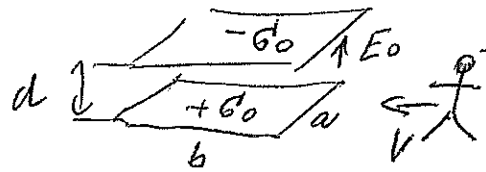
Magnetické pole (náboj je invariantní)

①

Přípravný vztah: pohyb vůči kondenzátoru a/nebo pohyb kondenzátoru

$d << (a)(b)$

Ⓐ



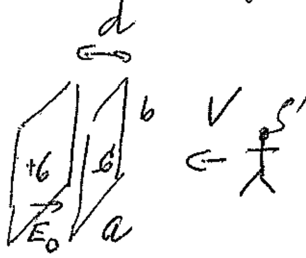
$E_0 = \frac{\sigma_0}{\varepsilon_0} \approx \frac{Q}{ab\varepsilon_0}$; $U_0 = E_0 d$

pohybuje-li se rychlostí V (vůči pozorovateli) pozorovatel „vidí" pole $E' = \frac{\sigma'}{\varepsilon_0} = \frac{Q}{a\frac{b}{\gamma_V}\varepsilon_0} = \frac{\sigma_0}{\varepsilon_0}\gamma_V = E_0\gamma_V$

(kontrakce délek)

$U' = E' d = U_0 \cdot \gamma_V$

Ⓑ



$E_0 = \frac{\sigma_0}{\varepsilon_0} = \frac{Q}{ab\varepsilon_0}$; $E' = \frac{\sigma_0}{\varepsilon_0} = \frac{Q}{ab\varepsilon_0} = E_0$

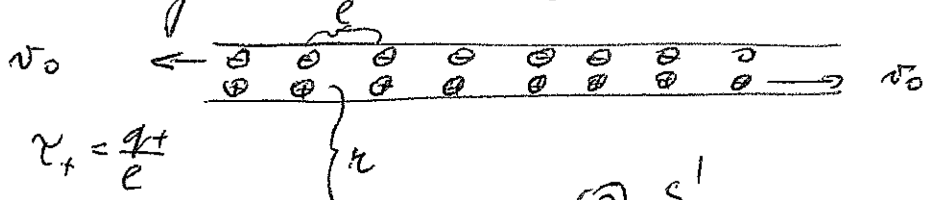
$U' = E' \cdot d' = E_0 d \gamma_V = U_0 \gamma_V$

Závěr: $E'_x = E_x$, $E'_y = E_y \gamma_V$, $E'_z = E_z \cdot \gamma_V$

pro pohyb uvnitř kondenzátoru — známe pouze pole!!!

Motivační model — mg. pole z kontrakce

Elektrolyt v trubici



$\tau_+ = \frac{q_+}{l}$

$\tau_- = \frac{q_-}{l}$ ① q $F(q) = Eq = 0$ ② S' $q \longrightarrow V$

$E = \frac{\tau}{2\pi r \varepsilon_0}$ z Gausse pro ∞ drát

$l = l_0\sqrt{1-\frac{v_0^2}{c^2}}$; $l_0 = \frac{l}{\sqrt{1-\frac{v_0^2}{c^2}}}$

(2)

$$\tau_+' = \frac{q_+}{l_+'} = \frac{q_+}{l_0\sqrt{1-\frac{v_+'^2}{c^2}}} = \frac{q_+\sqrt{1-\frac{v_0^2}{c^2}}}{l\sqrt{1-\frac{v_+'^2}{c^2}}}$$

$$\tau_-' = \frac{q_-}{l_-'} = \frac{q_-}{l_0\sqrt{1-\frac{v_-'^2}{c^2}}} = \frac{q_-\sqrt{1-\frac{v_0^2}{c^2}}}{l\sqrt{1-\frac{v_-'^2}{c^2}}}$$

$$\tau_+' + \tau_-' = \frac{q}{\gamma_{v_0} l}\left(\gamma_{v_+'} - \gamma_{v_-'}\right) = \boxed{-2\tau_+ \gamma_V \frac{V v_0}{c^2}}$$

$$\left(\gamma_{v_+'} - \gamma_{v_-'}\right) = \frac{1}{\sqrt{1-\frac{1}{c^2}\left(\frac{v_0-V}{1-\frac{v_0V}{c^2}}\right)^2}} - \frac{1}{\sqrt{1-\frac{1}{c^2}\left(\frac{v_0+V}{1+\frac{v_0V}{c^2}}\right)^2}} =$$

