The Inflationary Universe

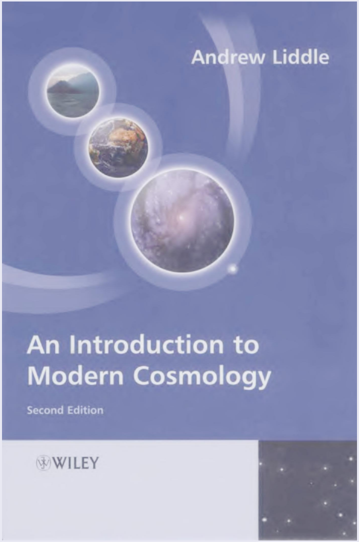
Michal Šumbera

April 30, 2018

CFRJS

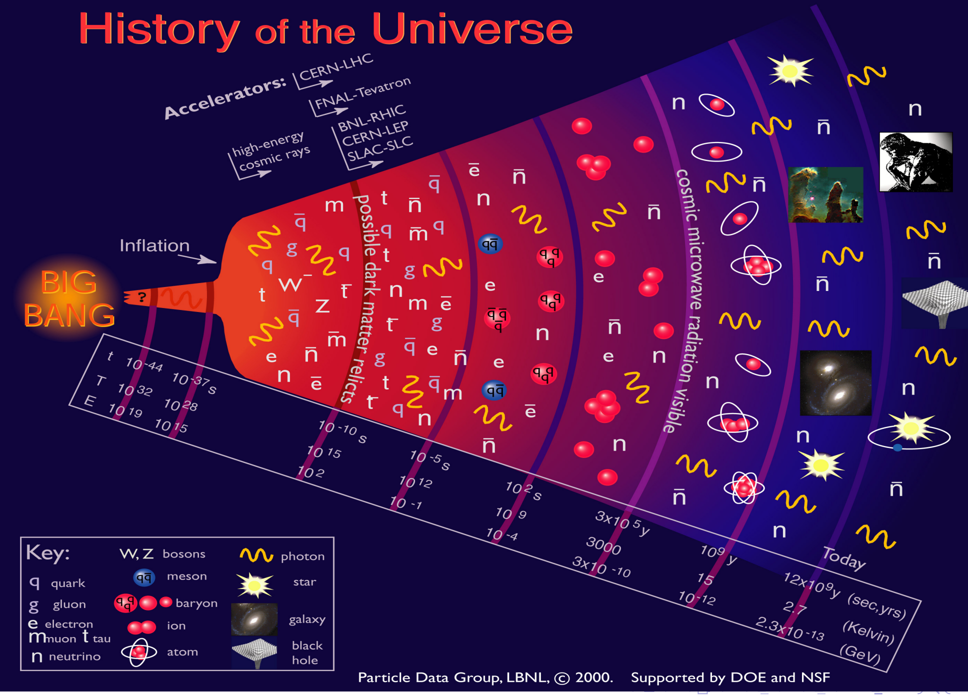
Literature

Our discussion is based on the book



The Inflationary Universe

- Problems with the Hot Big Bang
  - The Flatness Problem
  - The horizon problem
  - Relic particle abundances
- Inflationary expansion
- Solving the Big Bang problems
  - The flatness problem
  - The horizon problem
  - Relic particle abundances
- How much inflation?
- Inflation and particle physics
- Schwarzschild solution of Einstein equations and black holes
- Accelerating expansion of our universe: Non-dark energy scenario


History of the Universe
Accelerators:
CERN-LHC
FNAL-Tevatron
BNL-RHIC
CERN-LEP
SLAC-SLC
high-energy cosmic rays
Inflation
BIG BANG
?
possible dark matter relicts
cosmic microwave radiation visible
t 10 -44 10 -37 s
T 10 32 10 28
E 10 19 10 15
10 -10 s
10 15
10 2
10 -5 s
10 12
10 -1
10 2 s
10 9
10 -4
3x10 5 y
3000
3x10 -10
10 9 y
15
10 -12
Today
12x10 9 y (sec,yrs)
2.7 (Kelvin)
2.3x10 -13 (GeV)
Key:
q quark
g gluon
e electron
m muon t tau
n neutrino
W, Z bosons
meson
baryon
ion
atom
photon
star
galaxy
black hole
Particle Data Group, LBNL, © 2000. Supported by DOE and NSF

Inflation = solution to the Hot Big Bang scenario problems

- Despite all its successes, there remain some unsatisfactory aspects to the Hot Big Bang theory.
- Cosmological inflation – invented in 1981 by Alan Guth in a model of grand unification ("old inflation"). Scalar fields could get caught in a local minimum of the potential. The energy of the empty space would then have remained constant for a while as the Universe expanded $\Rightarrow$ $a(t) \propto \exp at$.
- Later substantially improved by Andrei Linde, Andreas Albrecht and Paul Steinhardt ("new inflation").
- Remains a hot research topic till nowadays.
- Not a replacement for the Hot Big Bang theory, but rather an extra add-on idea which is supposed to apply during some very early stage of the Universe's expansion.

Let us recall - Observational Parameters: Ω_0

- For which value of $H^2 = \frac{8}{3}\pi G\rho - ka^{-2}$ the Universe is flat?

  Flat $\Leftrightarrow$ $k = 0 \Rightarrow$
  $$\rho_c(t) = (3H^2)/(8\pi G) \quad (1)$$
- $G = 6.67 \times 10^{-11} m^3 kg^{-1} s^{-2} \Rightarrow$
  $\rho_c(t_0) = 1.88h^2 \times 10^{-26} kgm^{-3} = 2.78h^{-1} \times 10^{11} M_\odot/(h^{-1}Mpc)^3$
- Tiny density of matter is sufficient to halt and reverse the expansion of the Universe.
- $(10^{11} - 10^{12})M_\odot \simeq M_{galaxy}$, $R_{galaxy} \cong Mpc$
  $\Rightarrow$ the Universe cannot be far away from the critical density (within an order of magnitude or so).
- The Universe needs not be flat $\Rightarrow$ ρ_c is not necessarily the true density of the Universe, but it sets a natural scale for the density:
  $$\Omega(t) \equiv \rho/\rho_c \quad (2)$$
  where $\Omega(t)$ is (time-dependent) density parameter and $\Omega_0 \equiv \Omega(t_0)$ its present value.

