.2935	0.3597	0.3735

Table 3.3: Coefficients for the equation (3.31)

In Figures (3.4)-(3.6) are plotted both curves for energy ranges $1-100keV$. It is obvious that especially for the D-D reaction the BUCKY model is not correct.

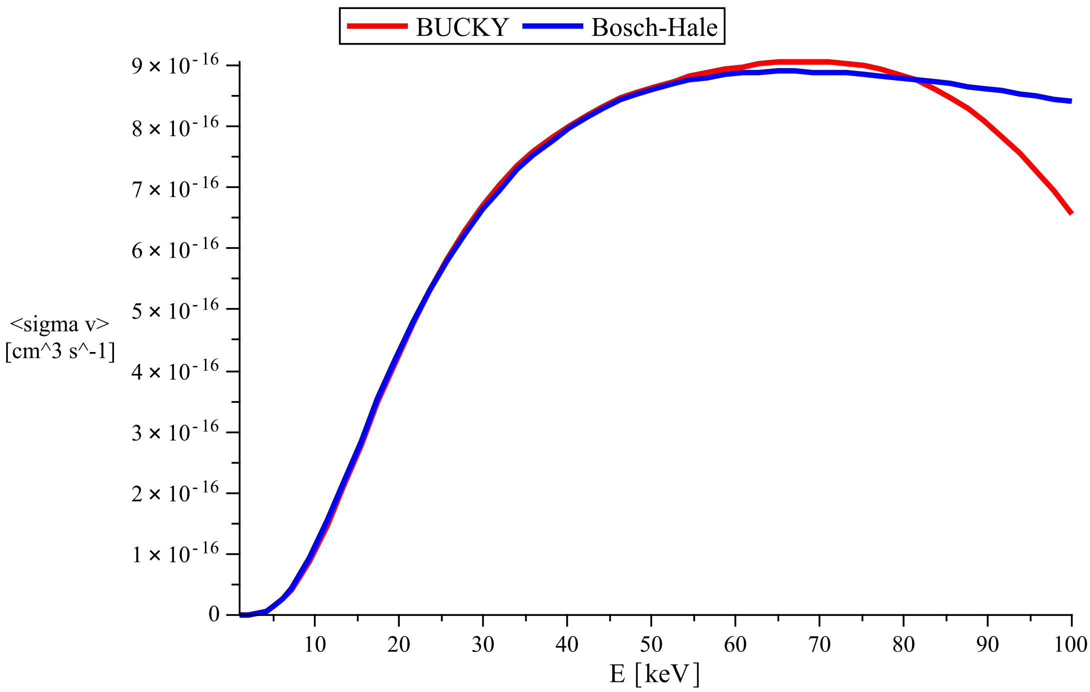

Figure 3.4: Comparison of BUCKY and Bosch-Hale for the D-T reaction

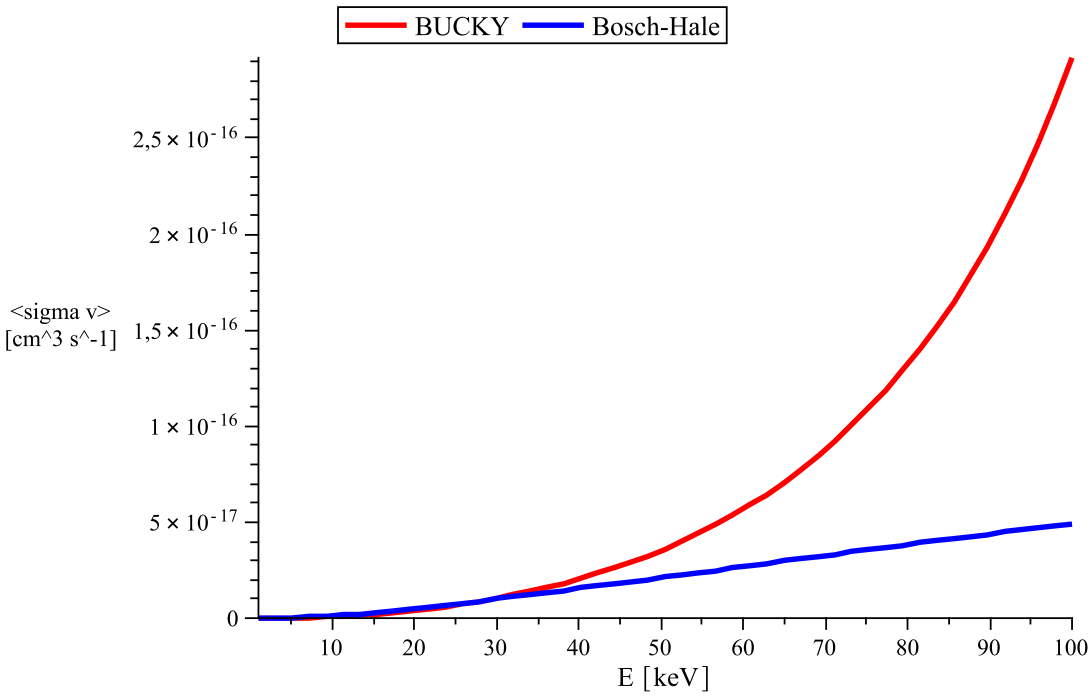

Figure 3.5: Comparison of BUCKY and Bosch-Hale for the D-D reaction

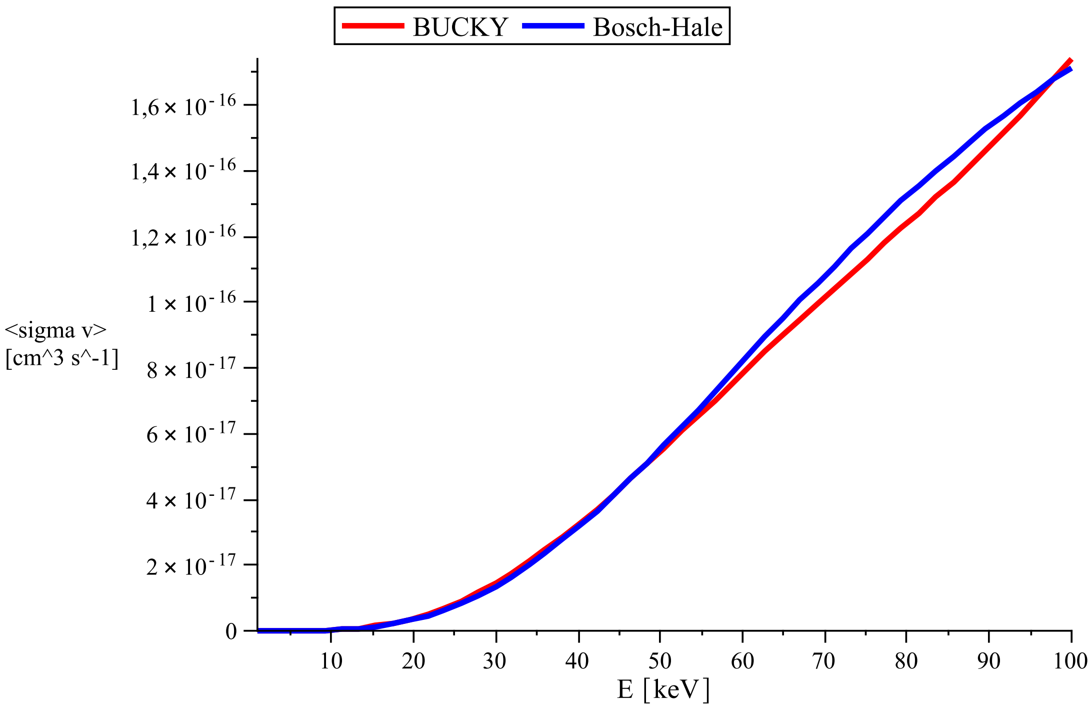

Figure 3.6: Comparison of BUCKY and Bosch-Hale for the D-He3 reaction

Chapter 4

Temporal Progress of Fusion Reactions

In this part, we will consider Maxwellian distribution of interacting particles as in the section 3.4.3 and reactions from the section 2.2. We will neglect all the other possible reactions. Furthermore, all the particles in the reaction will have the same temperature T. Under these conditions, we can use the data from Bosch-Hale fusion reactivity model.

According to the equation (3.26), for reaction among like particles the reaction rate can be expressed as:

$$-\frac{\mathrm{d}(n/2)}{\mathrm{d}t} = \langle\sigma v\rangle (n/2)^2 \Rightarrow -\frac{\mathrm{d}n}{\mathrm{d}t} = 1/2 \langle\sigma v\rangle n^2 \tag{4.1}$$

Now we can make a system of ordinary differential equations, which will describe temporal progress of particles densities of fusion reaction components for $\vec{x} = const$. We will assume that in time $t = 0$ we have plasma compound from D, T and He^3 particles (fuel-equations (4.2)-(4.4)). During the reactions, new particles will be generated (product-equations (4.5)-(4.7)).