skl. rychlostí

$$\beta_V = \frac{V}{c},\ \beta_0 = \frac{v_0}{c}$$

$$= \frac{1}{\sqrt{1-\left(\frac{\beta_0-\beta_V}{1-\beta_0\beta_V}\right)^2}} - \frac{1}{\sqrt{1-\left(\frac{\beta_0+\beta_V}{1+\beta_0\beta_V}\right)^2}} = \frac{1-\beta_0\beta_V}{\sqrt{1-\beta_0^2-\beta_V^2+\beta_0^2\beta_V^2}} - \frac{1+\beta_0\beta_V}{\sqrt{1-\beta_0^2-\beta_V^2+\beta_0^2\beta_V^2}} =$$

$$= \frac{-2\beta_0\beta_V}{\sqrt{(1-\beta_0^2)(1-\beta_V^2)}} = -2\beta_0\beta_V\gamma_{v_0}\gamma_V$$

$$E' = \frac{\tau_+' + \tau_-'}{2\pi\varepsilon_0 r'} \qquad r' = r\ ;\ \tau_+ = -\tau_-$$

$$\boxed{2\tau_+ v_0 = I}$$

$$E' = -\frac{2\tau_+\gamma_V V v_0}{2\pi r\varepsilon_0 c^2}.$$

$$\boxed{\mu_0 = \frac{1}{\varepsilon_0 c^2}}$$

z kondenzátoru

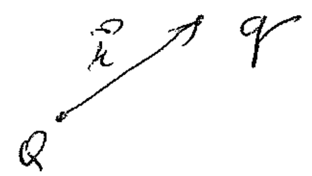
$$E = \frac{E'}{\gamma_V} = -\frac{2\tau_+ V v_0}{2\pi r\varepsilon_0 c^2} \Rightarrow F_y = -q\frac{\mu_0 I}{2\pi r}\cdot V$$

$$F \perp V$$

$$\vec F = q\left(\vec V\times\vec B\right) \quad \text{Lorentzova síla}$$

Ilustrační výpočet – z kontrakce délek Lorentzovu sílu. Pokud q v klidu ⊕ a ⊖ stejně husté. q v pohybu – transformace rychlostí.

Obecné síly $\vec{Q}$ na q

(1) 

$\vec v(Q) = 0 = \vec v(q)$

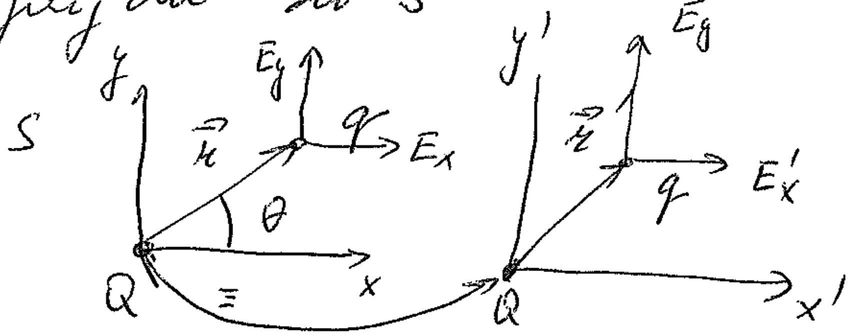
(2) $\vec v(Q) = 0$; $\vec v(q) \neq 0$

náboj se nemění při Lorentz. transf.

$$\vec F_c = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^3}\vec r$$

(3) $\vec v(Q) \neq 0$ $\quad \vec v(q) = 0$ (řešíme v rovině x, y. z je ekvivalentní y)

Zjistím v S' kde $\vec v'(Q) = 0$
a přejdu do S



$O \equiv O'$

$t = t' = 0$

L.T.: $x' = \gamma x$; $y' = y$

z „kondenzátoru"

v S'

$$E'_x = \frac{Q}{4\pi\varepsilon_0}\frac{x'}{r'^3} = \frac{Q}{4\pi\varepsilon_0}\frac{\gamma x}{r'^3} = E_x$$

$$E'_y = \frac{Q}{4\pi\varepsilon_0}\frac{y'}{r'^3} = \frac{Q}{4\pi\varepsilon_0}\frac{y}{r'^3} = \frac{E_y}{\gamma}$$

$\Downarrow \qquad\qquad \Downarrow$

$$\frac{E'_x}{E'_y} = \frac{x'}{y'} \qquad\qquad \frac{E_y}{E_x} = \frac{y}{x}$$

V obou případech je $\vec E$ radiální
(v obou soustavách souřadnic)

$$r'^3 = \left[(\gamma x)^2 + y^2+z^2\right]^{3/2} = \left[x'^2+y'^2+z'^2\right]^{3/2}$$

$$\vec E = (\vec E_x, \vec E_y, \vec E_z) = (E_x', \gamma E_y', \gamma E_z') = \gamma \frac{Q}{4\pi\varepsilon_0}\frac{(x,y,z)}{r'^3}$$

