Introduction to Quantum Chromodynamics

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Literature

Our discussion is based on

Quarks, partons and Quantum Chromodynamics

by Jiří Chýla

Available at http://www-hep.fzu.cz/ chyla/lectures/text.pdf

Additional material comes from

A modern introduction to particle physics

by Fayyazuddin & Riazuddin

World Scientific Publishing 2000 (Second edition)

Introduction

- The elements of the *constituent* quark model – sometimes also called *additive quark model* – will be introduced and some of its applications discussed.
- Beside the idea of quarks as fundamental building blocks of matter, the application of the quark model to the spectrum of hadrons had lead to introduction of another fundamental concept of present theory of strong interactions: the *color*.
- Color quantum number, which plays crucial role in the phenomenon of *quark confinement* – the fact that quarks do not exist in nature as isolated free objects like, for instance, electron or proton – has become the cornerstone of *Quantum Chromodynamics*, the theory of strong forces between colored objects, to be discussed in Chapter 6.

January 1964: birth of the quark model

- January 1964: even before the discovery of Ω^- and the confirmation of the *Eightfold way*, two theorist – Murray Gell-Mann and George Zweig published papers that heralded the birth of the quark model. Each of them have approached the problem from quite different directions.

MGM: *If we assume that the strong interactions of baryons and mesons are correctly described in terms of the broken "eightfold way", we are tempted to look for some fundamental explanation of the situation. A highly promised approach is surely dynamical "bootstrap" model for all strongly interacting particles within which one may try to derive isotopic spin and strangeness conservation and broken eightfold symmetry from self-consistency alone. Of course, with only strong interactions the orientation of the asymmetry in the unitary space cannot be specified; one hopes that in some way selection of specific components of the F-spin by electromagnetism and the weak interactions determines the choice of the isotopic spin and hypercharge directions.*

- For MGM quarks have always remained basically a mathematical concept, devoid of any physical reality,

January 1964: birth of the quark model

GZ: *Both mesons and baryons are constructed from a set of three fundamental particles, called aces. The aces break up into isospin doublet and singlet. Each ace carries baryon number* $1/3$ *and is fractionally charged.* SU_3 *(but not the Eightfold way) is adopted as a higher symmetry for the strong interactions. The breaking of this symmetry is assumed to be universal, being due to the mass difference among the aces. Extensive space-time and group theoretic structure is then predicted for both mesons and baryons, in agreement with existing experimental information. . . . An experimental search for the aces is suggested.*

- For GZ the starting point was ϕ meson discovery and its puzzling decay pattern. Contrary to phase space–based arguments ϕ preferred $\phi \to K\overline{K}$ (BR=83%) rather than $\phi \to \rho\pi$ (BR=12.9%) or $\phi \to \pi^+\pi^-\pi^0$ (BR=2.7%).
- To understand it GZ developed the phenomenological rule: $\phi = s\overline{s} \Rightarrow$ separation of s and $\overline{s}$ leads to creation of $u\overline{u}$ and $d\overline{d}$ which recombine with the "constituent" s and $\overline{s}$ quarks into kaons, hence the dominance of $K\overline{K}$.

SU(3) quark model

After the discovery of the Ω^- hyperon all observed hadrons could be arranged (identifying $H_2 = T_8 (= \frac{\sqrt{3}}{2} Y)$ into multiplets of the SU(3) group:

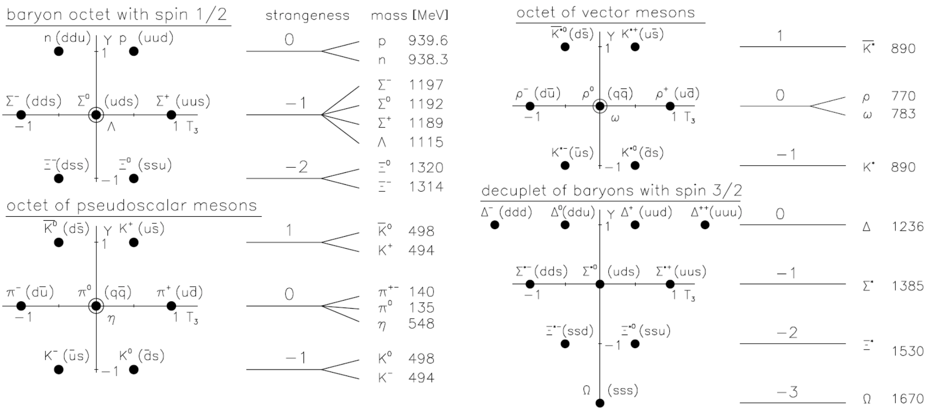


Figure 1: Basic SU(3) multiplets of baryons and mesons.

SU(3) quark model

The grouping of hadrons of the same J^P and B in SU(3) octets and decuplets was based essentially on their masses and isospin symmetry. Moreover, all evidence indicated that S was conserved by strong interactions exactly.

Few comments are in order:

1. Mass differences within the isospin multiplets are much smaller than those between the different SU(2) multiplets within one unitary multiplet, indicating that the full SU(3) symmetry is broken much more strongly than the subgroup of isospin symmetry.
2. The mean masses of isospin multiplets are *increasing* functions of the absolute value of the *strangeness*.
3. The mass pattern is especially simple in the baryon decuplet, where the four isospin multiplets are spaced nearly equidistantly, the mass separation being roughly 150 MeV.
4. Mass relations are much more complicated in the octets, in particular in the center (where, as we know, the state is **not** uniquely defined by its weight). In particular η, η' cause problems to accommodate their masses within the unitary multiplets.

SU(3) quark model

5. Strong interactions conserve T_3 and S exactly:
$$\Rightarrow \left[H_s, T^{\pm}\right] = [H_s, T_3] = [H_s, T_8] = 0. \quad (1)$$
6. In the middle of sixties serious effort were undertaken to find quarks. The fact that quarks should have fractional electric charges as well as fractional baryon numbers made them look exotic, but there was no obvious theoretical reason why these "exotics" should not exist in nature.
7. The went on until the late 1970, when it became increasingly clear that they do not exist in this way but are forever bound inside hadrons. The mechanism of this "quark confinement" will be discussed later.
8. In addition 4 basic multiplets there are many other, fully or partially, filled SU(3) multiplets. For meson octets, the states in the center are linear combinations of the three $q\overline{q}$ pairs: $u\overline{u}$, $d\overline{d}$, $s\overline{s}$.
9. Information on quark composition itself *doesn't* uniquely specify the corresponding hadron and so one needs to know more about its quantum numbers and/or wave function to distinguish, for instance, ω from ρ^0 or proton from Δ^+.

SU(3) quark model

- From the relations

$$\mathbf{3} \otimes \overline{\mathbf{3}} = \mathbf{8} \oplus \mathbf{1}, \qquad \mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{10}_{\mathrm{s}} \oplus \mathbf{8}_{\mathrm{ms}} \oplus \mathbf{8}_{\mathrm{ma}} \oplus \mathbf{1}_{\mathrm{a}} \tag{2}$$

  one sees why *three* quarks, and antiquarks, are needed to form the experimentally observed pattern of meson octets and baryon octet and decuplet.

- Natural question:
  Why the quarks don't form also other possible combinations, like diquarks (qq pairs), 4q configurations etc.?

- It took about a decade to answer it qualitatively and another decade to do so more quantitatively within the QCD. This will be addressed in the last section of this chapter together with the crucial feature of quark confinement. .

Quarks with flavor and spin: the SU(6) symmetry

- SU(6)= extension of SU(3) flavor symmetry taking into account spin 1/2 of the quarks ⇒ which thus exist in $3 \times 2 = 6$ different states.
- Typical differences ($\Delta m \approx 150 - 200$ MeV) between isospin multiplets within both the baryon octet and decuplet are about the same as the difference between the average masses of these SU(3) multiplets (see Fig. 1).
- ⇒ Assembling all the 56 baryonic states of different flavor-spin combinations ($4 \times 10 + 2 \times 8 = 56$) into one higher multiplet is justified.
- *Fully symmetric* multiplet **56**=(3,0) of SU(6) has just the right number of states 56, and decomposes with respect to the unitary and spin subgroups SU(3) and SU(2) as (**SU(3)**,**SU(2)**): $$\mathbf{56} = (\mathbf{10}, \mathbf{4}) \oplus (\mathbf{8}, \mathbf{2}). \qquad (3)$$
- Full decomposition of the direct product of three* quark sextets reads: $$\mathbf{6} \otimes \mathbf{6} \otimes \mathbf{6} = \mathbf{56}_s \oplus \mathbf{70}_{\mathrm{ms}} \oplus \mathbf{70}_{\mathrm{ma}} \oplus \mathbf{20}_{as}, \qquad (4)$$

  where, as in the case of the product of triplets of SU(3) group, the subscripts "ms" and "ma" denote representations with particular symmetry under the permutation of first two sextets.