Let us recall - Observational Parameters: Ω_0

- In the new variables the Friedman equation reads:

$$H^2 = \frac{8\pi G \rho_c \Omega}{3} - \frac{k}{a}^2 = H^2\Omega - \frac{k}{a}^2 \tag{3}$$

and so:

$$\Omega - 1 = \frac{k}{a^2 H^2} \tag{4}$$

- the case $\Omega = 1$ is very special, because in that case $k = 0$ and since k is a fixed constant the equation $\Omega = 1$ is true for all time.
- That's true independently of the type of matter in the Universe, and is often called a **critical-density Universe**.

- Multiplying Einstein equations by metrical tensor

$$g^{\mu\nu}\left[R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R\right] = R - 2R = -g^{\mu\nu}8\pi G T_{\mu\nu} = -8\pi G T^{\mu}_{\mu} \tag{5}$$

we obtain equivalent form:

$$R_{\mu\nu} = -8\pi G(T_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}T^{\lambda}_{\lambda}) \tag{6}$$

- In the vacuum energy-momentum tensor $T_{\mu\nu}$ vanishes $\Rightarrow$ Einstein field equations in empty space reduce to $R_{\mu\nu} = 0$, where

$$R_{\mu\nu} \equiv \partial^{\nu}\Gamma^{\lambda}_{\lambda\mu} - \partial^{\lambda}\Gamma^{\lambda}_{\mu\nu} + \Gamma^{\lambda}_{\mu\sigma}\Gamma^{\sigma}_{\nu\lambda} - \Gamma^{\lambda}_{\mu\nu}\Gamma^{\sigma}_{\nu\sigma} \tag{7}$$

and

$$\Gamma^{\mu}_{\nu\kappa} = \tfrac{1}{2}g^{\mu\lambda}(\partial_{\kappa}g_{\lambda\nu} + \partial_{\nu}g_{\lambda\kappa} + \partial_{\lambda}g_{\nu\kappa}) \tag{8}$$

- In a space-time of $N = 2$ or 3 dimensions this would imply the vanishing of the full curvature tensor $R_{\lambda\mu\nu\kappa}$ and the consequent absence of a gravitational field.
- **For $N \geq 4$ true gravitational fields can exist in empty space**.

- What about adding new terms to $G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$?
  - From $g_{\mu\nu}$ and its first derivatives no new tensor can be constructed $\Leftarrow$ at any point we can find a coordinate system in which the first derivatives of $g_{\mu\nu}$ vanish, so in this coordinate system the desired tensor must be equal to one of those that can be constructed out of the metric tensor alone, (e.g., $g_{\mu\nu}$ or $g^{\mu\nu}$ or $\varepsilon^{\mu\nu\lambda\eta}$ . . . , and so on), and since this is an equality between tensors it must be true in all coordinate systems.
  - The only possibility is: $$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \Lambda g_{\mu\nu} = -8\pi G T_{\mu\nu} \tag{9}$$
  - In the Friedmann equation Λ appears as an extra term, giving: $$H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} + \frac{\Lambda}{3} \tag{10}$$
  - **Einstein's original idea**: **use Λ and ρ to balance curvature k** $\Rightarrow$ $H(t) = 0$ **and hence a static Universe**. N.B. Such a balance proves to be unstable to small perturbations . . . .
  - Nowadays Λ is mostly discussed in the context of Universes with the flat Euclidean geometry $k = 0$.

- From the acceleration equation the effect of Λ can be seen more directly :
$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda}{3} \tag{11}$$
A positive cosmological constant gives a positive contribution to $\ddot{a}$, and so **acts effectively as a repulsive force**.
- For Λ sufficiently large, it can overcome the gravitational attraction represented by the first term in (11) $\Rightarrow$ **accelerating Universe**.
- Define new density parameter: $$\Omega_\Lambda \equiv \frac{\Lambda}{3H^2} \tag{12}$$
- Repeating the steps used to write the Friedmann equation in the form of (4), we find: $$\Omega + \Omega_\Lambda - 1 = ka^{-2}H^{-2} \tag{13}$$
- The flat Universe condition $k = 0$, generalizes to $\Omega + \Omega_\Lambda = 1$
  - Open Universe: $0 < \Omega + \Omega_\Lambda < 1$
  - Flat Universe: $\Omega + \Omega_\Lambda = 1$
  - Closed Universe: $\Omega + \Omega_\Lambda > 1$

Let us recall - Fluid description of Λ

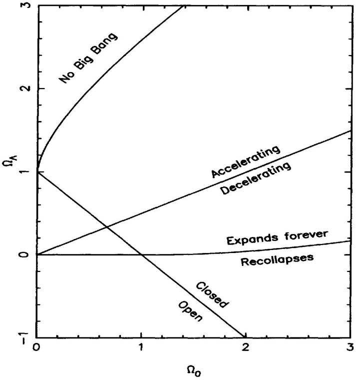
- Consider Λ as if it were a fluid with energy density ρ_Λ and pressure p_Λ. Defining $\rho_\Lambda \equiv \Lambda/(8\pi G)$ brings the Friedmann equation into the form
$$H^2 = \frac{8\pi G}{3}(\rho + \rho_\Lambda) - \frac{k}{a^2} \tag{14}$$
which automatically ensures that $\Omega_\Lambda \equiv \rho_\Lambda/\rho_c$
- Consider the fluid equation for Λ: $\dot{\rho_\Lambda} + 3\frac{\dot{a}}{a}\left(\rho_\Lambda + \frac{p_\Lambda}{c^2}\right) = 0$ (15)
  - $\rho_\Lambda = const.$ by definition $\Rightarrow$ $p_\Lambda = -\rho_\Lambda c^2$ (16)
  - Λ **has a negative effective pressure**.
  - As the Universe expands, work is done on the cosmological constant fluid.
  - Its energy density remains constant even though the volume of the Universe is increasing.
- Physical interpretation of Λ
  - In quantum physics: 'zero-point energy', which remains even if no particles are present. Unfortunately particle physics theories predict Λ $10^{124}\times$ larger than observations allow.

