Lagrangeovy rovnice 1. druhu $m_i \ddot{x}_i = F_i(\vec{x}, \dot{\vec{x}}, t) + \sum_{k=1}^{r} \lambda_k \frac{\partial f_k}{\partial x_i} + \underbrace{\sum_{k=1}^{r} T_i^{(k)}}_{=0} \quad \forall i \in \widehat{3N} \qquad f_k(\vec{x}, t) = 0 \quad \forall k \in \hat{r}$

Lagrangeovy rovnice 1. druhu $\quad m_i \ddot{x}_i = F_i(\vec{x},\dot{\vec{x}},t) + \sum_{k=1}^{r} \lambda_k \frac{\partial f_k}{\partial x_i} + \underbrace{\sum_{k=1}^{r} T_i^{(k)}}_{=0} \quad \forall i \in \widehat{3N} \qquad f_k(\vec{x},t) = 0 \quad \forall k \in \hat{r}$

1. Operátor úplné časové derivace

$$\frac{\hat{d}}{dt} : C^\infty(\mathbb{R}^{k\cdot s+1}) \longrightarrow C^\infty(\mathbb{R}^{(k+1)\cdot s+1}) \qquad \frac{\hat{d}}{dt} : F(\vec{x},\dot{\vec{x}},\dots,\vec{x}^{(k)},t) \longrightarrow \dot{F}(\vec{x},\dot{\vec{x}},\dots,\vec{x}^{(k+1)},t)$$

pro $\quad F : \mathbb{R}^{2s+1} \to \mathbb{R} \qquad F = F(\vec{x},\dot{\vec{x}},t)$

$\vec{x}, \dot{\vec{x}}, t$ nezávislé proměnné

$$\frac{\hat{d}F}{dt} = \frac{\partial F}{\partial x_i}\dot{x}_i + \frac{\partial F}{\partial \dot{x}_i}\ddot{x}_i + \frac{\partial F}{\partial t}\Big|_{(\vec{x},\dot{\vec{x}},t)}$$

$$\dot{F} = \dot{F}(\underbrace{\vec{x},\dot{\vec{x}},\ddot{\vec{x}},t}_{\text{nezávislé proměnné}}) \qquad \dot{\vec{x}} = \vec{v}, \ \ddot{\vec{x}} = \vec{a}$$

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1. **Operátor úplné časové derivace** $\frac{\hat{d}}{dt} : C^\infty(\mathbb{R}^{k \cdot s + 1}) \longrightarrow C^\infty(\mathbb{R}^{(k+1) \cdot s + 1}) \qquad \frac{\hat{d}}{dt} : F(\vec{x}, \dot{\vec{x}}, \dots, \vec{x}^{(k)}, t) \longrightarrow \dot{F}(\vec{x}, \dot{\vec{x}}, \dots, \vec{x}^{(k+1)}, t)$

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$\vec{x}, \dot{\vec{x}}, t$ nezávislé proměnné

nezávislé proměnné $\dot{\vec{x}} = \vec{v}$, $\ddot{\vec{x}} = \vec{a}$

2. parciální derivace podle času $\frac{\partial F}{\partial t} = \lim_{\Delta t \to 0} \frac{F(\vec{x}, \dot{\vec{x}}, t + \Delta t) - F(\vec{x}, \dot{\vec{x}}, t)}{\Delta t}$

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3. úplná derivace podle času

$\vec{x} = \vec{\tilde{x}}(t)$

$\dot{\vec{x}} = \dot{\vec{\tilde{x}}}(t)$ funkce času

$\tilde{F}(t) = F(\vec{\tilde{x}}(t),\dot{\vec{\tilde{x}}}(t),t)$

$$\dot{\tilde{F}} = \frac{d\tilde{F}}{dt} = \frac{\partial F}{\partial x_i}\dot{\tilde{x}}_i(t) + \frac{\partial F}{\partial \dot{x}_i}\ddot{\tilde{x}}_i(t) + \frac{\partial F}{\partial t}\Bigg|_{(\vec{\tilde{x}}(t),\dot{\vec{\tilde{x}}}(t),t)}$$

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$\vec{x} = \vec{\mathcal{X}}(t)$