(*)For the simpler case of isodublet of (u, d) with spin we have $\mathbf{4} \otimes \mathbf{4} \otimes \mathbf{4} = \mathbf{20}_s \oplus \mathbf{20}_{\mathrm{ms}} \oplus \mathbf{20}_{\mathrm{ma}} \oplus \mathbf{4}_{as}$.

- Analogously to (3) decompositions wrt to SU(3)⊗SU(2) of SU(6) reads:

$$\mathbf{70} = (\mathbf{10},\mathbf{2}) \oplus (\mathbf{8},\mathbf{4}) \oplus (\mathbf{8},\mathbf{2}) \oplus (\mathbf{1},\mathbf{2}), \quad (5)$$

$$\mathbf{20} = (\mathbf{8},\mathbf{2}) \oplus (\mathbf{1},\mathbf{4}), \quad (6)$$

For mesons we get: $$\mathbf{6} \otimes \overline{\mathbf{6}} = \mathbf{35} \oplus \mathbf{1}, \quad (7)$$

where the decomposition with respect to SU(3)⊗SU(2) subroup reads:

$$\mathbf{35} = (\mathbf{8},\mathbf{1}) \oplus (\mathbf{8},\mathbf{3}) \oplus (\mathbf{3},\mathbf{1}). \quad (8)$$

- SU(6) symmetry ⇒ wave functions of all baryons populating the **56**-plet, must be *fully symmetric* under any permutation of the constituent quarks.
- One way is to combine the known properties of SU(3) (flavor) and SU(2) (spin) subgroups. It is straightforward to show that the following expression for the states of the baryon **octet**

$$\Phi(\mathbf{8},\mathbf{2}) = \frac{1}{\sqrt{2}}\left(\Phi^{(8)}_{m,s}\Psi^{(2)}_{m,s} + \Phi^{(8)}_{m,a}\Psi^{(2)}_{m,a}\right) \quad (9)$$

is indeed *fully* symmetric with respect to any permutation of the quarks in the direct product (4).

SU(6) symmetry

- For the multiplet (**10**,**4**) the fully symmetric wave functions are simply products of corresponding SU(3) (flavor) decuplet and SU(2) (spin) quartet wave functions, which each *separately* are symmetric. In Table (1) the wave functions of all baryons in the octet with spin 1/2 are listed.

	$\Phi^{(8)}_{(m,s)}$	$\Phi^{(8)}_{(m,a)}$
p	$\frac{1}{\sqrt{6}}[(ud+du)u-2uud]$	$\frac{1}{\sqrt{2}}[(ud-du)u]$
n	$\frac{1}{\sqrt{6}}[(ud+du)d-2ddu]$	$\frac{1}{\sqrt{2}}[(ud-du)d]$
Σ^+	$\frac{1}{\sqrt{6}}[(us+su)u-2uus]$	$\frac{1}{\sqrt{2}}[(us-su)u]$
Ξ^0	$\frac{1}{\sqrt{6}}[(us+su)u-2ssu]$	$\frac{1}{\sqrt{2}}[(us-su)s]$
Σ^-	$\frac{1}{\sqrt{6}}[(ds+sd)d-2dds]$	$\frac{1}{\sqrt{2}}[(ds-sd)d]$
Ξ^-	$\frac{1}{\sqrt{6}}[(ds+sd)s-2ssd]$	$\frac{1}{\sqrt{2}}[(ds-sd)s]$
Σ^0	$\frac{1}{2\sqrt{3}}[s(ud+du)+(dsu+usd)-2(du+ud)s]$	$\frac{1}{2}[(dsu+usd)-s(du+ud)]$
Λ	$\frac{1}{2}[(dsu-usd)+s(du-ud)]$	$\frac{1}{2\sqrt{3}}[s(du-ud)+(usd-dsu)-2(du-ud)s]$

Table 1: The flavor part of wave functions of the baryon octet.

SU(6) symmetry

- In fact most of the expressions for the wave functions in Table 1 can be obtained exploiting the isospin and other two SU(2) subgroups of SU(3) and taking into account the following decomposition

$$\mathbf{2} \otimes \mathbf{2} = \mathbf{3}_s \oplus \mathbf{1}, \quad \mathbf{2} \otimes \mathbf{2} \otimes \mathbf{2} = \mathbf{4}_s \oplus \mathbf{2}_{m,s} \oplus \mathbf{2}_{m,a} \tag{10}$$

- To construct the fully symmetric quartet $\mathbf{4_s}$ as well as the remaining doublets of definite symmetry from the three fundamental doublets is a simple exercise in the Clebsh-Gordan coefficients (exercise 3.1).
- Note a difference between the proton wave function and that of the Δ^+ resonance although both these states have the same quark content *uud*. In the following we use the notation in which the third projection of the spin of a fermion (baryon or quark) is labeled by vertical arrow ($\uparrow$ for $\frac{1}{2}$ and $\downarrow$ for $-\frac{1}{2}$) to distinguish them from the third projection of the isospin.
- First 6 entries in Table are generalizations of the nucleon (p,n) wave functions, using three distinct SU(2) subgroups of SU(3), based on the pairs of (u, d), (u, s) and (d, s) quarks respectively.

SU(6) symmetry

- 2 states in the middle of the baryon octet require subtler arguments. They are both composed of the same combination (u, d, s) of quarks, but differ by SU(3) and SU(2) flavor quantum numbers:
  - The wave function of Σ^0 can be obtained from Σ^+ by application of the lowering operator $E_{-(\alpha^1+\alpha^2)}$, corresponding to the isospin subgroup.
  - the wave function of Λ is determined by the requirement that it has isospin 0, and must therefore be a linear combination of the following terms:
$$\frac{1}{\sqrt{2}}[(ud - du)s]; \quad \frac{1}{\sqrt{2}}[s(du - ud)]; \quad \frac{1}{\sqrt{2}}[dsu - usd], \qquad (11)$$
combined with the condition that it is *orthogonal* to the combination describing Σ^0 as well as to the SU(3) singlet:
$$\frac{1}{\sqrt{6}}(uds - dus + dsu - sdu + sud - usd) \qquad (12)$$
- The explicit expression for the wave function of the proton with spin projection $\frac{1}{2}$, displaying both flavor and spin structure, reads:

$$\begin{aligned} \mid p, \uparrow\rangle \quad = \quad & \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{6}} \mid (ud + du)u - 2uud\rangle \frac{1}{\sqrt{6}} \mid (\uparrow\downarrow + \downarrow\uparrow) \uparrow -2 \uparrow\uparrow\downarrow\rangle \right. \\ & \left. + \frac{1}{\sqrt{2}} \mid (ud - du)u\rangle \frac{1}{\sqrt{2}} \mid (\uparrow\downarrow - \downarrow\uparrow) \uparrow\rangle \right]. \qquad (13) \end{aligned}$$

Quarks with flavor and spin: magnetic moment

- The decompositions (3) and (6) imply that the baryon octet with spin 1/2 can be placed not only in fully symmetric 56-plet, but also into the fully antisymmetric 20-plet.
- For instance, fully antisymmetric wave function of the proton is:

$$\begin{aligned} |\, p, \uparrow\rangle = \tfrac{1}{\sqrt{6}} (|\, uud\rangle\, |\uparrow\downarrow\uparrow\rangle - |\, uud\rangle\, |\downarrow\uparrow\uparrow\rangle - |\, udu\rangle\, |\uparrow\uparrow\downarrow\rangle + \\ |\, udu\rangle\, |\downarrow\uparrow\uparrow\rangle + |\, duu\rangle\, |\uparrow\uparrow\downarrow\rangle - |\, duu\rangle\, |\uparrow\downarrow\uparrow\rangle) \end{aligned} \quad (14)$$

  and by interchanging $p \leftrightarrow n$ similarly for the neutron.
- Any of these wave functions can now be used to calculate one of the basic static properties of baryons, their magnetic moments. Table 2 contains the present experimental situation for baryons.
- In the NRQM the operator of the magnetic moment of a *pointlike* fermion with spin 1/2, electric charge (in units of positron charge) Z and mass m is defined as

$$\hat{\vec{\mu}} \equiv \frac{Ze}{2m}\vec{\sigma} = \frac{Ze}{m}\vec{s} \quad (15)$$

  where e is the positron charge in absolute units and $\vec{\sigma} \equiv (\sigma_1, \sigma_2, \sigma_3)$ is the vector of Pauli matrices.