- $\Lambda \neq 0$ greatly increases range of possible behaviors of the Universe.
  - Closed Universe ($k > 0$) does not need to recollapse, nor an open Universe needs to expand forever.
  - For Λ big enough there's even no need for a Big Bang: Universe begins in a collapsing phase followed by a bounce at finite size under the influence of the Λ. (Such models are now ruled out by observations).

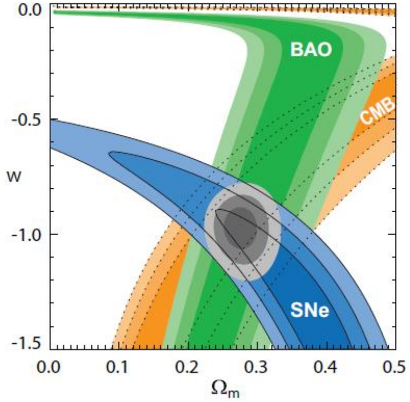
- Focus on $\Omega(t_0)$ and $\Lambda \Leftarrow H(t)$ provides only an overall scaling factor.
- Assume $p = 0$ matter.
- Pressureless Universe with $\Lambda \neq 0$ has:

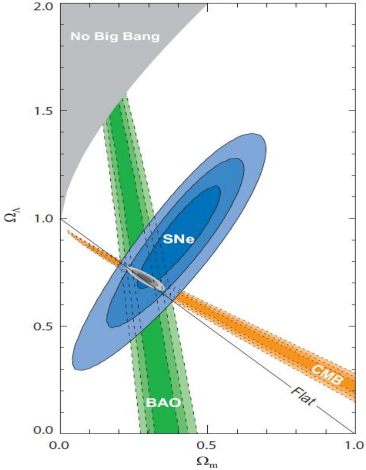
$$q_0 = \frac{1}{2}\Omega_0 - \Omega_\Lambda$$

- Assuming additionally $\Omega_0 + \Omega_\Lambda = 1$ (i.e. $k = 0$) this simplifies to $q_0 = \frac{3}{2}\Omega_0 - 1$ and acceleration occurs for $\Omega_\Lambda > 1/3$.
- For $\Omega_0 \leq 1$ the recollapse depends on $\Lambda \gtrless 0$, but for $\Omega_0 > 1$ the gravitational attraction of matter can overcome a small positive Λ and cause recollapse.



- M. Kowalski *et al.*, *Improved Cosmological Constraints from New, Old and Combined Supernova Datasets*, arXiv:0804.4142v1 [astro-ph] **Astrophysics Journal** 686:749-778,2008.
- N.B. $\Omega_m \equiv \Omega_0$, $p_\Lambda = -\, w\rho_\Lambda c^2$





The flatness problem

- Total density of material in the Universe (including dark energy) $\Omega_{tot} = \Omega_0 + \Omega_\Lambda$ is close to the critical density: $0.5 < \Omega_{tot} \leq 1.5$.
  $\Rightarrow$ The Universe is quite close to the flat (Euclidean) geometry.
- The Friedman equation can be rewritten as:

$$|\Omega_{tot}(t) - 1| = \frac{|k|}{a^2 H^2} \tag{17}$$

$\Rightarrow$ If Ω_{tot} is precisely equal to one, then it remains so for all times. But what if it is not?

- Neglecting curvature we have for
  - matter dominated Universe:

$$a(t) = \left(\frac{t}{t_0}\right)^{2/3}; \quad \rho(t) = \frac{\rho_0}{a^3} = \frac{\rho_0 t_0^2}{t^2} \Rightarrow a^2 H^2 \propto t^{-2/3} \tag{18}$$

  - radiation dominated Universe:

$$a(t) = \left(\frac{t}{t_0}\right)^{1/2}; \quad \rho(t) = \frac{\rho_0}{a^4} = \frac{\rho_0 t_0^2}{t^2} \Rightarrow a^2 H^2 \propto t^{-1} \tag{19}$$

The flatness problem

- Matter dominanted: $$|\Omega_{tot}(t) - 1| \propto t^{2/3} \quad (20)$$
- Radiation dominated: $$|\Omega_{tot}(t) - 1| \propto t \quad (21)$$
- $\Omega_{tot} - 1$ increases with time. $\Rightarrow$ The flat geometry is an unstable: if there is any deviation from flat geometry then the Universe will very quickly become more and more curved.
- For the Universe to be so close to flat geometry even at its large present age this means that at very early times it must have been extremely close to the flat geometry.
- N.B. The equations for $\Omega_{tot} - 1$ stop being valid once the curvature or cosmological constant terms are no longer negligible, since we used the a(t) solutions for the flat geometry to derive them. But they are fine to give us an approximate idea of what the problem is.

The flatness problem

- For simplicity assume Universe filled with radiation only. How close to one the density parameter $\Omega_{tot}(t)$ must have been at various early times t, based on the todays constraint (i.e. at $t_0 \simeq 4 \times 10^{17}$ sec)?

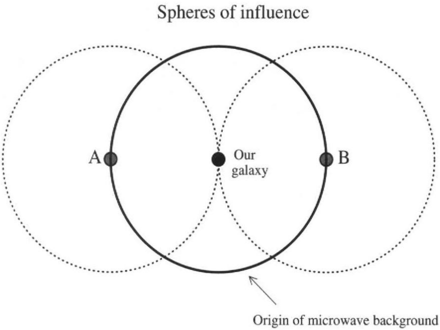
- At decoupling $t \simeq 10^{13}$sec we need $|\Omega_{tot}(t) - 1| \leq 10^{-5}$.
- At matter–radiation equality $t \simeq 10^{12}$sec, $|\Omega_{tot}(t) - 1| \leq 10^{-6}$.
- At nucleosynthesis $t \simeq 1$ sec, $|\Omega_{tot}(t) - 1| \leq 10^{-18}$.
- At the scale of EW symmetry breaking, which is the earliest known physics $t \simeq 10^{-30}$ sec wee need $|\Omega_{tot}(t) - 1| \leq 10^{-30}$!!!!!!