$$\frac{\mathrm{d}n_D}{\mathrm{d}t} = -1/2(\langle\sigma v\rangle_{DD-He} + \langle\sigma v\rangle_{DD-T})n_D^2 - \langle\sigma v\rangle_{DT} n_D n_T - \langle\sigma v\rangle_{DHe^3} n_D n_{He^3} \tag{4.2}$$

$$\frac{\mathrm{d}n_T}{\mathrm{d}t} = -\langle\sigma v\rangle_{DT} n_D n_T + 1/2 \langle\sigma v\rangle_{DD-T} n_D^2 \tag{4.3}$$

$$\frac{\mathrm{d}n_{He^3}}{\mathrm{d}t} = -\langle\sigma v\rangle_{DHe^3} n_D n_{He^3} + 1/2 \langle\sigma v\rangle_{DD-He} n_D^2 \tag{4.4}$$

$$\frac{\mathrm{d}n_\alpha}{\mathrm{d}t} = \langle\sigma v\rangle_{DT} n_D n_T + \langle\sigma v\rangle_{DHe^3} n_D n_{He^3} \tag{4.5}$$

$$\frac{\mathrm{d}n_p}{\mathrm{d}t} = \langle\sigma v\rangle_{DHe^3} n_D n_{He^3} + 1/2 \langle\sigma v\rangle_{DD-T} n_D^2 \tag{4.6}$$

$$\frac{\mathrm{d}n_n}{\mathrm{d}t} = \langle\sigma v\rangle_{DT} n_D n_T + 1/2 \langle\sigma v\rangle_{DD-He} n_D^2 \tag{4.7}$$

In Figures (4.1)-(4.8) are plotted solutions of this equations system for 2 energies $(20, 100keV)$ and several initial conditions. Values of $\langle\sigma v\rangle$ were obtained from Bosch-hale model in units $10^{-17} cm^3 s^{-1}$. System was numerically solved in program Maple 11 and all equations were divided by the initial particle density of deuterium $n_D(0) = n_{D_0}$. It means that all solutions of the particle densities are normalised to this initial particle density $n_D(0) = n_{D_0}$.

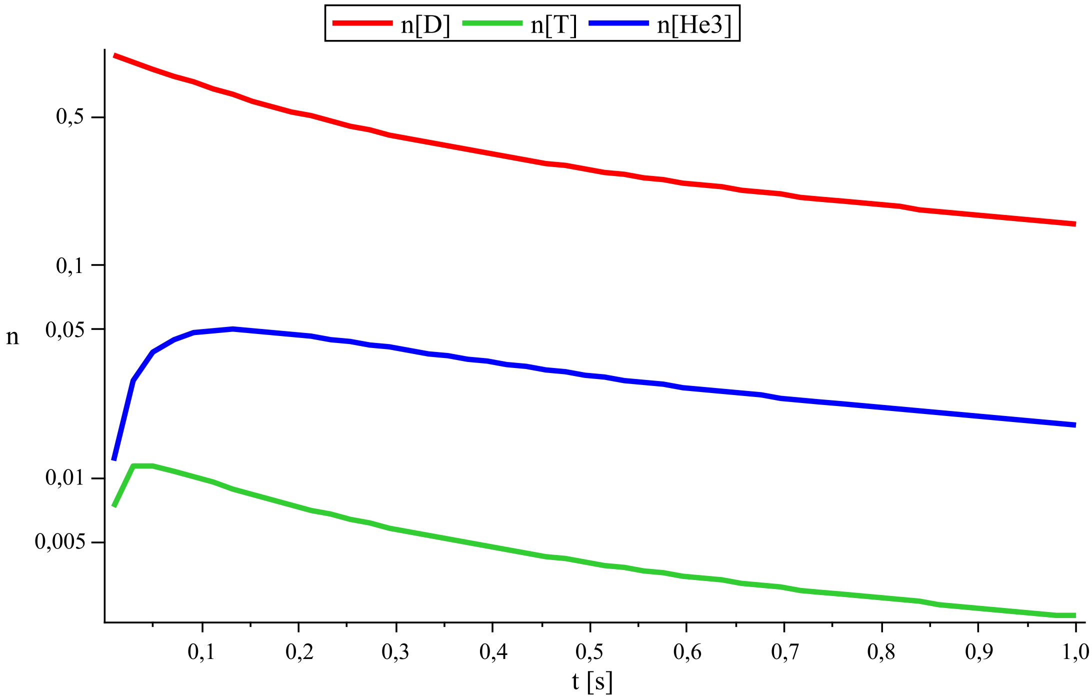

Figure 4.1: Fuel particle densities for $E = 100keV$ and ignition conditions $n_D(0) = 1, n_T(0) = n_{He^3}(0) = 0$