$$E^2 = \vec E\cdot\vec E = \left(\frac{Q}{4\pi\varepsilon_0}\right)^2 \gamma^2 \frac{x^2+y^2+z^2}{\left[(\gamma x)^2+y^2+z^2\right]^3} = \left(\frac{Q}{4\pi\varepsilon_0}\right)^2 \frac{x^2+y^2+z^2}{\gamma^4\left(x^2+(y^2+z^2) - \frac{V^2}{c^2}(y^2+z^2)\right)^3}$$

$$= \left(\frac{Q}{4\pi\varepsilon_0}\right)^2 \frac{(x^2+y^2+z^2)\left(1-\frac{V^2}{c^2}\right)^2}{(x^2+y^2+z^2)^3\left(1-\frac{\frac{V^2}{c^2}(y^2+z^2)}{x^2+y^2+z^2}\right)^3} = \left(\frac{Q}{4\pi\varepsilon_0}\right)^2 \frac{\left(1-\frac{V^2}{c^2}\right)^2}{r^4\left(1-\frac{V^2}{c^2}\sin^2\theta\right)^3}$$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}\underbrace{\frac{\left(1-\frac{V^2}{c^2}\right)}{\left(1-\frac{V^2}{c^2}\sin^2\theta\right)^{3/2}}}_{\text{Heavisideův faktor}} = \Gamma \cdot \frac{Q}{4\pi\varepsilon_0 r^2}$$

$\theta = 0$ – pole zeslabí

$\theta = \frac{\pi}{2}$ – pole zesílí

$$\vec E = \Gamma \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^3}\vec r$$

$$\phi = \oint \vec E\, d\vec S = \frac{Q}{\varepsilon_0}$$

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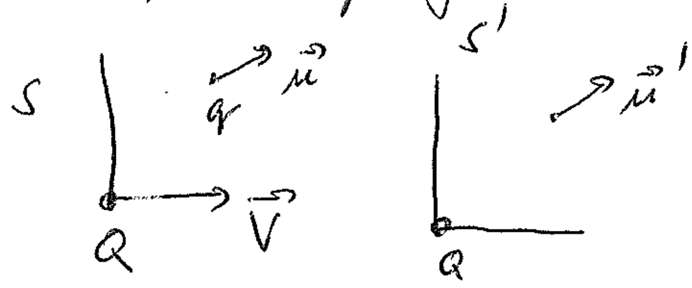
(4) Oba náboje v pohybu – složité

$r' = \gamma r$



$$\vec F' = \vec F_c'$$

S. S. tak, aby $\vec V \parallel x$

$$\vec F_c' = q\cdot \vec E' = q\cdot\frac{Q}{4\pi\varepsilon_0}\frac{\vec r'}{r'^3}$$

z STR

$$\vec F = (1-\gamma)\frac{\vec F'\cdot\vec V}{V}\cdot\frac{\vec V}{V} + \gamma\vec F' + \frac{\gamma}{c^2}\vec u\times(\vec V\times\vec F')$$

$$\vec{F} = \frac{qQ}{4\pi\varepsilon_0 r'^3}\left\{\left[(1-\gamma)\frac{x'V^2}{V^2}(1,0,0) + \gamma(x',y',z') + \frac{\gamma}{c^2}\left[\vec{u}\times(\vec{V}\times\vec{r}')\right]\right]\right\} =$$

$$= \frac{qQ}{4\pi\varepsilon_0 r'^3}\left[\underbrace{(x',\gamma y',\gamma z')}_{=\gamma\vec{r}} + \frac{\gamma}{c^2}\vec{u}\times(\vec{V}\times\vec{r}')\right] =$$

$$= \frac{qQ}{4\pi\varepsilon_0 r'^3}\gamma\cdot\vec{r} + \frac{qQ}{4\pi\varepsilon_0 r'^3}\gamma\frac{1}{c^2}\vec{u}\times(\vec{V}\times\vec{r}) =$$

$$= q\cdot\vec{E} + q\gamma\frac{1}{c^2}\vec{u}\times(\vec{V}\times\vec{E}') =$$

od pohybujícího se náboje v S $\boxed{\vec{E} = \frac{Q}{4\pi\varepsilon_0}\gamma\frac{\vec{r}}{r'^3}}$ v S'

$$\vec{V} = (V,0,0)\ ;\ (E_x, E_y, E_z) = (E_x', \gamma E_y', \gamma E_z')$$

$$\boxed{\vec{V}\times\gamma\vec{E}' = \vec{V}\times\vec{E}}$$

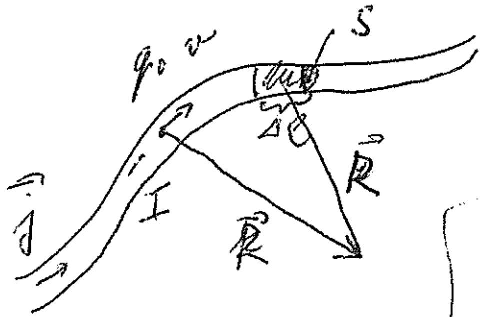
$$= q\vec{E} + q\frac{1}{c^2}\vec{u}\times(\vec{V}\times\vec{E})$$