Quarks with flavor and spin: magnetic moment

- The magnetic moment of a given fermion with spin $\frac{1}{2}$ is then defined as the expectation value of the, say, third component of (15) in the state with the projection of the spin pointing in this third direction.
- For pointlike fermions this expectation value is given simply as $Ze/2m$, while for particles with internal structure the situation is more complicated, as is clear from an example of the neutron, which has nonvanishing, negative magnetic moment, despite the fact that it has zero total electric charge.
- Assuming SU(6) symmetry of the baryon octet (i.e. $\Delta m = 0$, poor approximation outside isospin multiplets) and treating its members as pointlike fermions with spin $1/2$ we get for the baryon B of the mass m_B

$$\mu_B = |\vec{\mu}_B| \equiv \mu_N \frac{m_p}{m_B} \langle B, \uparrow| \, Q\sigma_3 \, | \, B, \uparrow\rangle, \quad \mu_N \equiv \frac{e}{2m_p} \tag{16}$$

  $Q = T_3 + Y/2$ is the operator of electric charge, $Q \epsilon SU(3) \subset SU(6)$.

- Consequently the operator $Q\sigma_3$ is an element of SU(6) algebra and thus its matrix elements in the baryon octet states are *related*.

Quarks with flavor and spin: magnetic moment

- Knowing how $Q\sigma_3$ acts on any state from $\mathcal{H}_{flavor} \otimes \mathcal{H}_{spin}$ we can evaluate its matrix elements in the wave functions of Table 1:

$$\langle p,\uparrow|\, Q\sigma_3 \,|\, p,\uparrow\rangle \;=\; \frac{4Q_u - Q_d}{3} = 1, \quad (17)$$

$$\langle n,\uparrow|\, Q\sigma_3 \,|\, n,\uparrow\rangle \;=\; \frac{4Q_d - Qu}{3} = -\frac{2}{3}, \quad (18)$$

$$\langle \Lambda,\uparrow|\, Q\sigma_3 \,|\, \Lambda,\uparrow\rangle \;=\; Q_s = -\frac{1}{3} \quad (19)$$

- First two numbers are in excellent agreement with experiment (i.e. they reproduce *ratio* of the measured magnetic moments of the proton and the neutron). For Λ due to significant mass difference between the nucleon doublet and Λ agreement is much worse.
- N.B. If baryon wave functions were given by the *ma* or *ms* parts of (9), the r.h.s. of (17-19) would equal Q_u, Q_d and Q_s, respectively. Assuming isospin symmetry ⇒ $\mu_n/\mu_p = -1/2$, in clear disagreement with experiment.
- So magnetic moments of baryons provide an independent strong argument for assigning the baryons into a fully symmetric **56**-plet of the SU(6) group.

Quarks with flavor and spin: magnetic moment

- In constituent quark model most of the SU(3) and SU(6) symmetry breaking is attributed to differences in quark masses. In this model not baryons, but constituent quarks u, d, s behave like a pointlike Dirac fermions, with masses m_u, m_d, m_s and magnetic moment operator $e_q\sigma_3/2m_q$.
- Isospin symmetry implies $m_u \doteq m_d \equiv m$, but as the SU(3) symmetry is violated in masses of hadrons at the level of a few hundreds of MeV, we expect similar difference between m_s and m.
- Under this assumptions magnetic moment of a baryon B (polarized in "third" direction) is then given simply as a sum of the magnetic moments of its constituent quarks:

$$\mu_B \equiv \sum_{u,d,s} \mu_q = \sum_{q=u,d,s} \langle B,\uparrow|\ \mu_3^q\ |\ B,\uparrow\rangle. \tag{20}$$

Quarks with flavor and spin: magnetic moment

- Using the explicit expressions for baryon wave functions given in Table 3.1, the above formula allows us to express baryon magnetic moments in terms of those of u, d and s quarks:

$$\begin{array}{rclrclrcl} \mu_p &=& (4\mu_u - \mu_d)/3, & \mu_n &=& (4\mu_d - \mu_u)/3, & \mu_\Lambda &=& \mu_s, \\ \mu_{\Sigma^+} &=& (4\mu_u - \mu_s)/3, & \mu_{\Sigma^-} &=& (4\mu_d - \mu_s)/3, & \mu_{\Sigma^0} &=& (2\mu_u + 2\mu_d - \mu_s) \\ \mu_{\Xi^0} &=& (4\mu_s - \mu_u)/3, & \mu_{\Xi^-} &=& (4\mu_s - \mu_d)/3, & \mu_{\Omega^-} &=& 3\mu_s. \end{array} \quad (21)$$

- Solving the first three equations for μ_u, μ_d, μ_s we get (in units of μ_N)

$$\mu_u = +1.852; \quad \mu_d = -0.972; \quad \mu_s = -0.613, \quad (22)$$

corresponding to $m_u = 338$ MeV, $m_d = 332$ MeV and $m_s = 510$ MeV. Quark masses obtained in this way are called **constituent** masses.

Quarks with flavor and spin: magnetic moment

- Using μ_u, μ_d and μ_s we can predict magnetic moments of five other baryons for which measurements are available. The results, displayed in Table 2, show a good, though not perfect, agreement with the data.

p	n	Λ
2.793	-1.913	-0.613±0.004
input	input	input

Σ^+	Σ^-	Ξ^0	Ξ^-	Ω
2.458±0.010	-1.160±0.025	-1.25±0.014	-0.651±0.003	-2.02±0.05
2.674	-1.092	-1.435	-0.494	-1.839

Table 2: Current status of experimental determination (first row) of and theoretical predictions (second row) for baryon magnetic moments in units of μ_N. N.B. Magnetic moments of the proton and neutron are known with large accuracy ($2 \cdot 10^{-9}$% and $2 \cdot 10^{-8}$% respectively).

Quarks with flavor and spin: magnetic moment

- N.B. Assigning the baryon octet to the fully antisymmetric 20-plet would lead to gross disagreement with data. For instance, the wave function of the proton given in (14) would imply

$$\mu_p = \mu_d < 0, \quad \mu_n = \mu_u > 0, \quad \Rightarrow \frac{\mu_p}{\mu_n} = \frac{\mu_d}{\mu_u} = -\frac{1}{2}, \tag{23}$$

  i.e. predicting thus both wrong signs and wrong magnitudes. Measurement of the magnetic moments of the baryon octet thus provides strong support for the assignment of baryons to the fully symmetric 56-plet of SU(6).

- $\Delta m \equiv m_u - m_s \doteq 180$ MeV is consistent with mass differences between isotopic mutliplets containing different number of strange quarks. In the additive quark model the proton therefore looks like a nucleus, with most of its mass concentrated in the masses of its weakly bound **constituent quarks**, each having the mass of about 330 MeV.

Spin structure of the baryons

- For baryon in a state with definite spin projection, say $\mid B, \uparrow\rangle$ introduce probabilities $P_q^B(\upuparrows)$, $P_q^B(\uparrow\downarrow)$ of finding in this state a quark q with the spin *parallel* or *antiparallel* to the spin of the baryon B.
- P_q^B are normalized to number of constituent quarks of the flavor q in the baryon B:
$$P_q^B(\upuparrows) + P_q^B(\uparrow\downarrow) = N^B(q). \quad (24)$$
- In terms of $\Delta^B(q)$ – fraction of the spin of the baryon B carried by the constituent quark q – we can write the sum rule: Spins of all quarks q add up to the spin of the baryon B:
$$\Delta^B(q) \equiv P_q^B(\upuparrows) - P_q^B(\uparrow\downarrow), \ \sum_q \Delta^B(q) = 1. \quad (25)$$
- P_q^B can be calculated using expressions for the baryon wave functions constructed above.
$$P_u^p(\upuparrows) = \tfrac{5}{3}, \ \ P_u^p(\uparrow\downarrow) = \tfrac{1}{3}, \ \ P_d^p(\upuparrows) = \tfrac{1}{3}, \quad (26)$$
$$P_d^p(\uparrow\downarrow) = \tfrac{2}{3} \Rightarrow \Delta^p(u) = \tfrac{4}{3}, \ \Delta^p(d) = -\tfrac{1}{3}, \quad (27)$$

  i.e. the up and down quarks carry the entire spin of the proton. This *contradicts with DIS* measurements as will be discussed in the next Chapter on the **parton model**. *This topic is currently one of the most interesting open problems in particle physics.* (STAR).

The Zweig rule

- Basic idea of OZI (Okubo, Zweig and Iizuka) rule is depicted in Fig. 2 which shows the so called "quark flow" diagrams for several decay channels of vector mesons ρ^0, ϕ as well as for process

$$\pi^+ + p \to \pi^+ + \pi^0 + p. \quad (28)$$

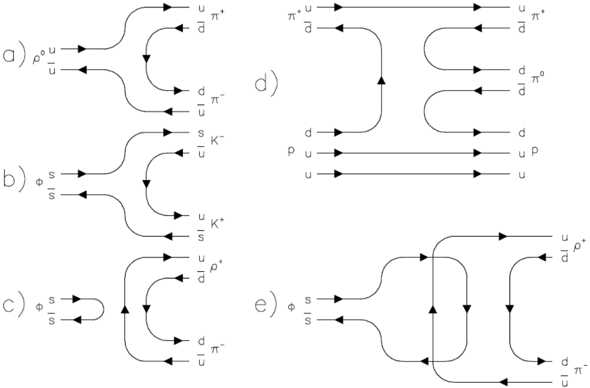


Figure 2: Examples of the OZI rule: the decay of ϕ meson and $\pi - N$ scattering. Solid lines describe the "flow" of quarks with definite flavour from the initial to final states or back.