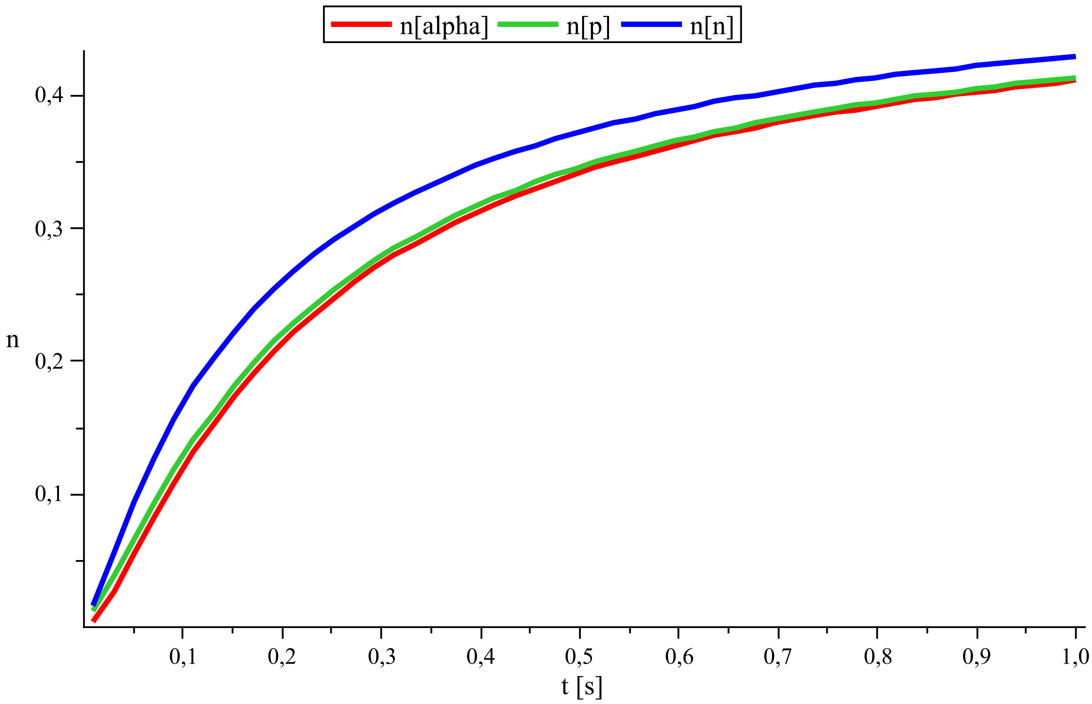

Figure 4.2: Product particle densities for $E = 100keV$ and ignition conditions $n_D(0) = 1, n_T(0) = n_{He^3}(0) = 0$