$$\vec{B} = \frac{1}{c^2}\vec{V}\times\vec{E} \;\text{"="}\; \frac{Q}{4\pi\varepsilon_0 c^2}\frac{1}{r^3}(\vec{V}\times\vec{r}) =$$

$$= \frac{\mu_0}{4\pi}\frac{Q}{r^3}(\vec{V}\times\vec{r}) \qquad \frac{1}{r'^3}\approx\frac{1}{r^3}$$

$\vec{F}$ – Lorentzova síla

Biot - Savartův zákon



$$\vec{B}_{q_0} = \frac{q_0 \mu_0}{4\pi R^3}(\vec{v}\times\vec{R})$$

$$\Delta\vec{B}_{\Delta l} = \frac{\mu_0 N q_0\, \Delta l\, S}{4\pi R^3}(\vec{v}\times\vec{R}) =$$

$\vec{v} \parallel \Delta\vec{l}$

$$② = \frac{\mu_0}{4\pi}\,\frac{\rho v S}{R^3}(d\vec{l}\times\vec{R}) =$$

$$= \frac{\mu_0}{4\pi}\,\frac{I}{R^3}(d\vec{l}\times\vec{R}) \text{ nebo}$$

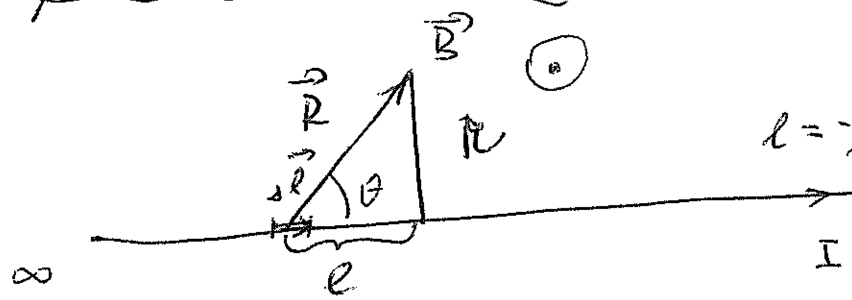
$$N q_0 \vec{v} = \rho\cdot\vec{v} = \vec{j}$$

①

$$\Delta\vec{B}_{\Delta l} = \frac{\mu_0}{4\pi}\,\frac{\Delta V}{R^3}(\vec{j}\times\vec{R})$$

$$\boxed{\vec{B} \,\text{"}=\text{"}\, \sum \Delta\vec{B} \,\text{"}=\, \frac{\mu_0}{4\pi} I \int_l \frac{d\vec{l}\times\vec{R}}{R^3} = \frac{\mu_0}{4\pi}\int_V \frac{\vec{j}\times\vec{R}}{R^3}\,dV}$$

Pole přímkového vodiče



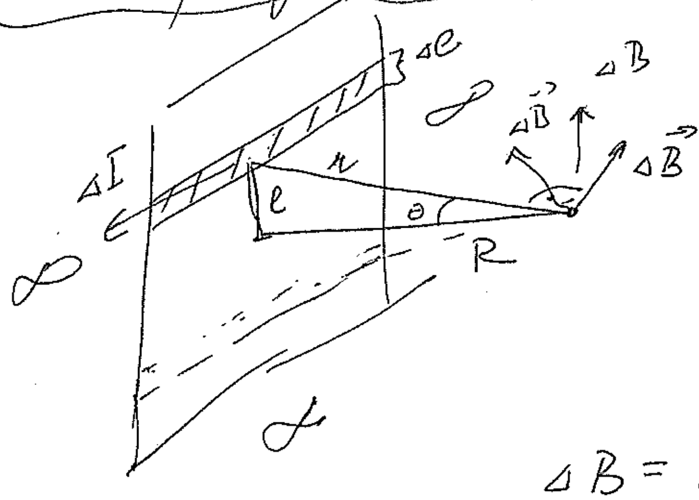
$$|\vec{R}| = \frac{r}{\sin\theta}$$

$$l = -\frac{r}{\operatorname{tg}\theta}, \quad dl = r\frac{d\theta}{\sin^2\theta}$$

$$|\vec{B}| = \frac{\mu_0}{4\pi} I \int_{-\infty}^{+\infty} \frac{d\vec{l}\times\vec{R}}{R^3} = \frac{\mu_0 I}{4\pi}\int_0^{\pi} \frac{\frac{r\,d\theta}{\sin^2\theta}\cdot\frac{r}{\sin\theta}\,\sin\theta}{\frac{r^3}{\sin^3\theta}}$$

$$= \frac{\mu_0 I}{4\pi r}\left[-\cos\theta\right]_0^{\pi}\vec{e}_\perp = \frac{\mu_0}{2\pi}\,\frac{I}{r}\,\vec{e}_\perp$$