- The easiest solution:
  Suppose that the Universe must have precisely the critical density.
  No reason to prefer this choice over any other.
  One needs an explanation of such a value.

The horizon problem: CMB isotropy and fluctuations

- The Universe has a finite age. $\Rightarrow$ The distance which light could have travelled during the lifetime of the Universe gives rise to a region known as observable universe.
- CMB is very nearly isotropic, i.e. light from all parts of the sky has the same temperture of $T = 2.725K$. $\Rightarrow$ Different regions of the sky have been able to interact and move towards thermal equilibrium.
- Observed light from the opposite sides of the sky was in contact at the time $t_{decoup} \sim 3 \times 10^5$ y $\ll 1.4 \times 10^{10}$ y and since it just reached us it can't have made it all the way across the opposite side of the sky.
- CMB waves have travelled to us since decoupling uninterrupted. $\Rightarrow$ Regions must have been interacting and thermalizing at $t < t_{decoup}$.
- CMB exhibits small fluctuations which are the 'seeds' from which the structures in the Universe start to grow.
  $\Rightarrow$ For the same reasons that one can not thermalize separate regions one can not also create irregularities.

The horizon problem: CMB isotropy



- We detect CMB radiation from points **A** and **B** on opposite sides of the sky. These points are well separated and would not have been able to interact at all since the Big Bang. Dotted lines indicate the extent of regions able to influence points **A** and **B** by the present – far less manage to interact by the time the microwave radiation was released.
- In the Hot Big Bang model it is impossible to explain why they have the same temperature to such accuracy.

The horizon problem: CMB fluctuations

- The present homogeneous, isotropic domain of the Universe is at least as large as the present horizon scale,
  $c \cdot t_0 = 3 \times 10^{10} cm \cdot s^{-1} \times 1.4 \times 10^{10} \times \pi \times 10^7$ s $\sim 10^{28}$ cm.
- Primordial radiation dominates at $t_i \sim t_{\mathcal{P}\ell}$: $T_{\mathcal{P}\ell} \equiv \sqrt{\frac{\hbar c^5}{G k_B^5}} \sim 10^{32}$ K.
  For adiabatic expansion ($aT \sim const.$) the size of this domain was initially smaller by the ratio of the corresponding scale factors, $a_i/a_0 \sim T_0/T_{\mathcal{P}\ell} \sim 10^{-32}$.
- Assuming that inhomogeneity cannot be dissolved by expansion $\Rightarrow$ the size of the homogeneous, isotropic region from which our universe originated at $t = t_i$ was larger than $\ell_i \sim c t_0 a_i/a_0$.
- Which should be compared to the size of a causal region $\ell_c \sim c t_i$:
  $$\frac{\ell_i}{\ell_c} \sim \frac{t_0}{t_i}\frac{a_i}{a_0} \sim \frac{10^{17}}{10^{-43}} 10^{-32} \sim 10^{28} \Rightarrow \frac{V_i}{V_c} \sim \left(\frac{\ell_i}{\ell_c}\right)^3 \sim 10^{84}$$

$\Rightarrow$ In 10^{84} causally disconnected regions the energy density was smoothly distributed with a fractional variation not exceeding $\delta\epsilon/\epsilon \sim 10^{-4}$!!!!!

Relic particle abundances

- The radiation density reduces with expansion as a^{-4}.
  $\Rightarrow$ Even if the Universe starts with just a very small amount of NR matter then its slower reduction in density $\sim a^{-3}$ will rapidly bring it to prominence. This is true notwhistanding the fact that the Universe remained radiation dominated till $t \sim 1000$ years.
- Original motivation for inflation were magnetic monopoles. Such particles are an inevitable consequence of models of unification of fundamental forces, the so-called Grand Unified Theories. It is predicted that they were produced with a high abundance at a very early stage in the Universe.
- They are extraordinarily massive $M \sim 10^{16}$ GeV. Such particles would be NR for almost all the Universe's history, giving them plenty of time to come to dominate over radiation.
- But todays Universe is not dominated by magnetic monopoles.
  $\Rightarrow$ Theories predicting them are incompatible with the standard Hot Big Bang model!

Solution: Inflationary expansion

- Inflation: a period in the evolution of the Universe during which the scale factor a was accelerating.
- What is needed for $\ddot{a} > 0$? The acceleration equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) > 0. \tag{22}$$

$$\rho > 0 \quad \Rightarrow \qquad p < -\tfrac{1}{3}\rho c^2. \tag{23}$$

- Classic example of inflationary expansion – Universe with a cosmological constant Λ. The full Friedman equation, including other matter terms and curvature is:

$$H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} + \frac{\Lambda}{3} \tag{24}$$

N.B. The expansion rapidly reduces the first two terms while the last one remains constant.

Inflationary expansion

- So after a while $H^2 = \frac{\Lambda}{3}$. Since $H \equiv \frac{\dot{a}}{a} \quad \Rightarrow \quad \dot{a} = \sqrt{\frac{\Lambda}{3}}a$
- Because Λ is constant we arive at:

$$a(t) = \exp\left(\sqrt{\frac{\Lambda}{3}}t\right) \tag{25}$$

- After some time, inflation must end, with the energy in the cosmological constant being converted into conventional matter.
- One way to think of this is to view it as a decay of the particles acting as the cosmological constant into normal particles.
- Provided all this happens when the Universe was extremely young the Big Bang can then proceed just as before and none of the successes of the Hot Big Bang model are lost.
- In typical models the Universe is extremely young when inflation is supposed to occur, $t \sim 10^{-34}$s i.e. about the time of the Grand Unification occuring at energy density $\sim 10^{16}$ GeV.