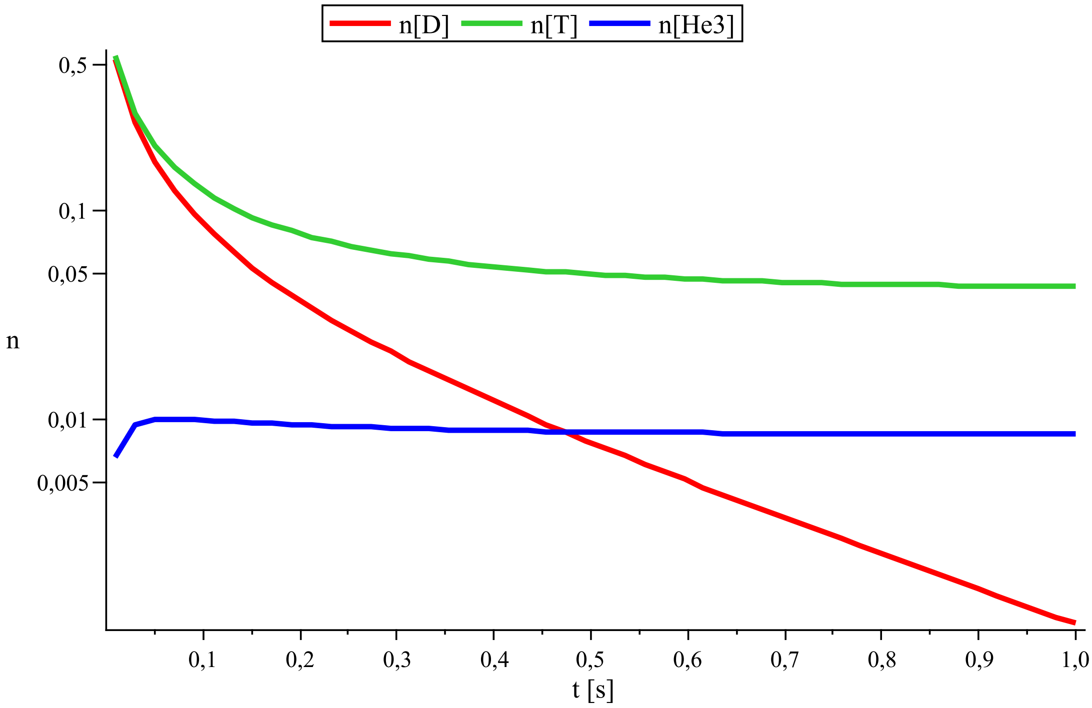

Figure 4.3: Fuel particle densities for $E = 100keV$ and ignition conditions $n_D(0) = 1, n_T(0) = 1, n_{He^3}(0) = 0$

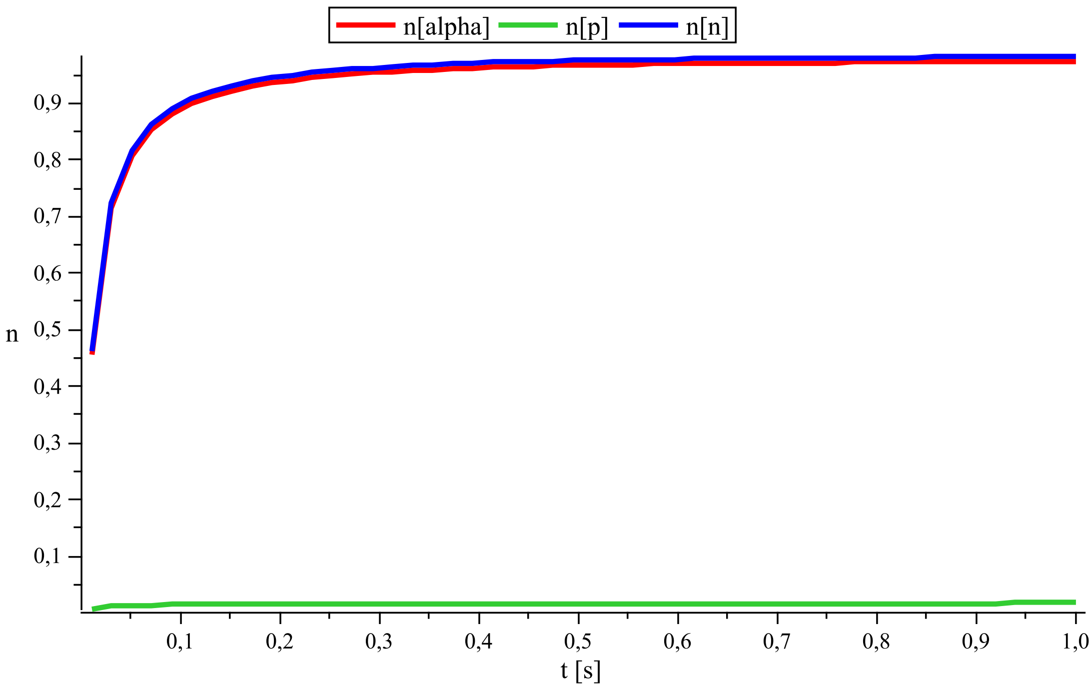

Figure 4.4: Product particle densities for $E = 100keV$ and ignition conditions $n_D(0) = 1, n_T(0) = 1, n_{He^3}(0) = 0$

