28) $\frac{1}{a} = \int_0^\infty e^{-as}\, ds$

$$(-\Delta+E)^{-1} = \int_0^\infty e^{\Delta s} e^{-Es}\, ds = \int_0^\infty e^{-Es}(4\pi s)^{-\frac{d}{2}} \exp\left(-\frac{|x-y|^2}{4s}\right) ds = \left[\begin{array}{l} s = |x-y|^2 \sigma \\ ds = |x-y|^2 d\sigma \end{array}\right]$$

$E>0$

$$= (4\pi)^{-\frac{d}{2}}|x-y|^{2-d}\int_0^\infty e^{-E|x-y|^2 \sigma} \sigma^{-\frac{d}{2}} \exp\left(-\frac{1}{4\sigma}\right) d\sigma = \left[\begin{array}{l} \sigma = u^{-2} \\ d\sigma = -2u^{-3}\, du \end{array}\right]$$

$$= 2(4\pi)^{-\frac{d}{2}}|x-y|^{2-d}\int_0^\infty \exp\left(-\frac{u^2}{4} - \frac{E|x-y|^2}{u^2}\right) u^{d-3}\, du$$

$$= 2(4\pi)^{-\frac{d}{2}}|x-y|^{2-d}\int_0^\infty \exp\left(-u^2\left(\frac14 - k\right) - \sqrt{k}\sqrt{E}\,|x-y| \underbrace{\left(\frac{\sqrt{k}\,u^2}{\sqrt{E}\,|x-y|} + \frac{\sqrt{E}\,|x-y|}{\sqrt{k}\,u^2}\right)}_{\geq 2}\right) u^{d-3}\, du$$

$$\le 2(4\pi)^{-\frac{d}{2}}|x-y|^{2-d}\int_0^\infty \exp\left(-u^2\left(\frac14-k\right)\right)\exp\left(-2\sqrt{k}\sqrt{E}\,|x-y|\right) u^{d-3}\, du$$

$$\le 2(4\pi)^{-\frac{d}{2}}|x-y|^{2-d}\exp\left(-2\sqrt{kE}\,|x-y|\right) J_{d-3}\left(\frac14 - k\right)$$

Gauss integral $J_{d-3}\left(\frac14-k\right)<\infty$ pokud $\frac14 > k$

$J_0(a) = \frac12\sqrt{\frac{\pi}{a}}$

$J_1(a) = \frac{1}{2a}$

$-J_n(a) = \frac{\partial}{\partial a} J_{n-2}(a)$

29) $$(-\Delta+E)^{-1} = 2(4\pi)^{-\frac{d}{2}}|x-y|^{2-d}\int_0^\infty \exp\left(-\frac{u^2}{4} - \frac{E|x-y|^2}{u^2}\right) u^{d-3}\, du$$

$\Rightarrow (-\Delta+E)^{-1}\ge 0$

$$(-\Delta+E)^{-1} \le |x-y|^{2-d} \exp\big(\underbrace{-2\sqrt{kE}\,|x-y|}_{0<2\sqrt{k}<1}\big)\, \underbrace{2(4\pi)^{-\frac{d}{2}}J_{d-3}\left(\tfrac14-k\right)}_{c_d}; \quad 0<k<\frac14 \quad d\ge3$$

30) $$|(V(-\Delta+E)^{-1}\varphi)(x)| = |V(x)|\left|\int_{\mathbb{R}^3} (-\Delta+E)^{-1}(x,y)\varphi(y)\,dy\right| \le |V(x)|\int_{\mathbb{R}^3}|(-\Delta+E)^{-1}(x,y)|\,|\varphi(y)|\,dy$$

$$\le |V(x)|\left\{\int_{\mathbb{R}^3}[(-\Delta+E)^{-1}(x,y)]^2\, dy\right\}^{\frac12}\|\varphi(y)\|_2$$ proto: $\|u*v\|_r = \|u\|_p\|v\|_q$, $1+\frac1r = \frac1p+\frac1q$

$$\int_{\mathbb{R}^3}|(-\Delta+E)^{-1}(x,y)|^2\, dy \le \int_{\mathbb{R}^3} c_3^2\, |x-y|^{-2}\exp\left(-(1-\varepsilon)2\sqrt{E}\,|x-y|\right) dy = \int_{\mathbb{R}^+} c_3^2\, 4\pi \exp\left(-2(1-\varepsilon)\sqrt{E}\,r\right) dr$$

$$\le c_3^2 \frac{4\pi}{\sqrt{E}\,2(1-\varepsilon)}$$

$$\|V(-\Delta+E)^{-1}\varphi\|_2 \le \|V\|_2\, c_3^2 \frac{\pi}{\sqrt E}\,\frac{2}{1-\varepsilon}\|\varphi\|_2 \xrightarrow[E\to\infty]{} 0$$

$$\|V(-\Delta+E)^{-1}\varphi\|_2 \le \|V\|_2\, \|(-\Delta+E)^{-1}\varphi\|_\infty \le \|V\|_2\, \|(-\Delta+E)^{-1}\|_2\,\|\varphi\|_2$$