The OZI rule

- All the processes can be divided into two classes:
  - **Zweig allowed** ones: those for which the *planar** flow diagrams are **connected**, i.e. the diagram cannot be separated into two parts without cutting some quark lines, like in Fig. 2a,b,d.
  - **Zweig forbidden** ones: those for which the diagrams are **disconnected**, like in Fig. 2c.
- Z.–forbidden processes are *suppressed* with respect to Z.–allowed ones. Best illustration is ϕ decay. From Z. rule and Fig. 2 we conclude that despite the fact that the K^+K^- channel is barely open (only about 30 MeV left for E_k of its decay products) it still dominates over $\rho\pi$ channel (with about 130 MeV left) by a factor of 40: $\phi \to K^+K^-; \quad \phi \to \rho^+\pi^-$ (29)
- Application of this rule to the decay of J/ψ meson played a crucial role in the discovery of the **charmed** quark (see the next Section).
- "Returning" lines in the quark diagrams like that in Fig. 2c can be interpreted as those in which some of the quarks *annihilate* to gluons (see later).

(*)diagrams are drawn in the plane and that there are *no crossings* of quark lines. By allowing crossings we could make any disconnected diagram connected simply by crossing some quark lines there and back, as shown in Fig. 2e.

Problems and puzzles of the Quark Model

Despite its overall success MGM GZ quark model has several serious problems:

- Quarks are **fermions** and their wave functions should therefore be *antisymmetric* with respect to the interchange of *all* their characteristics (space-time as well as internal) but the baryons were assigned to the *fully symmetric* **56**-plet of SU(6).
- Quarks had not been observed in nature as free particles. Neither had there been any indications for the existence of other "exotic" states, like those of the **sextet**, which would be populated by symmetric diquark combinations.
- There are also no signs of the existence of particles, which would correspond to states like $2q2\overline{q}$, or $4q\overline{q}$, formed by combinations of "allowed" states $q\overline{q}$ and 3q.

Problems and puzzles of the Quark Model

There were several suggestions how to solve the first, most pressing problem - that of quark statistics.

1. If quarks do not exist as free particles, the problem can simply be ignored. This was the attitude of Gell-Mann.
2. The spatial parts of the ground state wave functions could be antisymmetric. Although in principle possible, no realistic model of forces leading to such wave functions has been constructed.
3. In another attempt at the conventional explanation Sakita [**?**] suggested to assign the baryon octet to the fully antisymmetric 20-plet of SU(6), as in (14). This solves the statistics problem for the octet of baryons with spin $1/2$, but not for the decuplet of baryons with spin $3/2$. As shown in (23) it, moreover, leads to completely wrong values of baryon magnetic moments.
4. Very unconventional solution of the statistics problem had been proposed by O. Greenberg (and also by N. Bogolubov, B. Struminsky, A. Tavkhelidze) who assumed that quarks are not fermions, but the so called *parafermions* of rank three. This solved the statistics problem because there can be at most three such parafermions in each state.

Problems and puzzles of the Quark Model

- It became soon evident that the idea of quark parastatistics is similar, though not quite equivalent, to assuming that each quark flavour exists in **three** different **color** states* and the observed hadrons correspond to **white**, i.e. **colorless**, combinations.
- In the language of group theory quarks transform like the fundamental triplet of a *new* SU(3) group, called **color** SU(3) and denoted for distinction $SU_c(3)$. The observed hadrons are postulated to be **color singlets**.
- The wave functions of all baryons are thus of the form

$$| \,\mathrm{baryon}^{\alpha\beta\gamma}\rangle = \underbrace{\epsilon^{ijk} \mid q_i^{\alpha}\rangle \mid q_j^{\beta}\rangle \mid q_k^{\gamma}\rangle}_{\text{totally antisymmetric}}, \tag{30}$$

where $i, j, k = 1, 2, 3$ are the *color* indices while α, β, γ specify the *flavour*.

(*) Parastatistics corresponds to SO(3) group, rather than SU(3) as does color. It was A. Pais, who first used the terms "green", "red" and "yellow" to denote the three different states of a given quark flavor .

Problems and puzzles of the Quark Model

- As an example let us write down explicitly the wave function for the Δ^{++}:

$$\begin{aligned} |\,\Delta^{++}\rangle \;\; &= \;\; |\,u_1\rangle\,|\,u_2\rangle\,|\,u_3\rangle - |\,u_2\rangle\,|\,u_1\rangle\,|\,u_3\rangle + |\,u_2\rangle\,|\,u_3\rangle\,|\,u_1\rangle - \\ & \quad |\,u_3\rangle\,|\,u_2\rangle\,|\,u_1\rangle + |\,u_3\rangle\,|\,u_1\rangle\,|\,u_2\rangle - |\,u_1\rangle\,|\,u_3\rangle\,|\,u_2\rangle. \end{aligned} \quad (31)$$

- *To make color singlets for baryons, we need at least as many colors as there are quarks from which they are composed.* No diquark is therefore observable, but in principle the color singlet hypothesis doesn't by itself rule out the existence of states like $4q\overline{q}$ mentioned above.
- For mesons there is no such limitation, as for any number of colors the direct product of the fundamental representation and its complex conjugate contains the singlet **1**. Although the hypothesis of hadrons as color singlets proved extremely fruitful, it had not truly solved the puzzles of the quark model, but merely recast them into another question: **why are there only color singlet states in the nature?**

Problems and puzzles of the Quark Model

- The crucial step towards answering this question was taken by Yochiro Nambu in early 1965, shortly after the appearance of Greenberg's paper: Quark confinement follows from dynamics of what he called "superstrong interactions" between the "fundamental objects"*, i.e. quarks.
- The really fundamental ingredient of his model, which was phrased in terms of non–relativistic approximation of the corresponding field theory, was the assumption that the "super–strong force" acting between quarks is .... *mediated by an octet of gauge fields* $G_\mu, \mu = 1, ..., 8$, *coupled to the infinitesimal SU(3 generators (currents)* λ_μ *of the triplets, with the strength* g.

	u	d	s
Q_1	1	0	0
Q_2	1	0	0
Q_3	0	−1	−1

Table 3: Charge assignment in Hahn-Nambu model.

(*) Nambu was not aware of Greenberg's paper. Neither did he use the words "quark" or "ace" for members of his fundamental triplets.

Problems and puzzles of the Quark Model

- By introducing 8 gauge fields, Nambu had fully exploited the nonabelian nature of the underlying color SU(3) symmetry of quarks! By assuming that the force acting between quarks is mediated by 8 gauge bosons of color SU(3) symmetry, which themselves carry the color, Nambu had clearly laid down the foundations of QCD 8 years before its definite formulation as QFT.
- From the point of view of solving the problem of quark statistics this, however, was not necessary, as one could do with just one gauge field. Note that the above statement represents the explicit formulation of the essence the present day QCD!
- For Nambu his super–strong force was responsible for quark confinement and the exchange 8 gauge fields (i.e. gluons in QCD) was motivated by the fact that *For a system containing altogether N particles, the exchange of such fields between a pair then results in an interaction energy* that allowed him to explain why only color singlet states may exist in nature.

Problems and puzzles of the Quark Model

- Unfortunately, more than with the introduction of nonabelian nature of strong force, Nambu's name is connected with the so called Hahn-Nambu model of colored quarks with integral electric charges.
- In this model quarks with a given flavor but different colors were assigned different integer electric charges in such a way that their color-averaged values were equal to fractional electric charges of the Gell-Mann-Zweig quark model.
- The Hahn-Nambu model differed from the latter in several aspects (for instance, some of the gauge fields carried electric charge) but was identical to it as far strong interactions were concerned.
- Consequently, the fact that it has since been ruled out by experiments on deep inelastic scattering of leptons on nucleons and other processes to be described in the next Chapter, does nor change the fact that Nambu's model contains the very fundamental ingredient of QCD!

Color to the rescue: Quasinuclear colored model of hadrons

- Basic idea of color confinement was introduced by Nambu in the framework of NR QM. Interaction potential follows from assumption that the interaction is mediated by the exchange of the octet of colored gauge bosons and has, as we shall see, the following properties:
  - Quarks as individual "particles" are *infinitely* heavy and thus not observable.
  - The forces acting between quarks are *attractive* in color *singlet* channels, resulting in bound systems of *finite* mass.
  - In all other channels the forces are repelling and the systems thus infinitely heavy and unobservable.
  - The force $F_{q\bar{q}}$ between a $q\overline{q}$ pair in a meson is *twice* bigger than the force F_{qq} acting between each of three pairs of quarks inside any baryon.
- The binding energy thus *cancels* the infinite masses of constituent quarks in color singlets, as we want, but leaves the other states too heavy to be observable*.