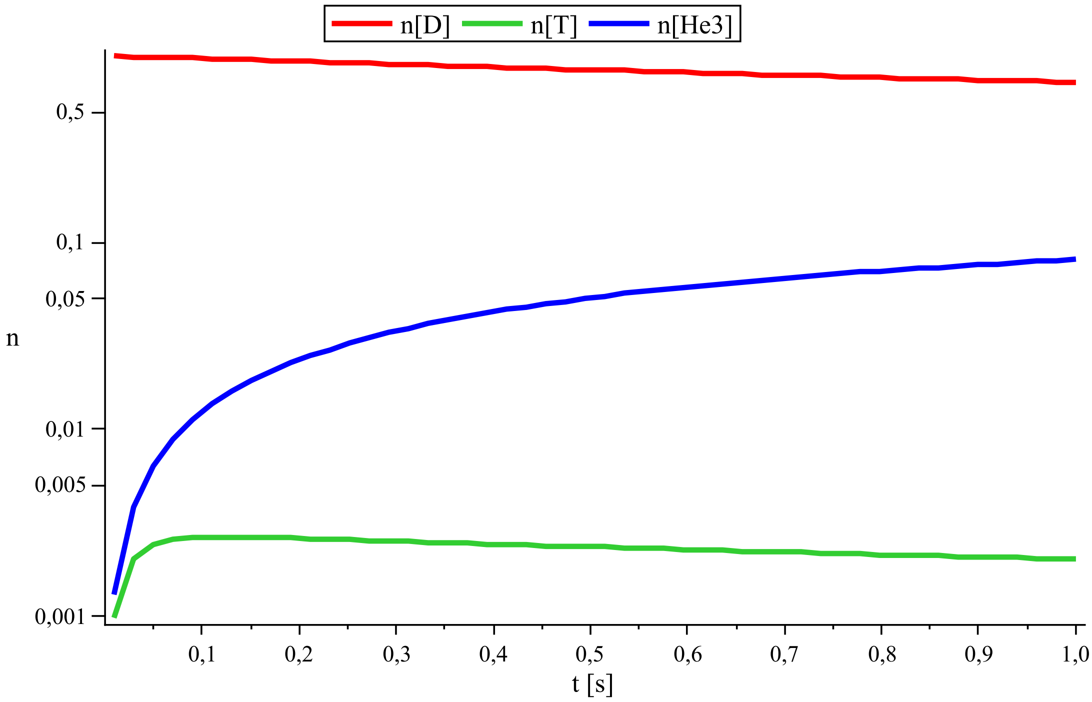

Figure 4.5: Fuel particle densities for $E = 20keV$ and ignition conditions $n_D(0) = 1, n_T(0) = n_{He^3}(0) = 0$

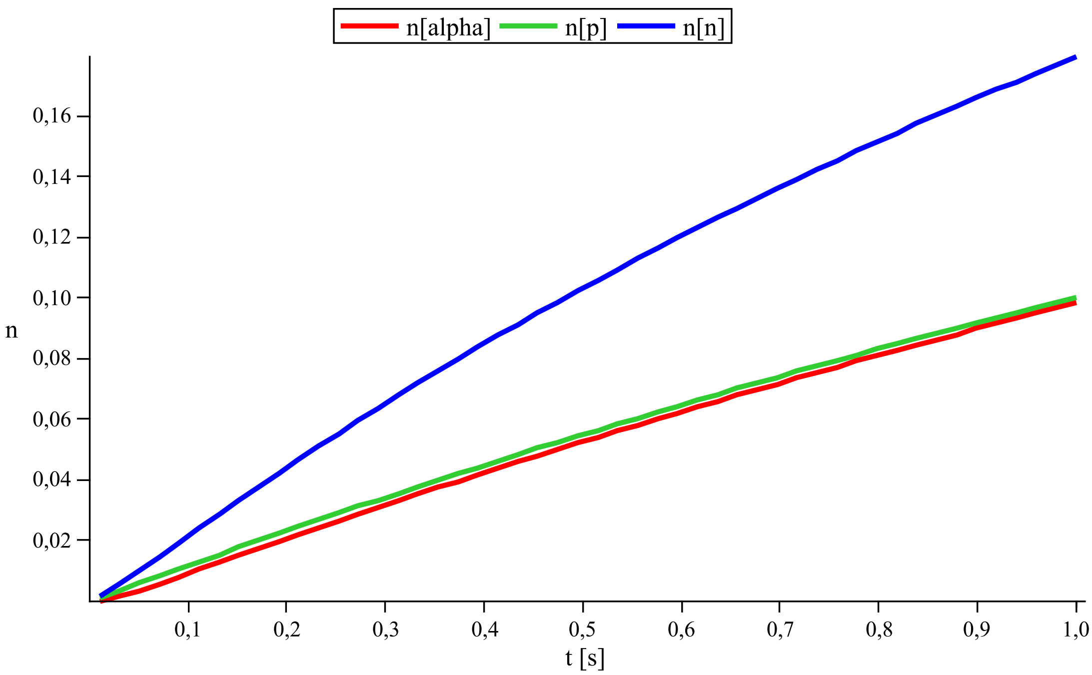

Figure 4.6: Product particle densities for $E = 20keV$ and ignition conditions $n_D(0) = 1, n_T(0) = n_{He^3}(0) = 0$