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$$\begin{tikzcd}
F(\vec{x},\dot{\vec{x}},t) \arrow[r, "\frac{\hat{d}}{dt}"] \arrow[d, "\vec{x}=\vec{\mathcal{X}}(t)"] & \dot{F}(\vec{x},\dot{\vec{x}},\ddot{\vec{x}},t) \arrow[d, "\vec{x}=\vec{\mathcal{X}}(t)"] \\
F(\vec{\mathcal{X}}(t),\dot{\vec{\mathcal{X}}}(t),t) \arrow[r, "\frac{d}{dt}"'] & \dot{F}(\vec{\mathcal{X}}(t),\dot{\vec{\mathcal{X}}}(t),\ddot{\vec{\mathcal{X}}}(t),t)
\end{tikzcd}$$

$= \tilde{F}(t) \qquad\qquad = \dot{\tilde{F}}(t)$

• upravíme levou stranu LR1D $\forall i \in \hat{s}$ (tj. bez sumace přes i)

$$m_i \ddot{x}_i = m_i \frac{\hat{d}}{dt}(\dot{x}_i) = \frac{\hat{d}}{dt}(m_i \dot{x}_i) = \frac{\hat{d}}{dt}\left(\frac{1}{2} m_i 2 \dot{x}_i\right) = \frac{\hat{d}}{dt}\left(\sum_j' \frac{1}{2} m_j 2 \dot{x}_j \delta_{ji}\right) = \frac{\hat{d}}{dt}\left(\sum_j' \frac{1}{2} m_j 2 \dot{x}_j \frac{\partial \dot{x}_j}{\partial \dot{x}_i}\right) = \frac{\hat{d}}{dt}\left(\frac{\partial}{\partial \dot{x}_i}\left(\underbrace{\frac{1}{2}\sum_j' m_j \dot{x}_j^2}_{T(\dot{\vec{x}})}\right)\right)$$

$$= \frac{\hat{d}}{dt}\left(\frac{\partial T}{\partial \dot{x}_i}\right)$$

kde $T(\dot{\vec{x}}) = \frac{1}{2}\sum_{j=1}^{s} m_j \dot{x}_j^2$ je kinetická energie soustavy

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- Síly vtištěné (akční) na pravé staně LR1D nahradíme potenciály → silové pole (síla) $\vec{F}$ se nazývá:

1) <u>konzervativní</u> $\vec{F} = \vec{F}(\vec{x})$ pokud $\exists U = U(\vec{x})$ potenciální energie tak, že $F_j(\vec{x}) = -\frac{\partial U(\vec{x})}{\partial x_j}$ $\forall j \in \hat{n}$

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2) <u>potenciální</u> $\vec{F} = \vec{F}(\vec{x}, t)$ pokud $\exists U = U(\vec{x}, t)$ potenciál tak, že $F_j(\vec{x}, t) = -\frac{\partial U(\vec{x}, t)}{\partial x_j}$ $\forall j \in \hat{n}$

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3) <u>zobecněná potenciální</u> $\vec{F} = \vec{F}(\vec{x}, \dot{\vec{x}}, t)$ pokud $\exists U = U(\vec{x}, \dot{\vec{x}}, t)$ tak, že zobecněný potenciál $F_j(\vec{x}, \dot{\vec{x}}, t) = -\frac{\partial U}{\partial x_j} + \frac{\hat{d}}{dt}\left(\frac{\partial U}{\partial \dot{x}_j}\right)$ $\forall j \in \hat{n}$

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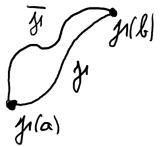
zobecněný potenciál

Pozn. práce konzervativních sil nezávisí na dráze $W = \int_\gamma \vec{F}(\vec{x}) \cdot d\vec{x} = -\int_\gamma \frac{\partial U}{\partial \vec{x}} d\vec{x} = -\int_\gamma 1\, dU = -\int_a^b 1\, dU(\gamma(t)) = U(\gamma(a)) - U(\gamma(b))$



Pozn. v $\mathbb{R}^3$ podmínka $\operatorname{rot} \vec{F} = 0 \Rightarrow \vec{F}(\vec{x}, t)$ je potenciální

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- harmonický ocilátor (elastické pole) $U(\vec{x}) = \frac{1}{2} K \left(\sqrt{\vec{x}^2} - a_0\right)^2$

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- Lorentzova síla (E.M. pole)

$$U(\vec{x}, \dot{\vec{x}}, t) = q\left(\varphi(\vec{x}, t) - \dot{\vec{x}}\cdot\vec{A}(\vec{x}, t)\right)$$

$$\vec{E} = -\nabla\varphi - \frac{\partial \vec{A}}{\partial t} \qquad \vec{B} = \nabla\times\vec{A}$$