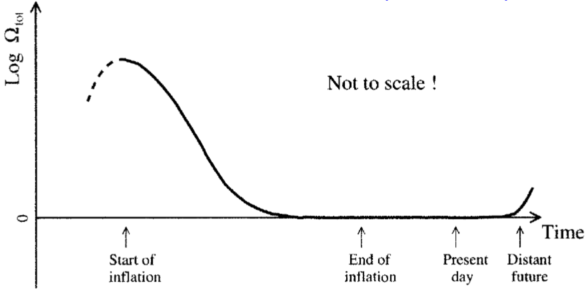
Solving the Big Bang problems – The flatness problem

- Inflation reverses the flatness problem:

$$\ddot{a} > 0 \quad \Rightarrow \quad 0 < \frac{d}{dt}(\dot{a}) = \frac{d}{dt}\left(a\frac{\dot{a}}{a}\right) = \frac{d}{dt}(aH) \tag{26}$$

- Inflation drives $\Omega_{tot}(t) \to 1$ rather than away from it!
- In the special case of perfect exponential expansion, the approach is particularly dramatic

$$|\Omega_{tot}(t) - 1| \propto \exp\left(-\sqrt{\frac{4\Lambda}{3}}t\right) \tag{27}$$



Solving the Big Bang problems – The flatness problem

- Analogy: balloon is very rapidly blown up to the size of the Sun; its surface would then look flat to us.
- The size of the portion of the Universe you can observe, given roughly by the Huble length cH^{-1} – (H^{-1} is roughly the age of the Universe and c the maximum speed) does not change while this happens. So very quickly you are unable to notice the curvature of the surface.
- By contrast, in the Big Bang scenario the distance you can see increases more quickly than the balloon expands, so you can see more of the curvature as time goes by.
- Inflation predicts a Universe extremely close to spatial flatness. Allowing for $\Lambda \neq 0$ in the present Universe, then a flatness requires

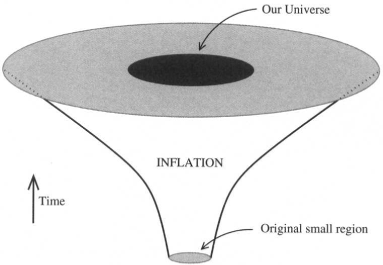
$$\Omega_0 + \Omega_\Lambda(t) = 1 \tag{28}$$

which is confirmed by the current observations.

Solving the Big Bang problems – The horizon problem

- Inflation greatly increases the size of a region of the Universe, while keeping its characteristic scale cH^{-1} fixed.

⇒ A small enough (to achieve thermalization before inflation) part of the Universe, can expand to be much larger than the size of our presently observable Universe.



- CMB microwaves coming from opposite sides of the sky really are at the same temperature because they were once in equilibrium.
- In a nutshell: due to inflation light can travel a much greater distance between the Big Bang and the time of decoupling than it can travel between decoupling and the present.

Solving the Big Bang problems – Relic particle abundances

- The dramatic expansion of the inflationary era dilutes away any unfortunate relic particles (magnetic monopoles *etc.*). Their density is reduced by the expansion more quickly than the cosmological constant.

- Provided enough expansion occurs, this dilution can easily make sure that the particles are not observable today; in fact, rather less expansion is needed than to solve the other problems.

- N.B. The decay of the cosmological constant which ends inflation must not regenerate the troublesome (i.e. extremely heavy) particles again $\Rightarrow$ the temperature of the Universe after inflation must not be too high, in order to make sure there is no new thermal production.

How much inflation?

Consider simplified model where

- Inflation ends at 10^{-34} sec.
- Expansion is strictly exponential.
- The Universe is perfectly radiation dominated all the way from the end of inflation to the present.
- The value of Ω_{tot} near the start of inflation is not hugely different from one.

How much inflation?

- The present age of the Universe is $H^{-1} = 4 \times 10^{17}\text{sec}$. During radiation domination $|\Omega_{tot}(t) - 1| \propto t$. So

$$|\Omega_{tot}(t) - 1| \leq 0.1 \Rightarrow |\Omega_{tot}(10^{-34}\mathrm{sec}) - 1| \leq 0.13 \times 10^{-53} \tag{29}$$

- During inflation $H = const. \Rightarrow \quad |\Omega_{tot}(t_0) - 1| \propto a^{-2}$

$\Leftrightarrow$ during inflation a is increased by a factor of at least 10^{27}!!

- Incredibly, by the standards of what comes out of inflation model building this isn't much at all. Expansion by factors like 10^{10^8} are not uncommon!
- Consider, for example, characteristic expansion time, $H^{-1} = 10^{-36}$ sec. Then between 10^{36} sec and 10^{34} sec, the expansion factor is

$$\frac{a_{final}}{a_{initial}} \backsimeq exp\left[H(t_{final} - t_{initial})\right] \backsimeq e^{99} \backsimeq 10^{43} \tag{30}$$

Inflation and particle physics

- Not to spoil nucleosynthesis the inflation must have happened before 1 sec. $\Rightarrow$ at $T \geq 10^{17}$ K.
- To describe such extreme physical conditions, when violent particle collisions are the norm, fundamental particle physics is required, and in particular theories of the fundamental interactions. Inflation is assumed to be driven by a new, as-yet-undiscovered, form of matter required by such theories.
- Key idea – phase transitions which are controlled by an unusual form of matter known as a scalar field φ. Depending on the precise nature of the transition scalar fields can behave with a negative pressure, and can satisfy the inflationary condition $\rho c^2 + 3p < 0$. That is, they behave like an effective cosmological constant.
- Once the phase transition comes to an end, the scalar field decays away and the inflationary expansion terminates, hopefully having achieved the necessary expansion by a factor of 10^{27} or more.