(*) This model doesn't in fact rely on actual infiniteness of free quark mass M_q. What is essential is the fact that M_q must be *large* compared to currently accessible energies so that free quarks cannot be observed even if the confinement were not exact.

Nambu's Quasinuclear colored model of hadrons

- The interquark potential is extension of a typical spin-spin interaction, where the forces between any system of fermions and antifermioms are described by two-body potentials $V_{ij}(r)$, acting between all pairs i, j of quarks, of the form

$$V_{ij} = v(\vec{r}_{ij})\vec{s}_i\vec{s}_j, \tag{32}$$

  where $\vec{s}_i$ are the usual spin matrices of SU(2) and the functions $v(\vec{r}_{ij})$ contain the dependence on space coordinates as well as all other quantum numbers except the spin.

- The full potential energy of a system of n quarks and antiquarks is then given as

$$V(n) = \sum_{i<j}\langle n \mid V_{ij} \mid n\rangle = \frac{v}{2}\langle n \mid \left[\left(\sum_{i=1}^{n}\vec{s}_i\right)^2 - \sum_{i}^{n}\vec{s}_i^2\right] \mid n\rangle = \frac{v}{2}\left[s(s+1) - \frac{3}{4}n\right], \tag{33}$$

  where $\mid n\rangle$ denotes the state vector of the system, $v \equiv \langle\mid v_{ij}(r_{ij})\mid\rangle$ is the mean value of the interquark potential taken in the space coordinates, s is total spin of the system and $3/4 = (1/2)(1/2+1)$ is just the square of the spin of each individual quark.

Nambu's Quasinuclear colored model of hadrons

- Analogously, introducing the *color* interaction in the form (λ_i are the familiar Gell-Mann matrices): $$\frac{1}{8}\sum_{i\neq j}^{n} v(\vec{r}_{ij})\vec{\lambda}_i\vec{\lambda}_j \quad (34)$$
  the full potential energy of a system is: $$V(n) = \frac{v}{2}(C - nc), \quad (35)$$
  where C and c are the eigenvalues of the **Casimir operator**
- Casimir operators are quadratic, cubic or higher order forms constructed from the generators of a given Lie algebra, which *commute* with all its elements $\Rightarrow$ they are **invariants** of any multiplet. The full set of these invariants can be used as another way of specifying the multiplets.
- In SU(3) there are *two* Casimirs, (36) and a cubic one, which, however, is much more complicated and has no simple use in the quark model.
  $$\mathbf{C} \equiv \sum_{a=1}^{8} T^a T^a \quad (36)$$
- In a given multiplet (p, q):
  $$C(p,q) \equiv \langle n \mid \mathbf{C} \mid n\rangle = \langle T_3^2\rangle + 2\langle T_3\rangle + \frac{3}{4}\langle Y\rangle^2 = \frac{1}{3}(p^2 + pq + q^2) + (p+q). \quad (37)$$

Nambu's Quasinuclear colored model of hadrons

	SU(3)	**C**	**C**-nc	**C**/**c**
q	triplet	4/3	0	1
$\overline{\mathrm{q}}$	antitriplet	4/3	0	1
qq	antitriplet	4/3	-4/3	1
qq	sextet	10/3	2/3	5/2
q$\overline{\mathrm{q}}$	singlet	0	-8/3	0
q$\overline{\mathrm{q}}$	octet	3	1/3	9/4

Table 4: The values of $C(p, q)$ for several important multiplets of SU(3)

- Expressing the total mass as the sum over M_q of and potential energy (35)

$$M(n) = nM_q + V(n) = n\left(M_q - \frac{1}{2}cv\right) + \frac{1}{2}Cv \qquad (38)$$

  the infinite quark masses M_q are seen to cancel by potential energy $V(n)$ provided $v = 2M_q/c$ and

$$M(n) = C\frac{v}{2} = \frac{C}{c}M_q. \qquad (39)$$

- Taking into account the dependence of $v(r_{ij})$ on its arguments but still assuming that it is the same for all pairs i, j leads to the same results, except for the fact that v in (39) and other equations is now the mean value of $v(r)$ in the wave function of the corresponding color multiplet.

Nambu's Quasinuclear colored model of hadrons

The resulting formula (39) has all the properties we need:

- In color singlet states $C = 0$ implying vanishing total mass* of the system, which makes it *observable*.
- In all nonsinglet channels, on the other hand, $C \neq 0$ and thus the system has a mass proportional to M_q. This makes it as unobservable as the quarks themselves.
- The forces between $\mathrm{q}\overline{\mathrm{q}}$ and qq pairs in mesons and baryons are in the relation: $F_{q\overline{q}} = 2F_{qq}$.

This simple model demonstrates the mechanism of color confinement in nonrelativistic quantum mechanics. It is much more difficult to show that it naturally follows from interaction of colored quarks in the relativistic quantum field theory as well. There are indications, based on numerical calculations in lattice gauge theory, that Quantum Chromodynamics, does, indeed, have this property.

(*)The potential doesn't have to cancel the whole mass M_q. What is important is that the remaining uncanceled part is *finite*.

- For the families of hadrons composed entirely of *light quarks*, there exists a relation between J and M^2 for Regge trajectories is given by:

$$J(M^2) = \alpha_0 + \alpha' M^2 \quad (40)$$

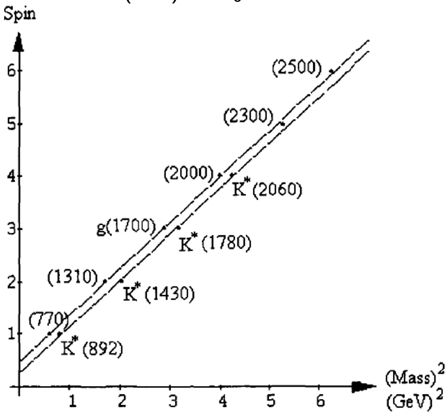
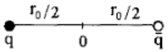


Figure 3: Chew-Frautschi plot showing the Regge trajectories for non-strange (I = 1) and strange (I = 1/2) bosons

Regge trajectories and $a\bar{q}$ strings

- Consider a massless (and for simplicity spinless) quark and antiquark connected by a string of length r_0, which is characterized by an *energy per unit length* σ.



  - For a given value of length r_0, the largest achievable angular momentum J occurs when the ends of the string move with the velocity of light.
  - In these circumstances, the speed at any point along the string at a distance r from the center will be:

$$v/c \equiv \beta(r) = 2r/r_0 \tag{41}$$

  - The total mass of the system and the orbital angular momentum of the string is:

$$\left.\begin{array}{l} M = 2\int_0^{r_0/2} \frac{dr\sigma}{\sqrt{1-\beta(r)^2}} = \sigma r_0 \frac{\pi}{2} \\ J = 2\int_0^{r_0/2} \frac{dr\sigma r\beta(r)}{\sqrt{1-\beta(r)^2}} = \sigma r_0^2 \frac{\pi}{8} \end{array}\right\} \Rightarrow J = \frac{1}{2\pi\sigma} M^2 \tag{42}$$

  suggesting that at a separation of the order of 1 fm, we may characterize the interquark interaction by the *linear potential*

$$V(r) = \sigma r \tag{43}$$

The arrival of charm

- Shortly after formulation of quark mode with three quark flavors u, d and s theorists started to speculate about the possible existence of the fourth, named *charm* by Bjorken and Glashow.
- Original argument – symmetry between quarks and leptons. In 1962 Lederman, Schwartz and Steinberger found ν_μ. This suggested that there might be the fourth quark, that would complete the second generation of the fundamental fermions.
- More urgent reasons were due to problems in theory of weak interactions:
  - Strong suppression of the so called *changing neutral currents* (FCNC), i.e. processes like $K^+ \to \pi^+ e^+ e^-$ (BR$=2.7 \cdot 10^{-7}$) with respect to usual charge current process $K^+ \to \pi^0 e^+ \nu_e$ (BR$=0.048$).
  - The problem with the so called axial anomalies.
- Problems resolved by postulating c-quark with $Q = 2/3$ and $I_3 = 1/2$ which forms an isospin doublet with the s-quark. m_c must not be too large if its contributions to the FCNC processes should solve the first problem, which gave the upper bound on m_c of about 2 GeV.

1974 November revolution: Charm discovery

- First glance very probably seen already in 1970 in an "beam dump" (all hadrons were absorbed in the heavy target) experiment carried out at BNL AGS. Lederman and col. were searching for the so called "heavy photons" and studied the distribution of $M_{\mu^+\mu^-}$ of muon pairs produced in:

$$p + U \rightarrow \mu^+\mu^- + \text{anything}, \quad (44)$$

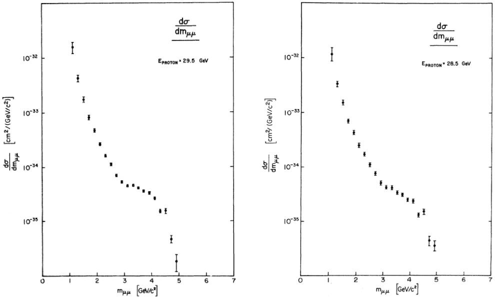


Figure 4: Lederman's $m_{\mu\mu}$ spectra

- The results of measurement at $E_p = 29.5$ and $E_p = 28.5$ GeV is reproduced on the left panel. Clear shoulder in the distribution of $m_{\mu\mu}$ is seen in the region 3 − 5 GeV, but poor mass resolution $\sim 0.1 m_{\mu\mu}$ prevented mode detailed study.

