

41) najdi najmenšie spektrum 1 častice

$$-\Delta\psi = E\psi$$

$$\Delta\psi + E\psi = 0$$

$$\psi(0) = 0$$

$$\Rightarrow \psi = \sin\left(\frac{\pi kx}{L}\right) \quad k \in \mathbb{N}$$

$$\psi(L) = 0$$

$$E_k = \frac{\pi^2}{L^2} k^2$$

Makladni' stav N fermionov: $\sum_{k=1}^N E_k = \frac{\pi^2}{L^2} \sum_{k=1}^N k^2$

$$k^3 - (k-1)^3 = 3k^2 - 3k + 1 \quad / \sum_k$$

$$N^3 = 3 \sum_{k=1}^N k^2 - 3 \sum_{k=1}^N k + \sum_{k=1}^N 1$$

$$\sum_{k=1}^N k^2 = \frac{N^3 + 3 \frac{N(N+1)}{2} - N}{3}$$

$$E_0^{(N)} = \frac{N^3}{3} + \frac{N(N+1)}{2} - \frac{N}{3} = \frac{N(N+1)(2N+1)}{6}$$

4.2) najpírod'nejšie 1 častice

$$-\Delta\psi = E\psi \Rightarrow \psi = \prod_{i=1}^3 \cos\left(\frac{\pi n_i x_i}{L}\right) \quad n_i \in \mathbb{N}_0$$

$$E_C = n_1^2 + n_2^2 + n_3^2$$

prít stavu s energií menší než C^2 :

$$\sum_{\substack{n_1, n_2, n_3 \\ n_1^2 + n_2^2 + n_3^2 \leq C}} 1 \sim \int\int\int_{\Omega} 1 \, dn_1 \, dn_2 \, dn_3 = 4\pi \frac{C^3}{3}$$

energie více stavu s energií menší než C^2 :

$$\sum_{\substack{n_1, n_2, n_3 \\ n_1^2 + n_2^2 + n_3^2 \leq C}} (n_1^2 + n_2^2 + n_3^2) \sim \int\int\int_{\Omega} n^2 \, dn_1 \, dn_2 \, dn_3 = 4\pi \frac{C^5}{5}$$

$$N = 4\pi \frac{C^3}{3} \quad C = \left(\frac{3N}{4\pi}\right)^{\frac{1}{3}}$$

$$E = KN^{\frac{5}{3}} \quad K \in \mathbb{R}^+ \quad K = \frac{4\pi}{5} \left(\frac{3}{4\pi}\right)^{\frac{5}{3}}$$

chyba v prít stavu - polhina okolo hromie sfery
 - modifikovaní / polhovaní v objemu $\frac{4}{3}\pi[(R+\sqrt{3})^3 - (R-\sqrt{3})^3] \sim C'R^2 \quad C' \in \mathbb{R}^+$