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- LR1D $\quad \dfrac{\hat{d}}{dt}\left(\dfrac{\partial T}{\partial \dot{x}_i}\right) = F_i^{(nep)} + \dfrac{\hat{d}}{dt}\left(\dfrac{\partial U}{\partial \dot{x}_i}\right) - \dfrac{\partial U}{\partial x_i} + \lambda_k \dfrac{\partial f_k}{\partial x_i}$

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$T = T(\dot{\vec{x}})$ tj $\dfrac{\partial T}{\partial x_i} = 0$

$$\frac{\hat{d}}{dt}\left(\frac{\partial (T-U)}{\partial \dot{x}_i}\right) - \frac{\partial (T-U)}{\partial x_i} = F_i^{(nep)} + \lambda_k \frac{\partial f_k}{\partial x_i}$$

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$\forall i \in \widehat{3N} \quad \boxed{\dfrac{\hat{d}}{dt}\left(\dfrac{\partial L}{\partial \dot{x}_i}\right) - \dfrac{\partial L}{\partial x_i} = F_i^{(nep)} + \lambda_k \dfrac{\partial f_k}{\partial x_i}} \qquad f_k(\vec{x}, t) = 0 \quad \forall k \in \hat{r}$

Lagrangeova funkce (v kartézských)

$$\boxed{\begin{aligned} &L = T - U \\ &L(\vec{x}, \dot{\vec{x}}, t) = \frac{1}{2}\sum_{j=1}^{3N} m_j \dot{x}_j^2 - U(\vec{x}, \dot{\vec{x}}, t) \end{aligned}}$$

- nejednoznačnost Lagrangeovy funkce $L' = L + \frac{\hat{d}}{dt} h(\vec{x}, t)$ $h = h(\vec{x}, t) \in C^{(2)}$ $\frac{\hat{d}h}{dt} = \frac{\partial h}{\partial x_j} \dot{x}_j + \frac{\partial h}{\partial t}$

- nejednoznačnost Lagrangeovy funkce $L' = L + \frac{\hat{d}}{dt} h(\vec{x}, t)$ $\quad h = h(\vec{x}, t) \in C^{(2)}$ $\quad \frac{\hat{d}h}{dt} = \frac{\partial h}{\partial x_j}\dot{x}_j + \frac{\partial h}{\partial t}$

$$\frac{\hat{d}}{dt}\left(\frac{\partial L'}{\partial \dot{x}_i}\right) - \frac{\partial L'}{\partial x_i} = \underbrace{\frac{\hat{d}}{dt}\left(\frac{\partial L}{\partial \dot{x}_i}\right) - \frac{\partial L}{\partial x_i}} + \frac{\hat{d}}{dt}\left(\frac{\partial h}{\partial x_i}\right) - \frac{\partial}{\partial x_i}\frac{\hat{d}h}{dt} =$$

$$\frac{\partial}{\partial \dot{x}_i}\left(\frac{\hat{d}h}{dt}\right) = \frac{\partial h}{\partial x_j}\frac{\partial \dot{x}_j}{\partial \dot{x}_i} = \frac{\partial h}{\partial x_i}$$

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$$= LS + \frac{\partial^2 h}{\partial x_j \partial x_i} \dot{x}_j + \frac{\partial^2 h}{\partial t \partial x_i} - \frac{\partial^2 h}{\partial x_i \partial x_j} \dot{x}_j - \frac{\partial^2 h}{\partial x_i \partial t} = LS$$

$\Rightarrow$ L a L' vedou na stejné LR1D

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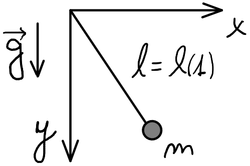
Př.



$$T = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2)$$

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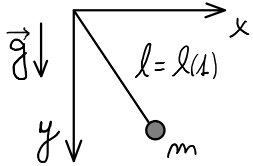
Př.



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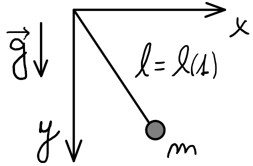
$\Rightarrow$ L a L' vedou na stejné LR1D

Př.