Rovinný proud



α – hustota plošná proudu (lineární)

$dI = \alpha \cdot dl \qquad r = \frac{R}{\cos\theta}$

$\Delta I = \alpha \cdot \Delta l$

$\Delta B = 2 \cdot \frac{\mu_0 \Delta I}{2\pi r} \cos\theta =$

$\boxed{\alpha = u \cdot \sigma}$

kondenzátor rychlostí u

$\sigma = \frac{\sigma_0}{\sqrt{1 - \frac{u^2}{c^2}}}$

$\boxed{l = R \cdot tg\,\theta;\ dl = \frac{R \cdot d\theta}{\cos^2\theta}}$

$= \frac{\mu_0 \Delta I}{\pi R} \cos^2\theta = \frac{\mu_0 \alpha}{\pi R} \Delta l \cos^2\theta$

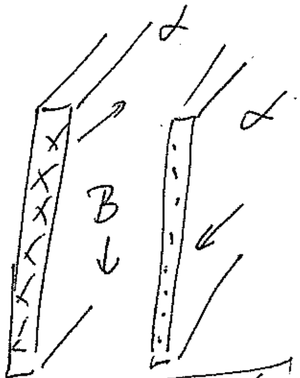
$\lim \Delta l \to 0$

$dB = \frac{\mu_0 \alpha}{\pi R} R\, d\theta$

$$B = \cancel{\int_0^\infty dB} = \int_0^{\frac{\pi}{2}} \frac{\mu_0 \alpha}{\pi} \cdot d\theta = \frac{\mu_0 \alpha}{2} \quad \left(\frac{\sigma}{2\varepsilon_0} \right)$$

mg μ_0

el $\frac{1}{\varepsilon_0}$

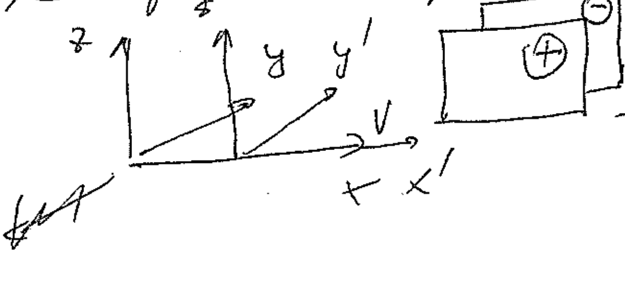


$B = 0$ $\qquad B = 0$

$\boxed{B = \mu_0 \alpha}$

Transformace polí

B, E – z kondenzátoru



v S $\begin{cases} \alpha = \sigma \cdot u \\ B_z = \mu_0 \alpha = \mu_0 \sigma u \\ E_y = \frac{\sigma}{\varepsilon_0} \end{cases}$

v S' $\quad u' = \dfrac{u - V}{1 - \frac{uV}{c^2}}$; $\quad$ když, v ... $\quad \sigma_0 = \sigma\sqrt{1-\frac{u^2}{c^2}} = \sigma'\sqrt{1-\frac{u'^2}{c^2}}$

$$\sigma' = \sigma \frac{\sqrt{1-\frac{u^2}{c^2}}}{\sqrt{1-\frac{(u-V)^2}{\left(1-\frac{uV}{c^2}\right)^2 c^2}}} = \sigma \frac{\sqrt{1-\frac{u^2}{c^2}}\left(1-\frac{uV}{c^2}\right)}{\sqrt{\left(1-\frac{uV}{c^2}\right)^2-\frac{(u-V)^2}{c^2}}}$$

$$= \sigma \frac{\sqrt{1-\frac{u^2}{c^2}}\left(1-\frac{uV}{c^2}\right)}{\sqrt{1+\left(\frac{u}{c}\frac{V}{c}\right)^2-\cancel{2\frac{uV}{c\,c}}-\frac{u^2}{c^2}-\frac{V^2}{c^2}+\cancel{2\frac{uV}{c^2}}}} =$$

$$= \sigma \frac{\cancel{\sqrt{1-\frac{u^2}{c^2}}}\left(1-\frac{uV}{c^2}\right)}{\sqrt{\cancel{\left(1-\frac{u^2}{c^2}\right)}\left(1-\frac{V^2}{c^2}\right)}} = \sigma\gamma_V\left(1-\frac{uV}{c^2}\right)$$

$$E_y' = \frac{\sigma'}{\varepsilon_0} = \gamma_V\left(\frac{\sigma}{\varepsilon_0} - \frac{Vu\sigma}{\varepsilon_0 c^2}\right) = \gamma_V\left(E_y - VB_z\right)$$