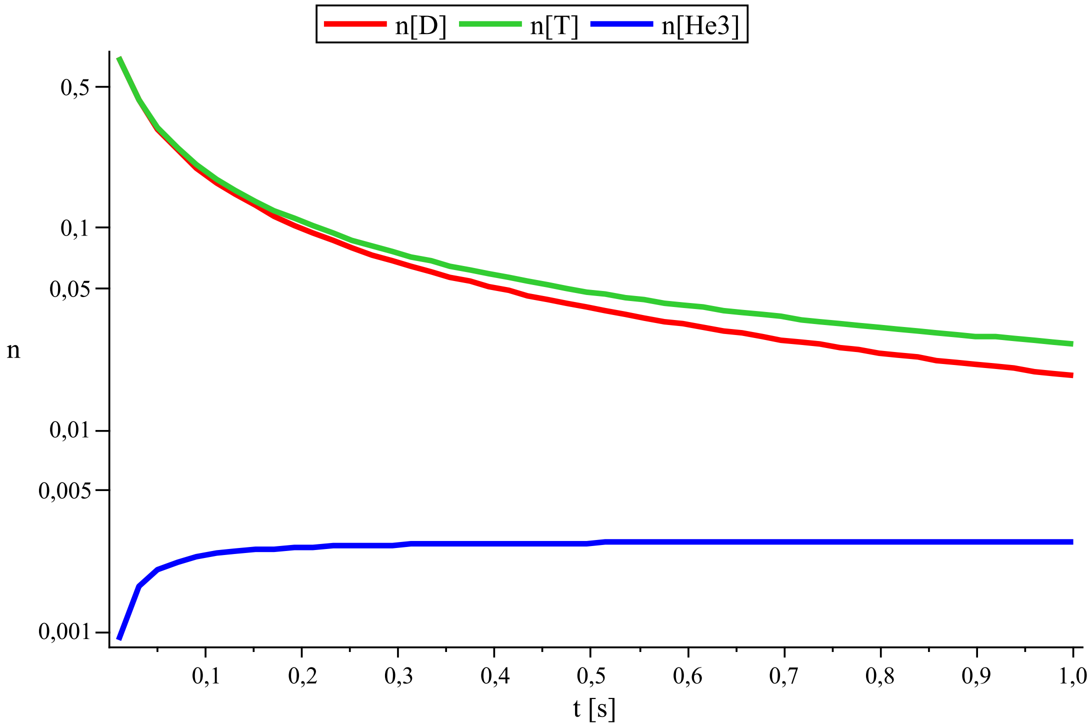

Figure 4.7: Fuel particle densities for $E = 20keV$ and ignition conditions $n_D(0) = 1, n_T(0) = 1, n_{He^3}(0) = 0$