- Authors conclude that data *exhibit no resonant structure* and in their 1973 paper claim that the distinct excess may be due to the production of a *resolution-broadened resonance* but it may also be interpreted as *merely a departure from the overly simplistic and arbitrarily normalized* $1/m^5$ *dependence*.

1974 November revolution: Charm discovery

- The group of S.C. Ting repeated "heavy photons" search at BNL but measuring e^+e^- instead. They used magnetic double arm spectrometer with Č counters for reliable identification of e^+ and e^- ($\Delta m_{e^+e^-} \approx 5$ MeV) and used Be-target. His detector, and was ready in late 1974.
- Knowing Lederman's results Ting could concentrate on the interesting region of m_{ee} between 3 and 5 GeV and got fast the first results shown on the right.
- The spectrum showed a clear evidence for a narrow resonance at 3.1 GeV, which Ting called J, and which is now know as J/ψ. Its observed width was compatible with experimental resolution, which implied that its true width had to be much smaller.

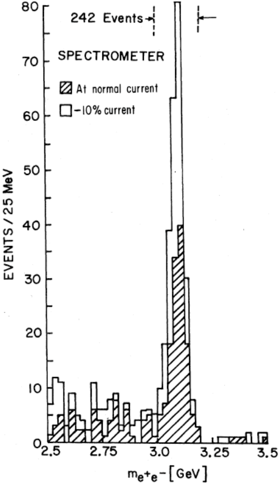


Figure 5: From J.J. Aubert, PRL 33 (1974)1404

- At about the same time first results from the new SPEAR e^+e^- collider at SLAC, commissioned in 1973, showed the rise of the total cross section for e^+e^- annihilation to hadrons (scaled by that of e^+e^- annihilation to $\mu^+\mu^-$ pairs) in the region up to $\sqrt{s} = 6$ GeV.
- The SLAC-LBL group lead by B. Richter found by a careful scan of the whole accessible region a peak in total cross section $\sigma(\mathrm{e^+e^-} \to \mathrm{hadrons})$ at 3.097 GeV, which they christened ψ.
- Zooming in on the nearby region, the SLAC-LBL group found soon also its first recurrence, called ψ', at 3.68 GeV and later the whole spectrum of states.

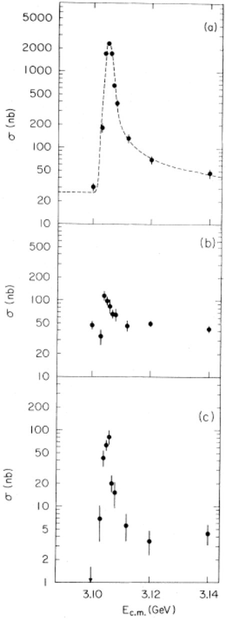


Figure 6: Cross section versus energy for (a) multihadron final states, (b) e^+e^- finals states, (c) $\mu^+\mu^-$, $\pi^+\pi^-$ and K^+K^- final states. From J.-E. Augustin *et al.*, PRL 33(1974)1406.

1974 November revolution: Charm discovery

- Interpretation of J/ψ (and ψ') as $c\bar{c}$ resonances was based on the remarkable fact that this heavy vector meson decays into hadrons via *strong* interactions about 1000 times slower than similar hadronic resonances like ρ etc. The unusually small width of only 70 keV (now this number stands at 87 keV) was explained by the OZI rule as a consequence of the fact that the *D*–mesons (not yet observed at the time of J/ψ discovery) are too heavy for J/ψ to decay via Zweig *allowed* channel into their pairs, and thus all decays have to go via Zweig *forbidden* ones, as illustrated in the lower left part of .

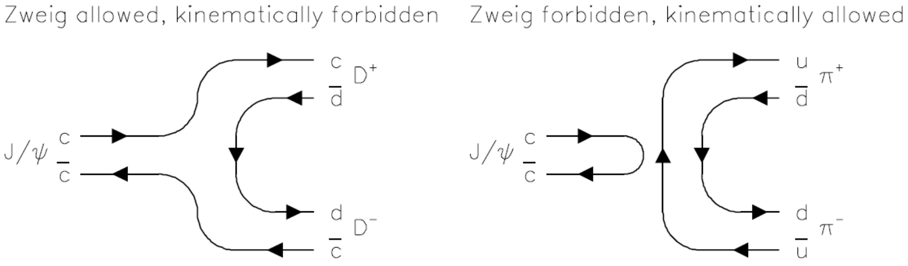


Figure 7: the OZI rule applied to decay of J/ψ

Open charm discovery

- Confirmation of the charm hypothesis as possible explanations of the observed J/ψ and ψ' had to wait until the early 1976, when *open charm mesons* were found at SLAC in reanalysis of older data. D, D^*, D_s, D_s^* are the bound states of c (or $\overline{c}$) with light quarks u, d and s or their antiquarks.
- $D^{\pm}$ were found as $D^{\pm} \to K^- \pi^+ \pi^+$ but not in a mode expected for conventional strange resonances, i.e. $D^{\pm} \to K^+ \pi^+ \pi^-$.
- *Charmed baryon* was found even earlier in νp collisions at BNL in early 1975. Hydrogen bubble chamber photograph, reminiscent of the discovery of Ω^-, in which three positive and two negative particles were accompanied by the unambiguous Λ, was convincingly interpreted as the process

$$\nu_\mu p \to \mu^- \Lambda \pi^+ \pi^+ \pi^+ \pi^- \tag{45}$$

which violates the sacred $\Delta S = \Delta Q$ rule of standard weak interactions. On the other hand such final states are expected if the primary process $\nu_\mu + d \to \mu^- + c$ or $\nu_\mu + s \to \mu^- + c$ creates the charm quark c which then hadronizes into charm mesons D or baryons Λ_c. As these hadrons contain the c-quark their weak decay proceeds as $c \to s + W^+$ and must thus contain strange particle K^- or $\overline{K}^0$ or Λ in the final state.

- With the discovery of the fourth quark the SU(3) flavor symmetry was extended into SU(4) one.

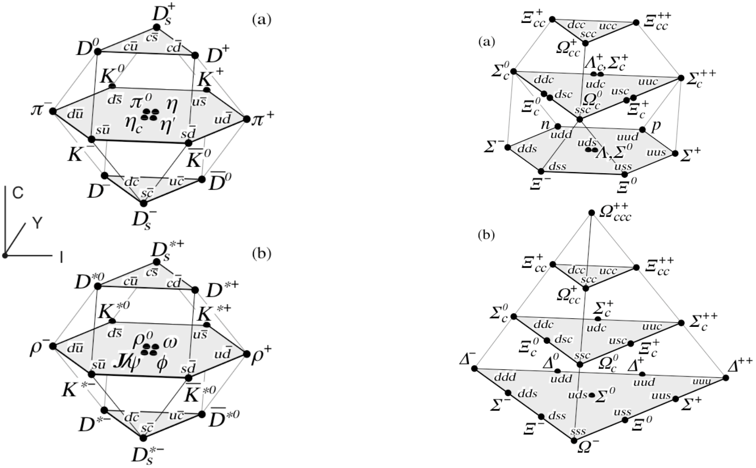


Figure 8: The basic SU(4) multiplets of mesons (left) and baryons (right) in the quark model.