podobně

$$E_z' = \gamma_V\left(E_z + VB_y\right) \quad ; \quad E_x' = E_x$$

$$B_z' = \mu_0\sigma' u' = \mu_0\gamma_V\sigma\left(1-\frac{uV}{c^2}\right)u' = \qquad u' \leftarrow \frac{u-V}{1-\frac{uV}{c^2}}$$

dosadit $\qquad \mu_0 = \frac{1}{\varepsilon_0 c^2}$

$$\cancel{= \mu_0\sigma\gamma_V\left(1-\frac{u}{c}\frac{V}{c}\right)\frac{u'}{c}c =}$$

$$\mu_0\sigma\gamma_V(u-V) = \gamma_V\left(B_z - \frac{V}{c^2}E_y\right) \qquad \vec{V} = (V,0,0)$$

$$B_y' = \gamma_V\left(B_y + \frac{V}{c^2}E_z\right) \quad ; \quad B_x' = B_x$$

$$\vec{E}' = \gamma\left(\vec{E} + \vec{V}\times\vec{B}\right) \quad ; \quad \vec{B}' = \gamma\left(\vec{B} - \frac{1}{c^2}\vec{V}\times\vec{E}\right)$$

Invariantní veličiny: $\vec{E}'\cdot\vec{B}' = \vec{E}\cdot\vec{B}$

pokud $E \perp B$ pak ve $\forall$ s.s. inerciálních

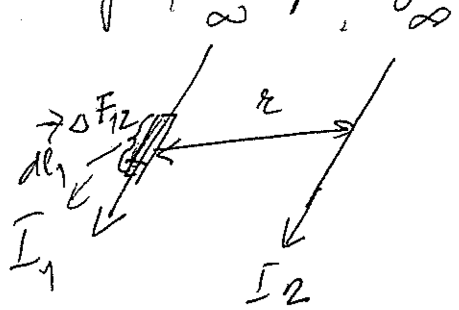
$$B'^2 - \frac{1}{c^2}E'^2 = B^2 - \frac{1}{c^2}E^2 \;\Big|\Big|\cdot\frac{1}{2\mu_0}\Big) = \Big(\frac{\varepsilon_0 E^2}{2} - \frac{B^2}{2\mu_0}$$

hustota energ

(q je invariantní

zmenší se objem a i energie v něm. Q ne!

Síly mezi proudy



$$\Delta\vec{F}_{12} = \rho\cdot dV(\vec{E} + \vec{u}\times\vec{B}) =$$

$$= \rho\, dV\vec{E} + (I_1\, d\vec{l}_1 \times \vec{B}_{1(2)}$$

pole od I_2 v místě

$$\rho\vec{u}\, dV = \vec{j}\,\Delta S\, dl = I\cdot d\vec{l}$$

neutr. $\rho_{celk} = 0$ – $\pm$ vykompenzováno

$$|\Delta F_{12}| = I_1\cdot dl_1\cdot\frac{\mu_0}{2\pi r}I_2\,;\quad F/_{1m} = \int_0^1 dF_{12} = \frac{\mu_0}{2\pi}\frac{I_1 \ldots}{r}$$

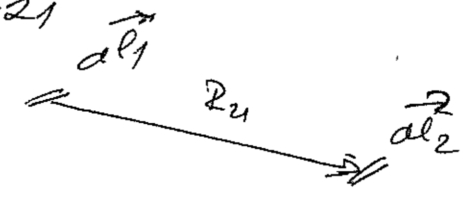
Definice ampéru – síla na 1m délky $= 2\cdot 10^{-7}$ N

B.-S.

Ampérův vzorec: $d\vec{F} = I\, d\vec{l}\times\vec{B}\,;\; d\vec{B} = \frac{\mu_0}{4\pi}I\frac{d\vec{l}\times \ldots}{R^3}$

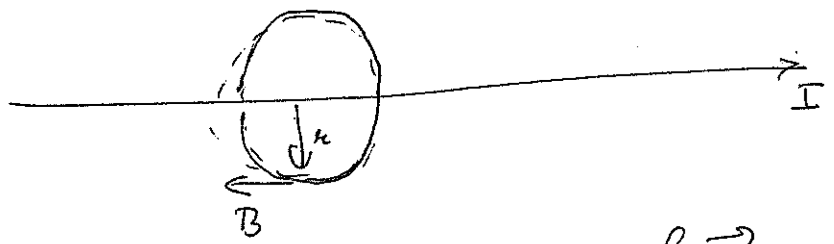
$$d\vec{F}'' = \frac{\mu_0}{4\pi}\frac{I_1 I_2}{R_{21}^3}\left[d\vec{l}_2\times(d\vec{l}_1\times\vec{R}_{21})\right]$$