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$$\frac{\hat{d}}{dt}\left(\frac{\partial L}{\partial \dot{y}}\right) - \frac{\partial L}{\partial y} = \frac{\hat{d}}{dt}(m\dot{y}) - mg = m\ddot{y} - mg = 2\lambda y$$

$$f(x, y, t) = x^2 + y^2 - l^2(t) = 0$$

Křivočaré souřadnice v afinním euklidovském prostoru E dimenze $m \in \mathbb{N}$

$\vec{X} = \hat{\vec{X}}(\vec{y}, t)$ $\quad X^i = \hat{X}^i(y^1, \ldots, y^m, t) \quad \forall i \in \hat{m} \quad \hat{X}^i : \mathbb{R}^{m+1} \to \mathbb{R}^m$ třídy $C^{(2)}$, regulární

kartézské složky polohového vektoru

souřadnice, n-tice čísel nejsou složky vektoru

Jacobián $\quad J = \det\left(\frac{\partial \hat{X}^i}{\partial y^j}\right) = \left|\frac{\partial(\hat{X}^1, \ldots, \hat{X}^m)}{\partial(y^1, \ldots, y^m)}\right| \neq 0$

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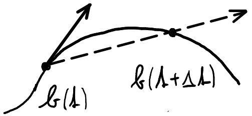
tečný vektor

$$\lim_{\Delta t \to 0} \frac{b(t+\Delta t) - b(t)}{\Delta t}$$



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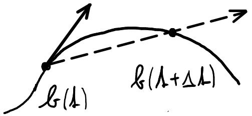

Báze $B = \mathbb{X} = (\vec{e}_1, \dots, \vec{e}_n)$ a $\tilde{B} = \mathbb{Y} = (\tilde{\vec{e}}_1, \dots, \tilde{\vec{e}}_n)$ tvořená tečnými vektory k souřadnicovým křivkám $y^j \to \hat{\vec{X}}(\vec{y}, t)$

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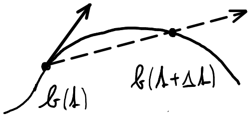

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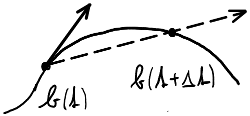

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Př. polární souřadnice (nenormalizované)

$x_1 = y_1 \cos y_2 \qquad y_1 \in (0, +\infty)$

$x_2 = y_1 \sin y_2 \qquad y_2 \in \langle 0, 2\pi)$

$$\mathbb{S} = \left(\frac{\partial x_i}{\partial y_j}\right) = \begin{pmatrix} \cos y_2 & -y_1 \sin y_2 \\ \sin y_2 & y_1 \cos y_2 \end{pmatrix}$$

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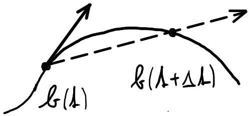

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$$y_1 = \sqrt{x_1^2 + x_2^2} \qquad y_2 = \operatorname{arctg}\left(\frac{x_2}{x_1}\right)$$

$$\mathbb{S}^{-1} = \left(\frac{\partial y_i}{\partial x_j}\right) = \begin{pmatrix} \frac{x_1}{\sqrt{x_1^2 + x_2^2}} & \frac{x_2}{\sqrt{x_1^2 + x_2^2}} \\ \frac{-x_2}{x_1^2 + x_2^2} & \frac{x_1}{x_1^2 + x_2^2} \end{pmatrix}$$

Newtonovy rovnice v křivočarých souřadnicích (pro jednu částici s hmotností m v prostoru $\mathbb{R}^m$)

$$m\ddot{x}^i = F^i(\vec{x},\dot{\vec{x}},t) \qquad x^i = \hat{x}^i(\vec{y},t)$$

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Newtonovy rovnice v křivočarých souřadnicích (pro jednu částici s hmotností m v prostoru $\mathbb{R}^m$)

$$m\ddot{x}^i = F^i(\vec{x},\dot{\vec{x}},t) \qquad x^i = \hat{x}^i(\vec{y},t) \qquad \dot{x}^i = \dot{\hat{x}}^i(\vec{y},\dot{\vec{y}},t) = \frac{d\,\hat{x}^i(\vec{y},t)}{dt} = \frac{\partial \hat{x}^i}{\partial y^j}\dot{y}^j + \frac{\partial \hat{x}^i}{\partial t} \qquad \forall i \in \hat{m}$$

$$\ddot{x}^i = \ddot{\hat{x}}^i(\vec{y},\dot{\vec{y}},\ddot{\vec{y}},t) = \frac{d}{dt}\left(\frac{\partial \hat{x}^i}{\partial y^j}\dot{y}^j + \frac{\partial \hat{x}^i}{\partial t}\right) = \frac{\partial^2 \hat{x}^i}{\partial y^k \partial y^j}\dot{y}^j\dot{y}^k + \frac{\partial \hat{x}^i}{\partial y^j}\frac{\partial \dot{y}^j}{\partial \dot{y}^k}\ddot{y}^k + \frac{\partial^2 \hat{x}^i}{\partial t\,\partial y^j}\dot{y}^j + \frac{\partial^2 \hat{x}^i}{\partial y^k \partial t}\dot{y}^k + \frac{\partial^2 \hat{x}^i}{\partial t^2} =$$

