$\boxed{8.43}$ Veličina $A^\mu A_\mu$ je Lorentzovsky invariantní. Je invariantní kalibračně?

Lorentzovská invariance

$$A'^\mu = \alpha^\mu{}_\nu A^\nu \qquad A'^\mu A'_\mu = g'_{\mu\nu} A'^\mu A'^\nu = g'_{\mu\nu}\, \alpha^\mu{}_\rho A^\rho \alpha^\nu{}_\sigma A^\sigma = \underbrace{g'_{\mu\nu}\alpha^\mu{}_\rho \alpha^\nu{}_\sigma}_{g_{\rho\sigma}} A^\rho A^\sigma = g_{\rho\sigma} A^\rho A^\sigma$$

líp $$A'^\mu A'_\mu = \alpha^\mu{}_\nu A^\nu A_\rho (\alpha^{-1})^\rho{}_\mu = \underbrace{(\alpha^{-1})^\rho{}_\mu \alpha^\mu{}_\nu}_{\delta^\rho_\nu} A^\nu A_\rho = \delta^\rho_\nu A^\nu A^\rho = A^\nu A_\nu \quad \checkmark$$

Kalibrační invariance

$$\tilde{A}^\mu = \begin{pmatrix} \frac{\tilde{\varphi}}{c} \\ \tilde{\vec{A}} \end{pmatrix} = \begin{pmatrix} \frac{\varphi}{c} - \frac{1}{c}\frac{\partial \Lambda}{\partial t} \\ \vec{A} + \operatorname{grad}\Lambda \end{pmatrix}$$

$$\tilde{A}^\mu \tilde{A}_\mu = \left(\frac{\tilde{\varphi}}{c}\right)^2 - \tilde{\vec{A}}^2 = \left(\frac{\varphi}{c}\right)^2 - \frac{2}{c^2}\varphi\frac{\partial\Lambda}{\partial t} + \frac{1}{c^2}\left(\frac{\partial\Lambda}{\partial t}\right)^2 - \vec{A}^2 - 2\vec{A}\operatorname{grad}\Lambda - (\operatorname{grad}\Lambda)^2 =$$

$$= A^\mu A_\mu \underbrace{- \frac{2}{c^2}\varphi\frac{\partial\Lambda}{\partial t} + \frac{1}{c^2}\left(\frac{\partial\Lambda}{\partial t}\right)^2 - 2\vec{A}\operatorname{grad}\Lambda - (\operatorname{grad}\Lambda)^2}_{\neq 0 \text{ obecně není kalibračně invariantní}}$$