Inflation from particle physics

- Consider the simplest theory of a one-component real scalar field φ with the mass m and coupling constant λ . The Lagrangian is:

$$L = \frac{1}{2}(\partial_\mu \varphi)^2 - \frac{m^2}{2}\varphi^2 - \frac{\lambda}{4}\varphi^4 \tag{31}$$

- Assume $\lambda \ll 1$. For φ small we can neglect the last term in (31) and the field satisfies usuall Klein–Gordon equation:

$$(\Box + m^2)\,\varphi \equiv \ddot{\varphi} - \Delta\varphi + m^2\,\varphi = 0 \tag{32}$$

whose general solution corresponds to the propagation of free (non–interacting) particles of mass m and momentum k:

$$\varphi(x) = (2\pi)^{-3/2} \int \frac{d^3 k}{\sqrt{2k_0}} \left[e^{i\,k\,x}\,a^+(\mathbf{k}) + e^{-i\,k\,x}\,a^-(\mathbf{k})\right] \tag{33}$$

- The field $\varphi(x)$ oscillates around $\varphi = 0$ which is also a minimum of the so-called Effective potential:

$$\mathrm{V}(\varphi) = \frac{1}{2}(\nabla\varphi)^2 + \frac{m^2}{2}\varphi^2 + \frac{\lambda}{4}\varphi^4 \tag{34}$$

Inflation from spontaneous symmetry breaking

- Consider now the same Lagrangian but with $m^2 = -\mu^2$:

$$L = \frac{1}{2}(\partial_\mu \varphi)^2 + \frac{\mu^2}{2}\varphi^2 - \frac{\lambda}{4}\varphi^4 \tag{35}$$

- The solution instead of oscillating around $\varphi = 0$ has modes that grow exponentially near $\varphi = 0$ when $\mathbf{k}^2 < m^2$:

$$\delta\varphi(\mathbf{k}) \sim \exp\left(\pm\sqrt{\mu^2 - \mathbf{k}^2}\, t\right) \cdot \exp(\pm i\, \mathbf{k}\, \mathbf{x}) \tag{36}$$

- The minimum of the effective potential:

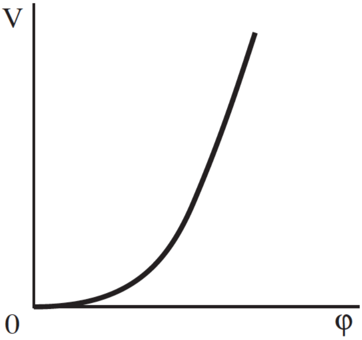
$$\mathrm{V}(\varphi) = \frac{1}{2}(\nabla\varphi)^2 - \frac{\mu^2}{2}\varphi^2 + \frac{\lambda}{4}\varphi^4 \tag{37}$$

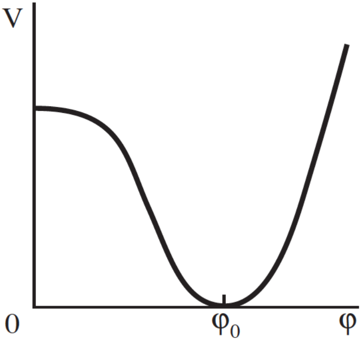
  now occurs at $\varphi_c = \pm\mu/\sqrt{\lambda} \neq 0$.

- Thus, even if the field is initially zero, it soon undergoes a transition (after $t \sim \mu^{-1}$) to a stable state $\varphi_c = \pm\mu/\sqrt{\lambda}$ a phenomenon known as spontaneous symmetry breaking (SSB)*.

---

*) Yochiro Nambu, Nobel prize 2008.





Effective potential $V(\varphi)$ in the simplest theories of the scalar field φ.

$V(\varphi)$ in the theory (31). $V(\varphi)$ in the theory (35).

Inflation from spontaneous symmetry breaking

- After SSB, excitations of the field φ near $\varphi_0 = \pm\mu/\sqrt{\lambda}$ can also be described by a solution which is a superposition of plane waves. In order to see that, let's make the change of variables

$$\varphi \to \varphi + \varphi_0 \tag{38}$$

- The Lagrangian (35) then takes the form:

$$\begin{aligned} \mathrm{L}(\varphi + \varphi_0) &= \frac{1}{2}(\partial_\mu(\varphi + \varphi_0))^2 + \frac{\mu^2}{2}(\varphi + \varphi_0)^2 - \frac{\lambda}{4}(\varphi + \varphi_0)^4 \\ &= \frac{1}{2}(\partial_\mu \varphi)^2 - \frac{3\,\lambda\,\varphi_0^2 - \mu^2}{2}\,\varphi^2 - \lambda\,\varphi_0\,\varphi^3 - \frac{\lambda}{4}\varphi^4 \\ &+ \frac{\mu^2}{2}\varphi_0^2 - \frac{\lambda}{4}\varphi_0^4 - \varphi\,(\lambda\varphi_0^2 - \mu^2)\,\varphi_0 \end{aligned} \tag{39}$$

- N.B. for $\varphi_0 \neq 0$, the effective mass squared of the field φ is not equal to $-\mu^2$, but rather $m^2 = 3\,\lambda\,\varphi_0^2 - \mu^2$, (40)

Inflation from spontaneous symmetry breaking

- The minimum of $V(\varphi) = \frac{1}{2}(\nabla\varphi)^2 - \frac{\mu^2}{2}\varphi^2 + \frac{\lambda}{4}\varphi^4$ (41)

  now occurs at $\varphi_0 = \pm\mu/\sqrt{\lambda} \Rightarrow m^2 = 3\lambda\frac{\mu^2}{\lambda} - \mu^2 = 2\mu^2 > 0$; (42)

  i.e. the mass squared of the field φ has now the correct sign.
- Reverting to the original variables, we can write the solution for φ as:

$$\varphi(x) = \varphi_0 + (2\pi)^{-3/2}\int \frac{d^3k}{\sqrt{2k_0}}\left[e^{ikx}a^+(\mathbf{k}) + e^{-ikx}a^-(\mathbf{k})\right]. \quad (43)$$

  corresponding to particles of the field φ with mass given by (42), propagating against the background of the constant classical field φ_0.
- The field φ_0 is constant over all space and there is no any preferred reference frame associated with it $\Rightarrow$ it is a classical field.
- The constant therm in (39) $\frac{\mu^2}{2}\varphi_0^2 - \frac{\lambda}{4}\varphi_0^4 - \varphi(\lambda\varphi_0^2 - \mu^2)\varphi_0$ can now be rewritten as $\frac{\mu^4}{2\lambda} - \frac{\mu^4}{4\lambda} = \frac{\mu^4}{4\lambda}$ thus represents an additive constant to the Lagrangian which is usually ignored.

