

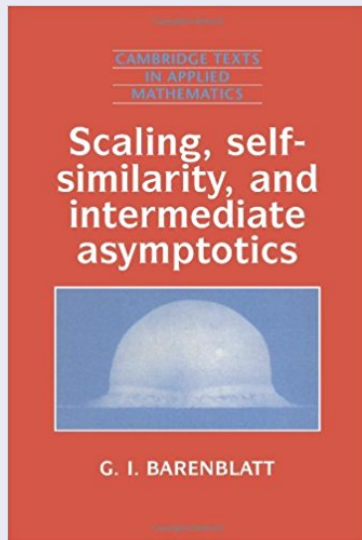
Nuclear Explosion

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Our discussion is based on the book



TNT equivalent: Wikipedia

- TNT equivalent – a convention for expressing energy, typically used to describe the energy released in an explosion.
- The energy liberated by 1g of TNT was arbitrarily defined as a matter of convention to be $4184 \text{ J} = 1 \text{ kcal} = 1.163 \text{ Wh}$.
- 1 kiloton (kt) and megaton (Mt) of TNT are traditional of energy release defined by that convention to be $4.184 \text{ TJ} (=1.163 \text{ kWh})$ and $4.184 \text{ PJ} (=1.163 \text{ MWh})$, which is the approximate energy released in the detonation of 1kg and 1000 kg of TNT, respectively.
- The kiloton (kt) and megaton (Mt) of TNT have traditionally been used to describe the energy output, and hence the destructive power, of a nuclear weapon.
- Using $E = mc^2$ we have $1\text{Mt TNT} = 46.5 \text{ g}$.
- The TNT equivalent appears in various nuclear weapon control treaties, and has been used to characterize the energy released in such other highly destructive events as an asteroid impact.

Nuclear and non-nuclear weapons examples

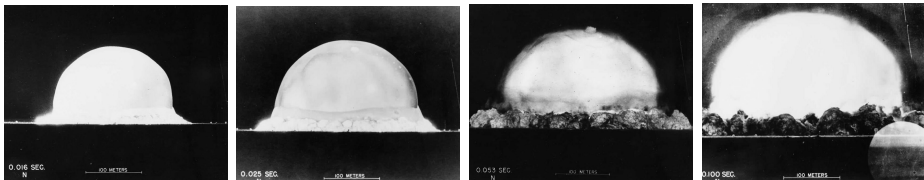
Nuclear weapons and the most powerful non-nuclear weapon examples			
Weapon	Total yield (kilotons of TNT)	Weight (kg)	R.E. ~
Davy Crockett (nuclear device)	0.022	23	1,000
Fat Man (dropped on Nagasaki) A-bomb	20	4600	4,500
Classic (one-stage) fission A-bomb	22	420	50,000
Hypothetical suitcase nuke	2.5	31	80,000
Typical (two-stage) nuclear bomb	500–1000	650–1120	900,000
W56 thermonuclear warhead	1,200	272–308	4,960,000
W88 modern thermonuclear warhead (MIRV)	470	355	1,300,000
B53 nuclear bomb (two-stage)	9,000	4050	2,200,000
B41 nuclear bomb (three-stage)	25,000	4850	5,100,000
Tsar nuclear bomb (three-stage)	50,000–56,000	26,500	2,100,000
GBU-57 bomb (Massive Ordnance Penetrator , MOP)	0.0035	13,600	0.26
Grand Slam (Earthquake bomb , M110)	0.0065	9,900	0.66
Bomb used in Oklahoma City (ANFO based on racing fuel)	0.0018	2,300	0.78
BLU-82 (Daisy Cutter)	0.0075	6,800	1.10
MOAB (non-nuclear bomb, GBU-43)	0.011	9,800	1.13
FOAB (advanced thermobaric bomb , ATBIP)	0.044	9,100	4.83

- The relative effectiveness (RE) factor relates an explosive's demolition power to that of TNT, in units of the TNT equivalent/kg (TNTe/kg).
- The RE factor is the relative mass of TNT to which an explosive is equivalent: The greater the RE, the more powerful the explosive.

Examples of even bigger explosions

Megatons of TNT	Energy [Wh]	Description
26.3	30.6 TWh	Megathrust earthquakes 2004 Indian Ocean earthquake
200	232 TWh	The total energy released by the eruption of Mt. Krakatoa in Indonesia in 1883.
540	628 TWh	The total energy produced worldwide by all nuclear testing and combat
1,460	1.69 PWh	The total global nuclear arsenal
62,500	73 PWh	The total solar energy received by Earth per minute
875,000	1,000 PWh	Approximate yield of the last eruption of the Yellowstone supervolcano .
6,000,000 = 6×10^6	6,973 PWh	The estimated energy at impact when the largest fragment of Comet Shoemaker–Levy 9 struck Jupiter
9.32×10^6	10,831 PWh	The energy released in the 2011 Tōhoku earthquake and tsunami
9.56×10^6	11,110 PWh	Megathrust earthquakes record huge M_w values, or total energy released. The 2004 Indian Ocean earthquake
1×10^8	116,222 PWh	The approximate energy released when the Chicxulub impact caused the mass extinction 66 million years ago
5.972×10^{15}	6.94×10^{27} Wh	The explosive energy of a quantity of TNT the mass of Earth .
7.89×10^{15}	9.17×10^{27} Wh	Total solar output in all directions per day.
1.98×10^{21}	2.3×10^{33} Wh	The explosive energy of a quantity of TNT the mass of the Sun .
$2.4 \times 10^{28} - 4.8 \times 10^{28}$	$2.8 - 5.6 \times 10^{40}$ Wh	A type 1a supernova
$2.4 \times 10^{30} - 4.8 \times 10^{30}$	$2.8 - 5.6 \times 10^{42}$ Wh	The largest type of supernova observed, gamma-ray bursts (GRBs)
1.3×10^{32}	1.5×10^{44} Wh	A merger of two black holes, first observation of gravitational waves