Newtonovy rovnice v křivočarých souřadnicích (pro jednu částici s hmotností m v prostoru $\mathbb{R}^n$)

$$m\ddot{x}^i = F^i(\vec{x},\dot{\vec{x}},t) \qquad x^i = \hat{x}^i(\vec{y},t) \qquad \dot{x}^i = \dot{\hat{x}}^i(\vec{y},\dot{\vec{y}},t) = \frac{\hat{d}\,\hat{x}^i(\vec{y},t)}{dt} = \frac{\partial \hat{x}^i}{\partial y^j}\dot{y}^j + \frac{\partial \hat{x}^i}{\partial t} \qquad \forall i \in \hat{n}$$

$$\ddot{x}^i = \ddot{\hat{x}}^i(\vec{y},\dot{\vec{y}},\ddot{\vec{y}},t) = \frac{\hat{d}}{dt}\left(\frac{\partial \hat{x}^i}{\partial y^j}\dot{y}^j + \frac{\partial \hat{x}^i}{\partial t}\right) = \frac{\partial^2 \hat{x}^i}{\partial y^k \partial y^j}\dot{y}^j\dot{y}^k + \frac{\partial \hat{x}^i}{\partial y^j}\frac{\partial \dot{y}^j}{\partial \dot{y}^k}\ddot{y}^k + \frac{\partial^2 \hat{x}^i}{\partial t\,\partial y^j}\dot{y}^j + \frac{\partial^2 \hat{x}^i}{\partial y^k \partial t}\dot{y}^k + \frac{\partial^2 \hat{x}^i}{\partial t^2} =$$

$$= \frac{\partial^2 \hat{x}^i}{\partial y^k \partial y^j}\dot{y}^j\dot{y}^k + \frac{\partial \hat{x}^i}{\partial y^j}\ddot{y}^j + 2\,\frac{\partial^2 \hat{x}^i}{\partial t\,\partial y^j}\dot{y}^j + \frac{\partial^2 \hat{x}^i}{\partial t^2}$$

Newtonovy rovnice v křivočarých souřadnicích (pro jednu částici s hmotností m v prostoru $\mathbb{R}^n$)

$$m\ddot{x}^i = F^i(\vec{x},\dot{\vec{x}},t) \qquad x^i = \hat{x}^i(\vec{y},t) \qquad \dot{x}^i = \hat{\dot{x}}^i(\vec{y},\dot{\vec{y}},t) = \frac{d\,\hat{x}^i(\vec{y},t)}{dt} = \frac{\partial \hat{x}^i}{\partial y^j}\dot{y}^j + \frac{\partial \hat{x}^i}{\partial t} \qquad \forall i \in \hat{n}$$

$$\ddot{x}^i = \hat{\ddot{x}}^i(\vec{y},\dot{\vec{y}},\ddot{\vec{y}},t) = \frac{d}{dt}\left(\frac{\partial \hat{x}^i}{\partial y^j}\dot{y}^j + \frac{\partial \hat{x}^i}{\partial t}\right) = \frac{\partial^2 \hat{x}^i}{\partial y^k \partial y^j}\dot{y}^j\dot{y}^k + \frac{\partial \hat{x}^i}{\partial y^j}\frac{\partial \dot{y}^j}{\partial \dot{y}^k}\ddot{y}^k + \frac{\partial^2 \hat{x}^i}{\partial t\,\partial y^j}\dot{y}^j + \frac{\partial^2 \hat{x}^i}{\partial y^k \partial t}\dot{y}^k + \frac{\partial^2 \hat{x}^i}{\partial t^2} =$$

$$= \frac{\partial^2 \hat{x}^i}{\partial y^k \partial y^j}\dot{y}^j\dot{y}^k + \frac{\partial \hat{x}^i}{\partial y^j}\ddot{y}^j + 2\,\frac{\partial^2 \hat{x}^i}{\partial t\,\partial y^j}\dot{y}^j + \frac{\partial^2 \hat{x}^i}{\partial t^2}$$