od $d\vec{B}$



Stacionární mg. pole (Ampérův zákon)

$$\frac{\partial}{\partial t} = 0$$

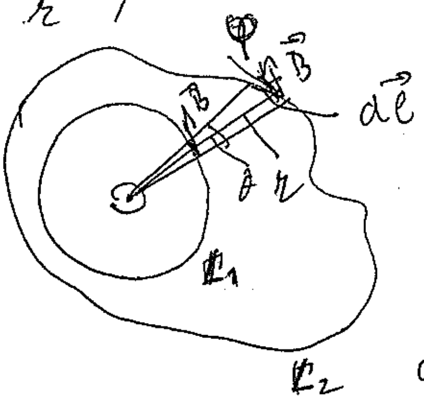


∞ drát

$$B = \frac{\mu_0}{2\pi}\frac{I}{r} \qquad \Gamma = \oint_L \vec{B}\cdot d\vec{l} = \oint_L \frac{\mu_0}{2\pi}\frac{I}{r}\,dl$$

$$= \mu_0 I$$

$B \sim \frac{1}{r}$; $l \sim 2\pi r$ (jako Gauss)



$$\vec{B}\cdot d\vec{l} = B\cdot dl\cdot\cos\varphi =$$

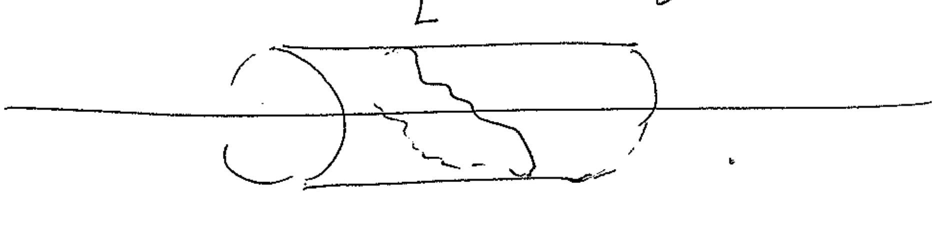
$$=$$

$$\cos\varphi\cdot dl \approx d\theta\cdot r$$

$$B\cdot dl\cdot\cos\varphi = \frac{B\cdot r\,d\theta}{\cos\varphi}\cos\varphi = \frac{\mu_0}{2\pi r} I r\,d\theta$$

$$\int_0^{2\pi} d\theta\,\frac{\mu_0}{2\pi}I\,d\theta = \mu_0 I$$

další tvar — válec



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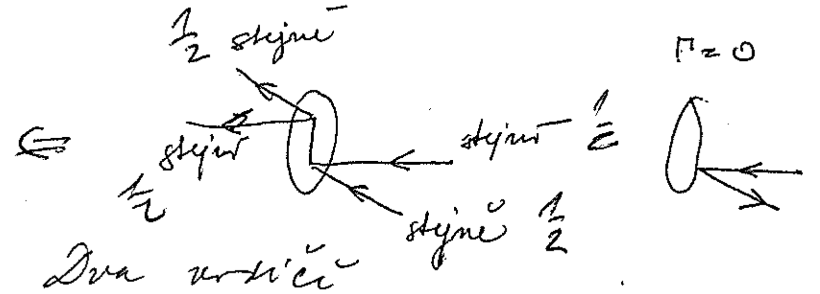
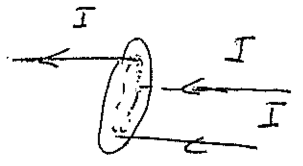
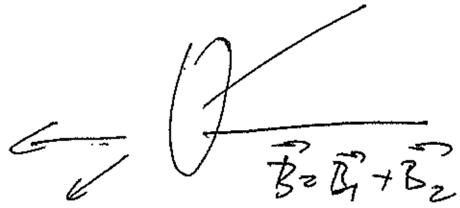




Dva vodiče

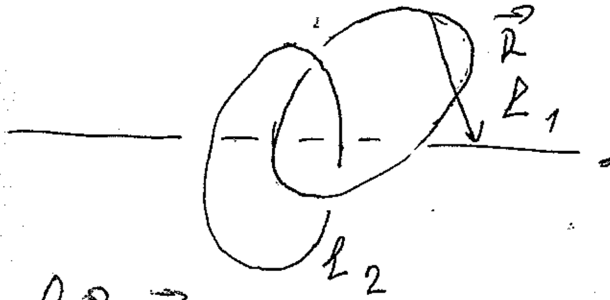


$$\Gamma = \mu_0 \sum_i I_i$$

$$\left(\phi = \frac{\sum Q}{\varepsilon_0} \right)$$

Vzájemnost

$I_1 = I_2 = I_\infty = \dots$



$$\Gamma_{12} = \oint_{L_1} \vec{B}_{12}\, d\vec{l}_1 =$$

$$= \oint_{L_1} \left[\oint_{\infty 2} \frac{\mu_0}{4\pi} I \frac{d\vec{l}_\infty \times \vec{R}}{R^3} \right] d\vec{l}_1 =$$