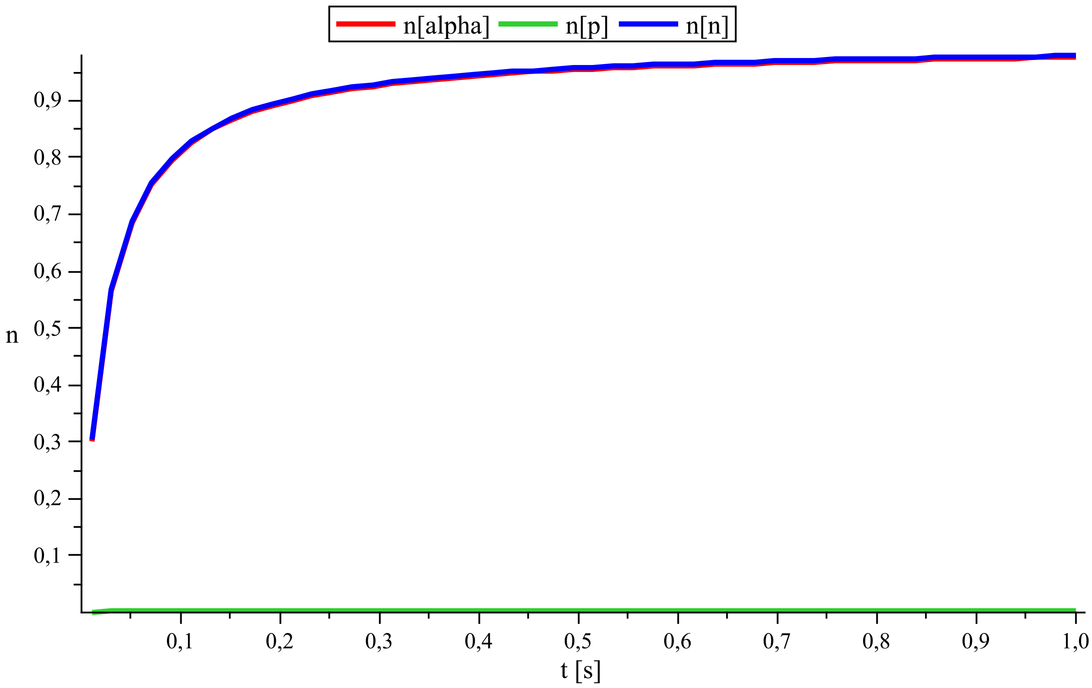

Figure 4.8: Product particle densities for $E = 20keV$ and ignition conditions $n_D(0) = 1, n_T(0) = 1, n_{He^3}(0) = 0$

Chapter 5

Advanced Fuels

Mixtures of hydrogen isotopes and light nuclei (Helium, Lithium, Boron) are called in the context of controlled fusion research the advanced fusion fuels[12]. These ractions produce only charged particles, whose energy can be converted directly to the electricity. Since the most-studied fusion reactions release up to 80% of their energy in neutrons, advanced fuels would remove problems associated with neutron radiation such as ionizing damage, neutron activation, and requirements for biological shielding, remote handling, and safety issues.

Advanced fuels come under aneutronic fusion, which is defined as any form of fusion power where no more than 1% of the total energy released is carried by neutrons[16]. Aneutronic fusion has manny advanteges, but physical conditions required for realizing these reactions are much harder than for e.g. $D - T$ reaction.

These are aneutronic reactions with the largest cross sections[16]:

$$D + Li^6 \rightarrow 2\alpha + 22.4MeV \tag{5.1}$$
$$p + Li^6 \rightarrow \alpha(1.7MeV) + He^3(2.3MeV) \tag{5.2}$$
$$He^3 + Li^6 \rightarrow 2\alpha + p + 16.9MeV \tag{5.3}$$
$$He^3 + He^3 \rightarrow \alpha + 2p + 12.86MeV \tag{5.4}$$
$$p + Li^7 \rightarrow 2\alpha + 17.2MeV \tag{5.5}$$
$$p + B^{11} \rightarrow 3\alpha + 8.7MeV \tag{5.6}$$

Conclusion

This bachelor's degree project consist of two parts. Major part (chapters 1,2,3 and 5) is a backgroud research of a given theme. In chapter 4 are obtained knowledges used to applied evaluations.

I would like to continue my work on the evaluation of reaction yields of nuclear processes in systems with non-maxwellian distributions, which was one of the given topics. However, I think that it would be suitable theme for a dissertation.

Bibliography

[1] http://physicsworld.com/cws/article/print/24295, Fusion: the way ahead, Richard Pitts, March 2006

[2] http://www.iop.org/Jet/fulltext/EFDR00001.pdf, Energy for future centuries, J.Ongena and G.Van Oost, January 2001

[3] J.Wesson, Tokamaks, third edition, Oxford university press, 2004

[4] http://www.world-nuclear.org/info/inf75.html, Supply of uranium

[5] http://www.iter.org/a/index_nav_4.htm, Resources needed for ITER Joint Implementation

[6] http://user.web.cern.ch/User/LHCCost/2001-10-16/LHCCostReview.html, LHC Cost Review to Completion

[7] http://www.spaceprojects.com/iss/, A "Station" in Space and NASA's Deceptions on Earth

[8] http://cs.wikipedia.org/wiki/Jadern%C3%A1_elektr%C3%A1rna_Temel%C3%ADn, JETE

[9] T.J.Dolan, Fusion Research Principles, Experiments and Technology-chapter 2, Pergamon press, 1980

[10] M.Venugopalan, Reactions under plasma conditions-chapter 20, Wiley, 1971

[11] J.Kvasnica, Statistick fyzika, Academia Praha, 1983

[12] S.Atzeni, J.Meyer-Ter-Vehn, The physics on inertial fusion-chapter 1, Clarendon press-Oxford, 2004

[13] NRL plasma formulary 2007, J.D.Huba, Beam Physics Branch, Plasma Physics Division, Naval Research Laboratory, Washington, DC 20375

[14] D.D.Clayton, Principles of stellar evolution and nucleosynthesis-chapter 4, McGraw-Hill, 1968

[15] T.A.Heltemes, G.A.Moses, J.F.Santarius, Analysis of an Improved Fusion Reaction Rate Model for Use in Fusion Plasma Simulations

[16] http://en.wikipedia.org/wiki/Aneutronic_fusion#cite_ref-5, Aneutronic fusion