SU(4)

There are many similarities between SU(3) and SU(4) groups, but there are several important differences as well

- SU(4) algebra has rank 3 $\Rightarrow$ its Cartan subalgebra has *three* elements. Their eigenvalues may be used to label the states of a given multiplet. Besides T_3 and Y, there is a generator C corresponding to the conserved charm quantum number.
- There are 3 simple roots and consequently *three* fundamental representations. One of them is the **defining** representation, **4**, formed by 4×4 matrices satisfying the appropriate commutation relations and describing the transformations of the basic quark quartet. Its complex conjugate representation, $\overline{\mathbf{4}}$, transforms the four antiquarks.
- Contrary to SU(3) there is *another* fundamental representation, which has no analogy in SU(3) group. This third fundamental quartet appears, for instance, in direct product of three quartets (see below) and is labeled $\mathbf{4}_{as}$ as it corresponds to fully antisymmetric combination of *three* quarks with *four* flavors. In SU(3) such an antisymmetric combination forms a *singlet*. The relation $Q = T_3 + \frac{1}{2}Y$ generalizes to $Q = T_3 + \frac{1}{2}(Y + C)$. (46)

SU(4)

- The most important direct products, describing systems of three quarks or $q\bar{q}$ pair read

$$\mathbf{4} \otimes \mathbf{4} \otimes \mathbf{4} = \mathbf{20}_s \oplus \mathbf{20}_{m,s} \oplus \mathbf{20}_{m,a} \oplus \mathbf{4}_{as}, \tag{47}$$

$$\mathbf{4} \otimes \overline{\mathbf{4}} = \mathbf{15} \oplus \mathbf{1}. \tag{48}$$

- The SU(4) multiplets can be graphically represented in three dimensional space by polyhedrons. According to (48) mesons form 15-plets, while baryons fill two kinds of 20-plets, a fully symmetric 20-plet $\mathbf{20}_s$ which contains the SU(3) baryon octet with spin 1/2 and another 20-plet, $\mathbf{20}_{m,s}$ with mixed symmetry, containing the SU(3) decuplet and corresponding to spin 3/2, see Fig. 8.
- Large mass splittings between charmed and noncharmed mesons and baryons indicate that except for the part of SU(4) symmetry responsible for the conservation of the charm quantum number in strong interactions, the rest of this symmetry is broken even more strongly than the SU(3) symmetry.

Charmonia

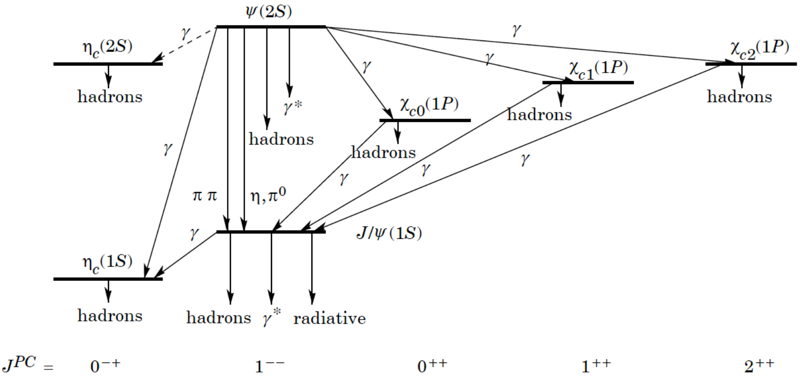


Figure 9: The spectrum of the lowest lying $c\bar{c}$ bound states.

NR QM description of Charmonia

- Discovery of c-quark has brought compelling evidence for the reality of quarks. The investigation of the properties of $c\overline{c}$ bound states, summarized in Fig 9, has shown that these states can be described to a good approximation by means of NR QM using the interquark potential $V(r)$ of the form
$$V(r) \equiv -\frac{4}{3}\frac{\alpha_s}{r} + \kappa r, \tag{49}$$
where α_s is the strong interaction coupling, analogous to α in Quantum Electrodynamics (see Chapter 7 for definition).
- The first part of (49), dominating at short distances, is motivated by perturbative QCD, whereas the second describes in a phenomenological manner the confinement.
- Obviously the heavy quarks behave in the way envisioned by Zweig. The discovery of charm with its rich spectrum of states that can easily be understood within NR QM has thus definitely buried the approach advocated by Gell-Mann. *Without treating quarks in a way reminiscent of nucleons inside nuclei, we would never be able to arrive at the predictions that agree so much with the experimental data!*

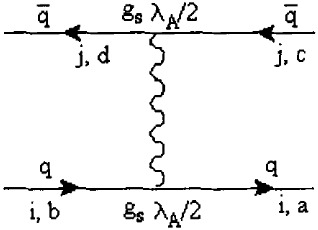


Figure 10: Diagram for one-gluon exchange potential for $q\bar{q}$ system.

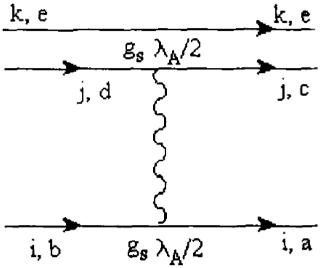


Figure 11: Diagram for one-gluon exchange potential for qqq.

$$V_{ij} = -\frac{g_s^2}{4\pi r}\sum_{A=1}^{8}\left(\frac{\lambda_A}{2}\right)_b^a \left(\frac{\lambda_A}{2}\right)_c^d \frac{1}{\sqrt{3}}\delta_a^c \frac{1}{\sqrt{3}}\delta_d^b \quad (50)$$

$\frac{1}{\sqrt{3}}\delta_a^c$ and $\frac{1}{\sqrt{3}}\delta_d^b$ in initial and final states arise due to normalized color singlet totally symmetric wave function for the $q\bar{q}$.

Recall $Tr(\lambda_A\lambda_B) = 2\delta_{AB},\ Tr(\lambda_A\lambda_A) = 16$

$$\Rightarrow V_{ij} = -\frac{4}{3}\frac{\alpha_s}{r},\ \alpha_s \equiv \frac{g_s^2}{4\pi} \quad (51)$$

$$V_{ij} = -\frac{g_s^2}{4\pi r}\frac{\varepsilon_{eac}}{\sqrt{6}}\frac{\varepsilon^{ebd}}{\sqrt{6}}\left(\frac{\lambda_A}{2}\right)_d^c \left(\frac{\lambda_A}{2}\right)_b^a \quad (52)$$

Factors $\frac{\varepsilon_{eac}}{\sqrt{6}}$ and $\frac{\varepsilon^{ebd}}{\sqrt{6}}$ arise because three-quark color wave function is totally antisymmetric in color indices. Using $\varepsilon_{eac}\varepsilon^{ebd} = \delta_a^b\delta_c^d - \delta_a^d\delta_c^b$

$$\Rightarrow V_{ij} = -\frac{2}{3}\frac{\alpha_s}{r} \quad (53)$$

NR QM description of Charmonia

- Thus we can write the two-body one-gluon exchange potential as

$$V_{ij} = k_s \frac{\alpha_s}{r}, \;\; k_s = \begin{cases} -\frac{4}{3} & q\bar{q} \\ -\frac{2}{3} & qq \end{cases} \qquad (54)$$

- N.B. Both in $q\bar{q}$ as well as in qq we get an attractive potential! For color singlet states $V_{ij}^{q\bar{q}} = 2V_{ij}^{qq}$.
- Since (running) coupling constant α_s becomes smaller as we decrease the distance, the effective potential V_{ij} approaches the lowest order one-gluon exchange potential given in (54) as $r \to 0$.
- We can write the potential for small distances ($r < 0.1$ fm) in momentum space, as*

$$V(\mathbf{q}^2) = \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} e^{-i\mathbf{q}\cdot\mathbf{r}} V_{ij} d^3q = k_s 4\pi\alpha_s/\mathbf{q}^2 \qquad (55)$$

(*)

$$\int_{-\infty}^{\infty} \frac{e^{i\mathbf{q}\cdot\mathbf{r}}}{\mathbf{q}^2} d^3q = 2\pi \int_0^{\infty} \int_0^{\pi} e^{i|\mathbf{q}|rcos\theta} \frac{1}{\mathbf{q}^2} |\mathbf{q}^2| d|\mathbf{q}| sin\theta d\theta$$

$$= \frac{4\pi}{r} \int_0^{\infty} \frac{sin|\mathbf{q}|r}{r} d|\mathbf{q}| = \frac{4\pi}{r} \int_0^{\infty} \frac{sinx}{x} dx = \frac{4\pi}{r} \frac{\pi}{2} = \frac{2\pi^2}{r}$$

Discovery τ lepton: Who has ordered it?

- For a few months after the discovery of J/ψ there seemed to be just two complete generations of fundamental fermions.
- However, already during 1975, evidence of new physics began to emerge from analysis of unlike sign dimuon events observed in $\mathrm{e}^+\mathrm{e}^-$ annihilations at SLAC by a group led by M. Perl. Events were interpreted as coming from the production of pairs of heavy leptons $\tau^\pm$, with mass of about 1.8 GeV, followed by their decays satisfying the universality of weak interactions:

$$\mathrm{e}^+\mathrm{e}^- \to \tau^+\tau^-, \quad \tau^- \to \nu_\tau \mu^- \overline{\nu}_\mu \quad \text{or} \qquad (56)$$
$$\tau^- \to \nu_\tau \mathrm{e}^- \overline{\nu}_e, \quad \tau^+ \to \overline{\nu}_\tau \mu^+ \nu_\mu \ \text{ or } \tau^+ \to \nu_\tau \mathrm{e}^+ \nu_e,$$

where ν_τ is the conjectured neutrino associated with the charged $\tau^\pm$.

- Experimentally the τ-lepton had been identified by the presence in the final state of combinations $\mu^+\mathrm{e}^-$ or $\mu^-\mathrm{e}^+$. The existence and identity of this third neutrino has finally been established only two years ago. The discovery of the τ-lepton opened the gates to the third generation and led to searches for its conjectured quark members.