$$m\left[\frac{\partial^2 \hat{x}^i}{\partial y^k \partial y^j}\dot{y}^j\dot{y}^k + \frac{\partial \hat{x}^i}{\partial y^j}\ddot{y}^j + 2\,\frac{\partial^2 \hat{x}^i}{\partial t\,\partial y^j}\dot{y}^j + \frac{\partial^2 \hat{x}^i}{\partial t^2}\right] = F^i(\vec{x},\dot{\vec{x}},t) \qquad \Big/ (S^{-1})^l{}_i = \frac{\partial y^l}{\partial x^i} \qquad \frac{\partial y^l}{\partial x^i}\frac{\partial \hat{x}^i}{\partial y^j} = \frac{\partial y^l}{\partial y^j} = \delta^l{}_j$$

Newtonovy rovnice v křivočarých souřadnicích (pro jednu částici s hmotností m v prostoru $\mathbb{R}^n$)

$$m\ddot{x}^i = F^i(\vec{x},\dot{\vec{x}},t) \qquad x^i = \hat{x}^i(\vec{y},t) \qquad \dot{x}^i = \dot{\hat{x}}^i(\vec{y},\dot{\vec{y}},t) = \frac{d\,\hat{x}^i(\vec{y},t)}{dt} = \frac{\partial \hat{x}^i}{\partial y^j}\dot{y}^j + \frac{\partial \hat{x}^i}{\partial t} \qquad \forall i \in \hat{n}$$

$$\ddot{x}^i = \ddot{\hat{x}}^i(\vec{y},\dot{\vec{y}},\ddot{\vec{y}},t) = \frac{d}{dt}\left(\frac{\partial \hat{x}^i}{\partial y^j}\dot{y}^j + \frac{\partial \hat{x}^i}{\partial t}\right) = \frac{\partial^2 \hat{x}^i}{\partial y^k \partial y^j}\dot{y}^j\dot{y}^k + \frac{\partial \hat{x}^i}{\partial y^j}\frac{\partial \dot{y}^j}{\partial \dot{y}^k}\ddot{y}^k + \frac{\partial^2 \hat{x}^i}{\partial t\,\partial y^j}\dot{y}^j + \frac{\partial^2 \hat{x}^i}{\partial y^k \partial t}\dot{y}^k + \frac{\partial^2 \hat{x}^i}{\partial t^2} =$$

$$= \frac{\partial^2 \hat{x}^i}{\partial y^k \partial y^j}\dot{y}^j\dot{y}^k + \frac{\partial \hat{x}^i}{\partial y^j}\ddot{y}^j + 2\,\frac{\partial^2 \hat{x}^i}{\partial t\,\partial y^j}\dot{y}^j + \frac{\partial^2 \hat{x}^i}{\partial t^2}$$

$$m\left[\frac{\partial^2 \hat{x}^i}{\partial y^k \partial y^j}\dot{y}^j\dot{y}^k + \frac{\partial \hat{x}^i}{\partial y^j}\ddot{y}^j + 2\,\frac{\partial^2 \hat{x}^i}{\partial t\,\partial y^j}\dot{y}^j + \frac{\partial^2 \hat{x}^i}{\partial t^2}\right] = F^i(\vec{x},\dot{\vec{x}},t) \quad \Big/ (S^{-1})^l{}_i = \frac{\partial y^l}{\partial x^i} \qquad \frac{\partial y^l}{\partial x^i}\frac{\partial \hat{x}^i}{\partial y^j} = \frac{\partial y^l}{\partial y^j} = \delta^l{}_j$$

$$m\,\frac{\partial y^l}{\partial x^i}\left[\frac{\partial^2 \hat{x}^i}{\partial y^k \partial y^j}\dot{y}^j\dot{y}^k + 2\,\frac{\partial^2 \hat{x}^i}{\partial t\,\partial y^j}\dot{y}^j + \frac{\partial^2 \hat{x}^i}{\partial t^2}\right] + m\ddot{y}^l = \frac{\partial y^l}{\partial x^i}F^i(\vec{x},\dot{\vec{x}},t) = \tilde{F}^l(\vec{y},\dot{\vec{y}},t) \qquad \forall l \in \hat{n}$$