Inflation and particle physics

- Introduction of a constant homogeneous scalar field φ_0 represents a restructuring of the vacuum. Space filled with a constant scalar field φ_0 remains "empty" — the constant scalar field does not carry a preferred reference frame with it, it does not disturb the motion of objects passing through the space that it fills, and so forth.
- In general relativity, however, the constant field affects the properties of space-time:

$$\mathrm{R}_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu}\, \mathrm{R} = 8\,\pi\,\mathrm{G}\,\mathrm{T}_{\mu\nu} = 8\,\pi\,\mathrm{G}\,[\tilde{\mathrm{T}}_{\mu\nu} + g_{\mu\nu}\,\mathrm{V}(\varphi)], \tag{44}$$

  where $\tilde{\mathrm{T}}_{\mu\nu}$ is energy-momentum tensor of particles and $g_{\mu\nu}\,\mathrm{V}(\varphi)]$ is the energy-momentum tensor of the vacuum.
- N.B. The "pressure" exerted by the vacuum and its energy density have opposite signs, $p = -\rho = -\mathrm{V}(\varphi)$.

Estimate of the vacuum energy: QFT

- Uncertainty principle: any coordinate localization in (t, x) has a spread in (E, p) value $\Rightarrow$ vacuum has an energy.

$\Rightarrow$ The cosmological constant Λ, as the energy density of the vacuum, naturally has a non-zero value (Zeldovich 1968).

- The simplest estimate: boson field. Normal modes are simply a set of harmonic oscillators. The vacuum energy (zero–point energy) is:

$$E_\Lambda = \sum_i \frac{1}{2}\hbar\omega_i \quad , \tag{45}$$

where $\sum_i = \int d^3x d^3p (2\pi\hbar)^{-3}$ is understood.

- Energy density of the vacuum $\rho_\Lambda = E_\Lambda / V$, where $\int d^3x = V$ is position–independent (i.e. it is constant over the space).

Estimate of the vacuum energy: Bosonic QFT

- Momentum integration:

$$\rho_\Lambda = \frac{E_\Lambda}{V} = \int_0^{p_{\mathcal{P}\ell}} \frac{4\pi p^2 dp}{(2\pi\hbar)^3} \left(\frac{1}{2} \sqrt{p^2 c^2 + m^2 c^4} \right) \tag{46}$$

where the upper limit is given by the Planck momentum:

$$p_{\mathcal{P}\ell} = \frac{E_{\mathcal{P}\ell}}{c} = \sqrt{\frac{\hbar c^3}{G}} \backsimeq 10^{19} GeV/c. \tag{47}$$

The integration yields:

$$\rho_\Lambda \backsimeq \frac{1}{16\pi^2} \frac{E^4_{\mathcal{P}\ell}}{\hbar c^3} \backsimeq \frac{(3 \times 10^{27} \mathrm{e}V)^4}{\hbar c^3} \tag{48}$$

- Normalizing this to the value of critical density

$$\rho_c(t) = (3H^2)/(8\pi G) \backsimeq \frac{(2.5 \times 10^{-3} \mathrm{e}V)^4}{\hbar c^3} \tag{49}$$

one obtains for $\Omega_\Lambda \equiv \rho_\Lambda / \rho_c$:

$$(\Omega_\Lambda)_{QFT} = 10^{120} \qquad \text{vs.} \qquad (\Omega_\Lambda)_{obs} = 0.75 \qquad !!!!!!$$

Estimate of the vacuum energy: Supersymmetric QFT

- For fermion field the normal modes are also a set of harmonic oscillators.
$$E_\Lambda = \sum_i \left(-\frac{1}{2} + n_i\right)\hbar\omega_i \quad , \quad n_i = 0, 1 \tag{50}$$
- The vacuum energy for fermion field is negative!
$\Rightarrow \exists$ mechanism for cancelation between the bosonic and fermionic vacuum energy contributions.
- Supersymmetry (SUSY) – extension of the Standard Model predicts equality between bosonic and fermionic d.o.f. $\Rightarrow$ Vacuum energy of the system with exact supersymmetry must vanish!
- SUSY predicts $m_b = m_f$. In reality the supersymmetry is badly broken $\Delta m^2 \equiv m_f^2 - m_b^2 \gtrsim (10^2 GeV/c^2)^2$. The first–order contribution from broken SUSY is still 80–90 orders short of the required $\mathcal{O}(10^{-120})$:

$$(\rho_\Lambda)_{SUSY} = \frac{1}{16\pi^2}\frac{E_{\mathcal{P}\ell}^4}{\hbar c^3}\left(\frac{\Delta m^2}{E_{\mathcal{P}\ell}^2}\right) = \frac{1}{16\pi^2}\frac{E_{\mathcal{P}\ell}^4}{\hbar c^3} \times 10^{-36} \tag{51}$$

Schwarzschild solution of Einstein eqs. and black holes

In 1915 Schwarzschild solving $G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -8\pi G T_{\mu\nu}$

where $R_{\mu\nu} \equiv \frac{\partial\Gamma^{\lambda}_{\lambda\mu}}{\partial x^{\nu}} - \frac{\partial\Gamma^{\lambda}_{\mu\nu}}{\partial x^{\lambda}} + \Gamma^{\lambda}_{\mu\sigma}\Gamma^{\sigma}_{\nu\lambda} - \Gamma^{\lambda}_{\mu\nu}\Gamma^{\sigma}_{\nu\sigma}$, $R \equiv R^{\mu}_{\mu}$

and $\Gamma^{\mu}_{\nu\kappa} = \frac{1}{2}g^{\mu\lambda}(\partial_\kappa g_{\lambda\nu} + \partial_\nu g_{\lambda\kappa} + \partial_\lambda g_{\nu\kappa})$