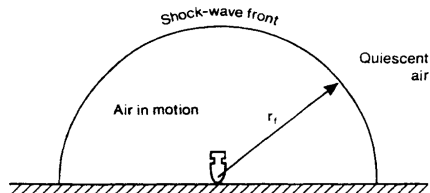
Trinity, birth of the atomic age



- At 5:30 AM on the morning of July 16, 1945, the pre-dawn stillness of the New Mexico desert was shattered by the most momentous, man-made explosion of all time. At a site called Trinity, a plutonium bomb was assembled atop a 100 foot steel tower.
- The bomb was detonated, producing an intense flash and a fireball that expanded to 600 meters in two seconds. The explosive power was equivalent to 18.6 kilotons of TNT. It grew to a height of more than 12 kilometers, boiling up in the shape of a mushroom. Forty seconds later, the blast of air from the bomb reached the observation bunkers, along with a long and deafening roar of sound. And so began the ATOMIC AGE...

www.atomcentral.com/trinity-explosion

Shock-wave description of nuclear explosion



- What are the quantities on which the shock wave radius r_f depends?
 - E – energy of explosion, $[E] = M \cdot L^2 \cdot T^{-2}$
 - ρ_0 – initial density of exploding matter, $[\rho_0] = M \cdot L^{-3}$
 - r_0 – initial radius of exploding matter, $[r_0] = L$
 - p_0 – initial pressure, $[p_0] = M \cdot L^{-1} T^{-2}$
 - t – time when we measure r_f , $[t] = T$

Shock-wave description: G.I. Taylor solution 1941

- Assumption (1): A finite amount of energy is suddenly released in an infinitely concentrated form $\Rightarrow r_0 = 0$.
- Assumption (2): At time t when we measure r_f the motion is at the stage when the maximum pressure of the moving gas reached at the shock-wave front $p_{max} \gg p_0 \Rightarrow p_0 = 0$.
- Assumption (3): At intermediate stage of a nuclear explosion, when a very intense shock wave propagates in the atmosphere, the gas motion inside **the shock wave can be considered as adiabatic**.

\Rightarrow Can use adiabatic index $\gamma = c_p/c_V = (f + 2)/f$, where f equals #d.o.f. of the gas.

- N.B. For a diatomic gas (terrestrial air), with $f = 5$ d.o.f. (3 translational and 2 rotational d.o.f.) $\Rightarrow \gamma = 7/5$.

Dimensional analysis

- Neglecting r_0 and ρ_0 let's construct from E , t and ρ_0 quantity \mathfrak{R} having the same dimension as $[r_f] = L$, i.e. the length:

$$\mathfrak{R} = \left(\frac{Et^2}{\rho_0} \right)^{1/5} \quad (1)$$

- Explosion is point-like ($r_0 = 0$) \Rightarrow The dimensionless quantity $l = r_f / \mathfrak{R}$ is unchanged when we perform the scale transformation

$$r_f \rightarrow \lambda r_f; \quad \lambda > 0 \quad (2)$$

- The (dimensionless) quantity l then can be expressed as

$$l = r_f / \mathfrak{R} = F(\mathfrak{R}, \rho_0, t, \gamma) \quad (3)$$

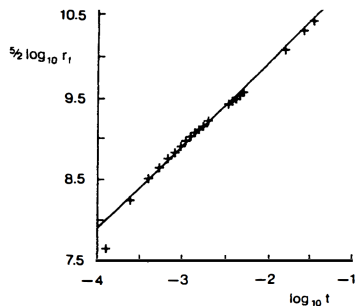
where F is a certain function.

Dimensional analysis

- Crucial are the assumptions (1) and (2) that r_0 and ρ_0 are negligibly small.
- The first 3 arguments of $F(\mathcal{R}, \rho_0, t, \gamma)$ have independent dimensions. But units of time are absent in the dimensions of \mathcal{R} and ρ_0 .
- By varying the time scale we will vary the numerical value of t while leaving the values of $l = F(\mathcal{R}, \rho_0, t, \gamma)$ and its arguments \mathcal{R} , ρ_0 and γ (last is a fixed abstract number). $\Rightarrow l$ is independent on t .
- By the same arguments l is independent on ρ_0 and \mathcal{R} .

Scaling law for the radius of the shock wave

- Thus from (1) one obtains: $r_f = C(\gamma)E^{1/5}t^{2/5}\rho_0^{-1/5}$ (4)



- Measuring position of the shock wave front at several different times we can determine the coefficients $C(\gamma), E^{1/5}, \rho_0^{-1/5}$ and hence the energy of nuclear explosion.
- Using the same arguments one can obtain the scaling laws

$$p_f \sim E^{2/5} \rho_0^{3/5} t^{-6/5}, \quad \rho_f = \rho_0, \quad v_f = E^{1/5} \rho_0^{-1/5} t^{-3/5} \quad (5)$$