Newtonovy rovnice v křivočarých souřadnicích (pro jednu částici s hmotností m v prostoru $\mathbb{R}^n$)

$$m\ddot{x}^i = F^i(\vec{x},\dot{\vec{x}},t) \qquad x^i = \hat{x}^i(\vec{y},t) \qquad \dot{x}^i = \dot{\hat{x}}^i(\vec{y},\dot{\vec{y}},t) = \frac{d\,\hat{x}^i(\vec{y},t)}{dt} = \frac{\partial \hat{x}^i}{\partial y^j}\dot{y}^j + \frac{\partial \hat{x}^i}{\partial t} \qquad \forall i \in \hat{n}$$

$$\ddot{x}^i = \ddot{\hat{x}}^i(\vec{y},\dot{\vec{y}},\ddot{\vec{y}},t) = \frac{d}{dt}\left(\frac{\partial \hat{x}^i}{\partial y^j}\dot{y}^j + \frac{\partial \hat{x}^i}{\partial t}\right) = \frac{\partial^2 \hat{x}^i}{\partial y^k \partial y^j}\dot{y}^j\dot{y}^k + \frac{\partial \hat{x}^i}{\partial y^j}\frac{\partial \dot{y}^j}{\partial \dot{y}^k}\ddot{y}^k + \frac{\partial^2 \hat{x}^i}{\partial t\,\partial y^j}\dot{y}^j + \frac{\partial^2 \hat{x}^i}{\partial y^k \partial t}\dot{y}^k + \frac{\partial^2 \hat{x}^i}{\partial t^2} =$$

$$= \frac{\partial^2 \hat{x}^i}{\partial y^k \partial y^j}\dot{y}^j\dot{y}^k + \frac{\partial \hat{x}^i}{\partial y^j}\ddot{y}^j + 2\,\frac{\partial^2 \hat{x}^i}{\partial t\,\partial y^j}\dot{y}^j + \frac{\partial^2 \hat{x}^i}{\partial t^2}$$

$$m\left[\frac{\partial^2 \hat{x}^i}{\partial y^k \partial y^j}\dot{y}^j\dot{y}^k + \frac{\partial \hat{x}^i}{\partial y^j}\ddot{y}^j + 2\,\frac{\partial^2 \hat{x}^i}{\partial t\,\partial y^j}\dot{y}^j + \frac{\partial^2 \hat{x}^i}{\partial t^2}\right] = F^i(\vec{x},\dot{\vec{x}},t) \quad \Big/ (S^{-1})^l{}_i = \frac{\partial y^l}{\partial x^i} \qquad \frac{\partial y^l}{\partial x^i}\frac{\partial \hat{x}^i}{\partial y^j} = \frac{\partial y^l}{\partial y^j} = \delta^l{}_j$$

$$m\,\frac{\partial y^l}{\partial x^i}\underbrace{\left[\frac{\partial^2 \hat{x}^i}{\partial y^k \partial y^j}\dot{y}^j\dot{y}^k + 2\,\frac{\partial^2 \hat{x}^i}{\partial t\,\partial y^j}\dot{y}^j + \frac{\partial^2 \hat{x}^i}{\partial t^2}\right]}_{\text{polynom 2. stupně v rychlostech } \stackrel{?}{=} 0} + m\ddot{y}^l = \frac{\partial y^l}{\partial x^i}F^i(\vec{x},\dot{\vec{x}},t) = \tilde{F}^l(\vec{y},\dot{\vec{y}},t) \qquad \forall l \in \hat{n}$$

$\Rightarrow$ Galileiho transformace je lineární funkci souřadnic a času

Newtonovy rovnice v křivočarých souřadnicích (pro jednu částici s hmotností m v prostoru $\mathbb{R}^m$)

$$m\ddot{x}^i = F^i(\vec{x},\dot{\vec{x}},t) \qquad x^i = \hat{x}^i(\vec{y},t) \qquad \dot{x}^i = \hat{\dot{x}}^i(\vec{y},\dot{\vec{y}},t) = \frac{\hat{d}\,\hat{x}^i(\vec{y},t)}{dt} = \frac{\partial \hat{x}^i}{\partial y^j}\dot{y}^j + \frac{\partial \hat{x}^i}{\partial t} \qquad \forall i \in \hat{m}$$