To be or not to be? Υ discovery

- Using 400 GeV proton beam at Fermilab the group of L. Lederman was looking again for dilepton pairs produced in p+Be collisions. Employing double arm magnetic spectrometer ($\Delta m_{l^+l^-}$ was 2%) between January 1976 and September 1977 the group published 4 quite conflicting results.
- Reporting results on e^+e^- pairs they claimed to have found cluster of events with $5.8 < m_{ee} < 6.2$ suggesting *that the data contain a new resonance at 6 GeV*. Repeating their investigation using $\mu^+\mu^-$, they found no evidence for the 6 GeV resonance . . . .

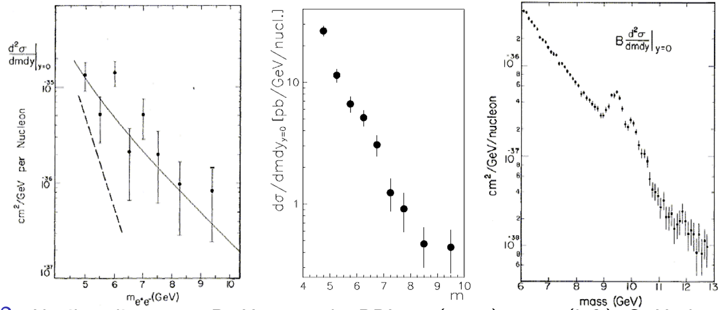


Figure 12: Upsilon discovery: D. Hom et al.: PRL 36 (1976), 1236 (left), S. Herb et al.: PRL 37 (1976), 1374 (middle), W. Innes et al.: PRL 39 (1977), 1240 (right).

To be or not to be? Υ discovery

- Improving their detector they had finally reported the clear and compelling evidence for two new heavy resonances at $m_{\mu\mu} = 9.41$ GeV and $m_{\mu\mu} = 10.06$ GeV, with $\Gamma \sim 1$ GeV, compatible with $\Delta m_{l^+l^-}$ of their detector.
- The states – Υ and Υ' – are actually very narrow: their widths, 52 and 44 keV respectively, are comparable to those of J/ψ and ψ'. Contrary to the charm family, there are, however, three narrow $b\bar{b}$ states, the third one, Υ'', at 10.44 GeV.
- Since then a lot of experimental attention has been paid to the measurement of the spectrum of $b\bar{b}$ bound states, the so called *bottomonia*. The results, summed up in Fig. 13, show a spectrum of states similar to that of charmonium, which turns out to be even better described within the framework of nonrelativistic quantum mechanics based on the potential (49) than the $c\bar{c}$ bound states.

Bottomonia

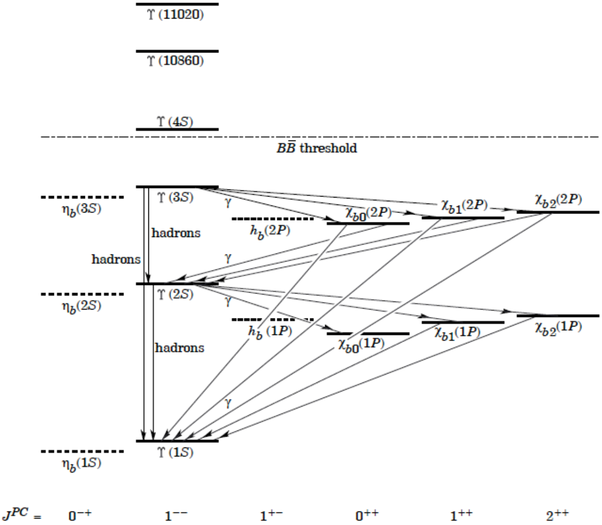


Figure 13: The spectrum of the lowest lying $b\bar{b}$ bound states.

Lone at the top

- Since the discovery of the fifth quark in 1977 the search went on to find the sixth quark, called *top*, which would complete the third generation. Many theorists tried to estimate its mass taking into account the known pattern of quark masses, but all failed to forecast its *huge value*.
- However, precise LEP measurements of the properties of the Z and W bosons combined with the theoretical predictions of the Standard Model (which depend on m_t through radiative corrections) had finally lead to the prediction of the top quark mass in the range $170 - 180$ GeV.
- This allowed experimentalists at Fermilab to narrow their search until they found it among complicated multijet final states originating from

$$q + \overline{q} \to t + \overline{t} \to b + \overline{b} + W^+ + W^-, \quad W_1 \to 2jets \;\; W_2 \to l + \nu \qquad (57)$$

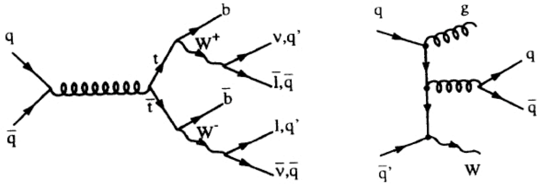


Figure 14: The basic mechanism of the top quark production at TEVATRON energies (left) and the main background process (right).

Lone at the top

- The search concentrated on the channel, called "lepton and jets", in which one of the W's decays leptonically and the other W into two jets. This process had in principle lead to four jets, two of them coming from the b-quark, and one charged lepton (with the accompanying neutrino remaining undetected).
- This channel, which accounts for roughly 35% of all final states has manageable background coming predominantly from Standard Model process of associated production of W and jets, described by diagrams like that in the right part of Fig. 14.
- Statistically significant excess of events with 3 and 4 jets and a charged lepton over the Standard model prediction, shown in Fig. 59, had been observed. Performing kinematic fits of the 7 events with four jets to the hypothesis of four jets (two of which had to be identified as b-quark jets by the presence of secondary vertex signalling the decay of a B-meson), one charged lepton and one massless missing particle (the associated neutrino) allowed the CDF Collaboration to determine the mass of the top quark in each event.

Lone at the top

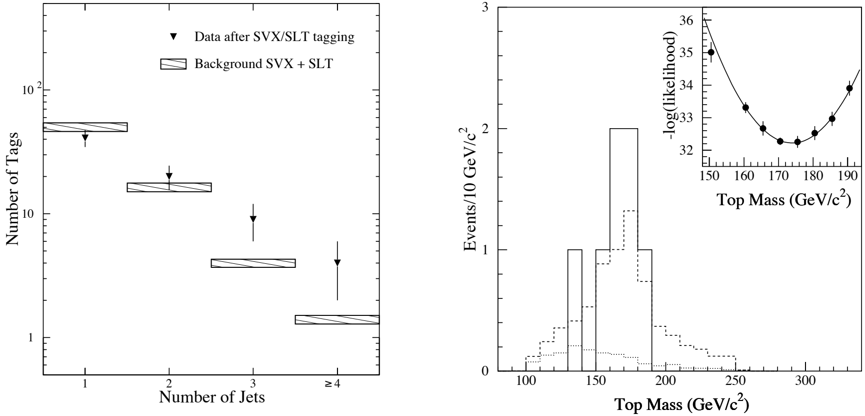


Figure 15: The first evidence for the top quark as observed by the CDF Collaboration at FNAL (F. Abe et al.: PRL 73(1994)225). Top mass distribution for the data (solid histogram), the W+jets background (dots) and the sum of background plus Monte Carlo $t\bar{t}$ for $M_{top} = 175$ GeV (dashed). The inset shows the logarithmic likelihood fit used to determine the top mass.

Lone at the top

- The resulting distribution, shown in the right part of Fig. 59, could best be fitted assuming the top quark mass to be $m_t = 174 \pm 10(stat) \pm 12(syst)$ GeV, remarkably close to the predicted value mentioned above. The discovery of the top quark was thus a triumph of the Standard Model.
- Compared to the lighter quarks, the top quark has one property that singles it out: due to its huge mass, its lifetime,

$$\tau_t \doteq \tau_\mu \left(\frac{m_\mu}{m_t}\right)^5 \doteq 2 \cdot 10^{-22} s \tag{58}$$

  is so short, in fact comparable to that of strongly decaying resonances, that it decays via weak interaction before it can hadronize!
- This implies there are no $t\bar{t}$ bound states, analogous to the rich spectrum of $c\bar{c}$ or $b\bar{b}$ states, or bound states of the top quark with the other quarks, similar to charmed (or bottom) mesons and baryons.
- Note that for c- or b-quarks the approximate formula (58) yields, taking into account the (small!) CKM matrix elements describing the transitions $b \to u + W^-$ and $b \to c + W^-$, lifetimes in the range of picoseconds.

Exercises

1. Derive (13).
2. Construct wave function of Σ^0 from that of Σ^+.
3. Prove (17) – (19).
4. Evaluate magnetic moments of all baryons in Table 3.2, assuming $m_u = m_d \equiv m$, but $m_s \neq m$.
5. Derive (27).
6. Show that the Casimir operator (36) commutes with all generators of SU(3).
7. Derive (37).