31) $\varphi_n \in V$, $\varphi_n \to \varphi$; $\varphi \in \mathcal{D}(A)$

$A\varphi_n \to \psi_A$ A uzavřený $\psi_A = A\varphi$

$B\varphi_n \to \psi_B$ B omezený

$U(A\varphi_n) \to \psi_{AU}$

posčítáme

$(UA+B)\varphi_n \to \psi_{AU} + \psi_B$

$U\varphi_n = \alpha_n$

$U\varphi_n \to U\varphi$

$\alpha_n \to \alpha$

$A\alpha_n \to \eta = \psi_{AU} + \psi_B$

A uzavřený $\Rightarrow \alpha \in \mathcal{D}(A) \Rightarrow U\varphi_n \in \mathcal{D}(A)$

propagátor k $H = \sum -m_j \Delta_j$:

$$(U(\Lambda)\psi)(x) = \lim_{k\to\infty} \prod_{j=1}^{n} \left(\frac{m_j}{2\pi i \Lambda}\right)^{\frac{3}{2}} \int_{\mathbb{R}^{3n}} \exp\left(\frac{i}{2\Lambda}\sum_{j=1}^{n} m_j |x_j - y_j|^2\right) \psi(y) f_k(y)\, dy$$

$|f_k| \le 1$, $f_k \xrightarrow{k\to\infty} 1$

pro fixní x vezmeme

$$\psi(y) = \exp\left(-\frac{i}{2\Lambda}\sum_{j=1}^{n} m_j |x_j - y_j|^2\right) \chi_M(y)$$

$$\chi_M(y) = 0 \qquad y \notin M$$

$$= 1 \qquad y \in \{x \in M \mid d(x, \partial M) > \varepsilon\}; \qquad \varepsilon > 0$$

$$\chi_M \in C_0^\infty(\mathbb{R}^{3n})$$

pak

$$(U(\Lambda)\psi)(x) = \prod_{j=1}^{n} \left(\frac{m_j}{2\pi i \Lambda}\right)^{\frac{3}{2}} \int_{\mathbb{R}^{3n}} \chi_M(y)\, dy$$

$$|(U(\Lambda)\psi)(x)|^2 = \prod_{j=1}^{n} \left(\frac{m_j}{2\pi \Lambda}\right)^{3} \left(\int_{\mathbb{R}^{3n}} \chi_M(y)\, dy\right)^2 > 0$$ pokud je M neprázdná

33) 1) V_D unitární propagátor, tzn.

$$V_D(t,r) = V_D(t,s)\, V_D(s,r)$$

$$U_0(r,t)\, U(t,r) = U_0(s,t)\, U(t,s)\, U_0(r,s)\, U(s,r) \qquad \Big/ \cdot U(r,s)$$

$$U_0(r,t)\, U(t,s) = U_0(s,t)\, U(t,s)\, U_0(r,s) \qquad \Big/ \cdot U_0(t,s) \cdot \rightarrow$$

$$U_0(t,s)\, U_0(r,t)\, U(t,s) = U(t,s)\, U_0(r,s)$$

$$U_0(t,s)\, U_0(r,t) = U_0^{-1}(s,t)\, U_0^{-1}(t,r) = U_0^{-1}(s,r) = U_0(r,s)$$

$$\Rightarrow \quad U_0(r,s)\, U(t,s) = U(t,s)\, U_0(t,s)$$

2) U unitární propagátor $\Longleftrightarrow$ a) $U(t,s) = U(t,r)\, U(r,s)$

b) $s,t \rightarrow U(s,t)$ silně spojité v $\mathbb{R}^2$

a) $$U_\tau(t,s) \overset{?}{=} U_\tau(t,r)\, U_\tau(r,s)$$
$$\overset{?}{=} U_0(\tau,t)\, U(t,\cancel{\tau})\, U_0(\cancel{\tau},\tau)\, U_0(\tau,r)\, U(r,s)\, U_0(s,\tau)$$
$$\overset{?}{=} U_0(\tau,t)\, U(t,\cancel{\tau})\, U_0(\cancel{\tau},r)\, U(r,s)\, U_0(s,\tau)$$
$$\overset{?}{=} U_0(\tau,t)\, U(t,r)\, U(r,s)\, U_0(s,\tau)$$
$$\overset{?}{=} U_0(\tau,t)\, U(t,s)\, U_0(s,\tau)$$
$$\Downarrow$$

b) $U_\tau(t,s) = U_\tau(t,r)\, U_\tau(r,s)$

$U_\tau(t,s)$ je složení silně spojitých zobrazení $\Rightarrow$ $U_\tau(t,s)$ silně spojité