for a spherical distribution of mass M and radius R found:

$$g_{tt} = f(r),\ g_{rr} = 1/f(r),\ g_{\theta\theta} = r^2,\ g_{\phi\phi} = r^2 sin^2\theta \tag{52}$$

where $f(r) = (1 - \frac{R_s}{r})$ and $R_s = 2GM$ is Schwarzschild radius

$$ds^2 = f(r)dt^2 - \frac{1}{f(r)}dr^2 - r^2 d\Omega^2 \tag{53}$$

or in Eddington-Finkelstein coordinates $p = t + r + R_s \log(\frac{|r-R_s|}{R_s})$:

$$ds^2 = f(r)dp^2 - 2dpdr - r^2 d\Omega^2 \tag{54}$$

Radial light rays follow paths determined by solving $f(r)dp^2 = 2dpdr$. Light rays along $dp = 0$ are always ingoing. Light rays along $f(r)dp = 2dr$ are are outgoing for $r > R_s$ and ingoing for $r < R_s$.

Temperature, acceleration and the black holes

- Combining Gravitation (G), Thermodynamics (T) and quantum mechanics ($\hbar$) Stephen Hawking calculated the temperature of non–rotating black hole with mass M as*:

$$T_{BH} = \frac{\hbar c^3}{8\pi k_B GM} = \frac{\hbar c}{4\pi GR_s} = \frac{1}{4\pi GR_s} \tag{55}$$

where $R_s = 2GM/c^{\,2}$ is the Schwarzschild radius.

- Unruh effect: Vacuum seen by the uniformly accelerated observer with acceleration a looks to him like as a heath bath of temperature**:

$$T = \frac{\hbar \mathrm{a}}{2\pi c k_B} = \frac{\mathrm{a}}{2\pi}. \tag{56}$$

---

*) For derivation of (55) see e.g. A. Zee: Einstein gravity in a nutshell. From dimensional analysis one gets $T_{BH} \sim 1/(GM)$.

**) For derivation of (58) see e.g. A. Zee: Einstein gravity in a nutshell or Wikipedia.

Black Hole Entropy - Bekenstein 1972

Leonard Susskind: The Black Hole War

- What is an increase in energy (mass) of the black hole when we add a single bit of information carried by a single photon?
- Photon in order to carry a single bit of information must have its location as uncertain as possible, provided only that it gets into the black hole. i.e. its wavelenght should be close to BH diameter $\lambda = 2R_s$ and so its energy must be $E_\gamma = 2\pi\hbar c/(2R_s) = \pi\hbar c/(R_s)$.
- Corresponding change of the BH mass is $\Delta M = \pi\hbar/R_s c$ and the change in its radius equals $\Delta R_s = 2G\Delta M/c^2 = 2\pi\hbar G/(R_s c^3)$.
- Change in the BH entropy is given by the first law of thermodynamics $\Delta E = \Delta(Mc^2) = T_{BH}\Delta S = \hbar c^3 \Delta S/8\pi GM$
  $\Rightarrow \; S = 4\pi GM^2/(\hbar c) = 4\pi R_s^2 c^3/(4\hbar G) = A/(4\ell_{P\ell}^2)$.
- Adding one bit of information will increase the area of the horizon (and so also the entropy) of any black hole by one square Planck unit.

Non-dark energy scenario: Entropic accelerating universe

- In (Damien A. Easson, Paul H. Frampton, George F. Smoot; Phys.Lett. B696 (2011) 273) an alternative interpretation of accelerating Universe was proposed based on the entropy and temperature intrinsic to the horizon of the universe.
- The acceleration is due to an entropic force* naturally arising from the information storage on the horizon surface screen.
- Entropy = the information holographically stored to the horizon (t'Hooft, Susskind).
- The assumption is that cosmological horizon has both a temperature and entropy associated with it as was first shown by Gary Gibbons and Stephen Hawking.

*) Entropic force comes from to the entire system's thermodynamical tendency to increase its entropy and not from particular underlying microscopic force. For example pressure of an ideal gas has an entropic origin.

Non-dark energy scenario: Entropic accelerating universe

- Let's take now instead of Black Hole Schwarzschild radius take the apparent horizon of the universe (Hubble radius) $R_H = c/H$.
  The associated temperature, $T_{Horizon} \equiv T_H$ can be estimated as:

$$T_H = \frac{\hbar}{k_B}\frac{H}{2\pi} \sim 3 \times 10^{-30} K \tag{57}$$

- The temperature of the horizon is associated with the acceleration:

$$a_H = \frac{2\pi c k_B T_H}{\hbar} = cH \sim 10^{-9} ms^{-2} \tag{58}$$

$\Rightarrow$ The ambiguous dark energy component is non-existent.
Instead there is an entropic force contribution acting at the horizon and pulling outward towards the horizon to create the appearance of a dark energy component.

Non-dark energy scenario: Entropic accelerating universe

- The entropy on the Hubble Horizon, e.g. the Hubble radius $R_H = H^{-1}$, is
$$S_H = \pi \ell_{P\ell}^2 R_H^2 k_B \sim (2.6 \pm 0.3) \times 10^{122} k_B \tag{59}$$
- Increasing the radius R_H, by Δr, increases the entropy by ΔS_H according to
$$\Delta S_H = 2\pi \ell_{P\ell}^2 R_H \Delta r \tag{60}$$
- The entropic force is simply
$$F_r = -\frac{dE}{dr} = -T\frac{dS}{dr} = -T_H \frac{dS_H}{dr} = \frac{H}{2\pi G}\frac{2\pi}{H} = -\frac{1}{G} \tag{61}$$
the minus sign indicates pointing in the direction of increasing entropy, which in this case is the horizon.
- The pressure from entropic force exerted is
$$p = \frac{F_r}{A} = -\frac{1}{4\pi R_H^2} T_H \frac{dS_H}{dr} = -\frac{H^2}{4\pi}\frac{1}{G} = -\frac{2}{3}\rho_c \tag{62}$$
This is close to the value of the currently measured dark energy/cosmological constant negative pressure!!!