$$\ddot{x}^i = \hat{\ddot{x}}^i(\vec{y},\dot{\vec{y}},\ddot{\vec{y}},t) = \frac{\hat{d}}{dt}\left(\frac{\partial \hat{x}^i}{\partial y^j}\dot{y}^j + \frac{\partial \hat{x}^i}{\partial t}\right) = \frac{\partial^2 \hat{x}^i}{\partial y^k \partial y^j}\dot{y}^j\dot{y}^k + \frac{\partial \hat{x}^i}{\partial y^j}\frac{\partial \dot{y}^j}{\partial \dot{y}^k}\ddot{y}^k + \frac{\partial^2 \hat{x}^i}{\partial t\,\partial y^j}\dot{y}^j + \frac{\partial^2 \hat{x}^i}{\partial y^k \partial t}\dot{y}^k + \frac{\partial^2 \hat{x}^i}{\partial t^2} =$$

$$= \frac{\partial^2 \hat{x}^i}{\partial y^k \partial y^j}\dot{y}^j\dot{y}^k + \frac{\partial \hat{x}^i}{\partial y^j}\ddot{y}^j + 2\,\frac{\partial^2 \hat{x}^i}{\partial t\,\partial y^j}\dot{y}^j + \frac{\partial^2 \hat{x}^i}{\partial t^2}$$

$$m\left[\frac{\partial^2 \hat{x}^i}{\partial y^k \partial y^j}\dot{y}^j\dot{y}^k + \frac{\partial \hat{x}^i}{\partial y^j}\ddot{y}^j + 2\,\frac{\partial^2 \hat{x}^i}{\partial t\,\partial y^j}\dot{y}^j + \frac{\partial^2 \hat{x}^i}{\partial t^2}\right] = F^i(\vec{x},\dot{\vec{x}},t) \quad \Big/ (S^{-1})^l{}_i = \frac{\partial y^l}{\partial x^i} \qquad \frac{\partial y^l}{\partial x^i}\frac{\partial \hat{x}^i}{\partial y^j} = \frac{\partial y^l}{\partial y^j} = \delta^l_j$$

$$m\,\frac{\partial y^l}{\partial x^i}\underbrace{\left[\frac{\partial^2 \hat{x}^i}{\partial y^k \partial y^j}\dot{y}^j\dot{y}^k + 2\,\frac{\partial^2 \hat{x}^i}{\partial t\,\partial y^j}\dot{y}^j + \frac{\partial^2 \hat{x}^i}{\partial t^2}\right]}_{\text{polynom 2. stupně v rychlostech } \overset{?}{=} 0} + m\ddot{y}^l = \frac{\partial y^l}{\partial x^i}F^i(\vec{x},\dot{\vec{x}},t) = \tilde{F}^l(\vec{y},\dot{\vec{y}},t) \qquad \forall l \in \hat{m}$$

$\Rightarrow$ Galileiho transformace je lineární funkcí souřadnic a času

Př. polární souřadnice (nenormalizované)

$$\frac{\partial x_i}{\partial t} = 0 \qquad \left(\frac{\partial^2 x_1}{\partial y_k \partial y_j}\right) = \begin{pmatrix} 0 & -\sin y_2 \\ -\sin y_2 & -y_1\cos y_2 \end{pmatrix} \qquad \left(\frac{\partial^2 x_2}{\partial y_k \partial y_j}\right) = \begin{pmatrix} 0 & \cos y_2 \\ \cos y_2 & -y_1\sin y_2 \end{pmatrix} \qquad \left(\frac{\partial y_l}{\partial x_i}\right) = S^{-1} = \begin{pmatrix} \cos y_2 & \sin y_2 \\ -\frac{\sin y_2}{y_1} & \frac{\cos y_2}{y_1} \end{pmatrix}$$

$$m\begin{pmatrix} \cos y_2 & \sin y_2 \\ -\frac{\sin y_2}{y_1} & \frac{\cos y_2}{y_1} \end{pmatrix}\begin{bmatrix} -2\dot{y}_1\dot{y}_2\sin y_2 - y_1\dot{y}_2^2\cos y_2 \\ 2\dot{y}_1\dot{y}_2\cos y_2 - y_1\dot{y}_2^2\sin y_2 \end{bmatrix} + m\begin{pmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{pmatrix} = \begin{pmatrix} F_1\cos y_2 + F_2\sin y_2 \\ -\frac{F_1}{y_1}\sin y_2 + \frac{F_2}{y_1}\cos y_2 \end{pmatrix} \Rightarrow \begin{aligned} m(\ddot{y}_1 - y_1\dot{y}_2^2) &= \tilde{F}_1 \\ m\left(\ddot{y}_2 + \frac{2}{y_1}\dot{y}_1\dot{y}_2\right) &= \tilde{F}_2 \end{aligned}$$