$$= \oint_{\infty L_2} \left[- \oint_{L_1} \frac{\mu_0}{4\pi} I \frac{d\vec{l}_1 \times \vec{R}}{R^3} \right] d\vec{l}_{\infty 2}$$

$$= \int_{\infty} \vec{B}_{\infty 1}\, d\vec{l}_\infty = -\Gamma_{\infty 2}$$

$B_{\infty 1}$ na přecpovodem I ve smyčce

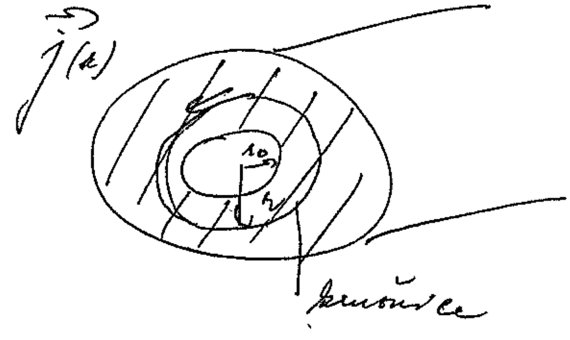
$$\boxed{\vec{a} \cdot (\vec{b} \times \vec{c}) = \vec{b} \cdot (\vec{c} \times \vec{a}) = \vec{c} \cdot (\vec{a} \times \dots}$$

$$= \int_\infty \vec{B} \cdot d\vec{l}$$

$$\Gamma_2 = \Gamma_\infty$$

$|\Gamma_{12}| = |\Gamma_{21}|$ pro stejný proud

Ampér + symetrie počítání



$$\Gamma = \oint \vec{B} \cdot d\vec{l} = B \cdot 2\pi r$$

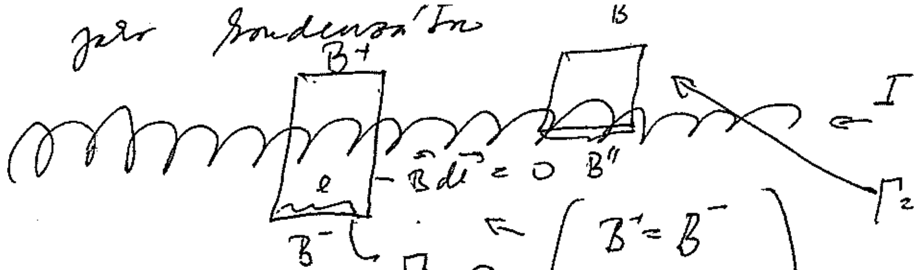
$$= \mu_0 \int_S \vec{j}\, d\vec{S} =$$

$\vec{j} = \text{konst}$

$$= \mu_0\, j \cdot \pi (r^2 - r_0^2)$$

$$\Rightarrow B =$$

jako kondenzátor



$-\vec{B}\,d\vec{l} = 0$

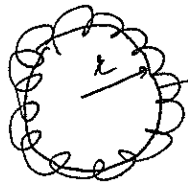
$\Gamma = 0$ $\left(\begin{array}{c} B^+ = B^- \\ 2B^+ l = 0 \\ B^+ = 0 \end{array}\right)$

$\Gamma = B'' \cdot l = \mu_0 I_c = \mu_0 \cdot N I$

$B'' = \mu_0 \frac{N}{l} I$

hustota závitů

Toroid



$B\, 2\pi r = \mu_0 N \cdot I$

$$\boxed{B = \frac{\mu_0 N \cdot I}{2\pi r}}$$

Ampér $\oint_L \vec{B}\,d\vec{l} = \int_S \text{rot}\,\vec{B}\,d\vec{S} \Rightarrow$

$= \mu_0 I$

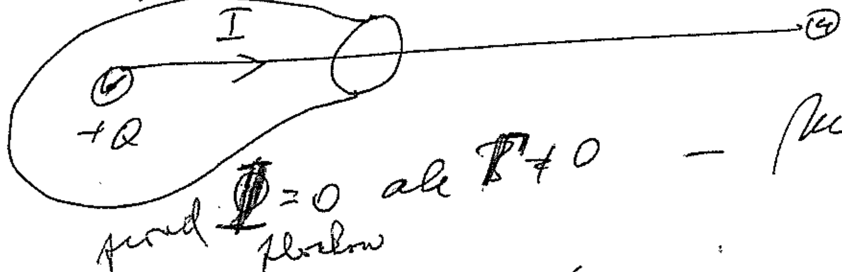
$$\int_S \text{rot}\,\vec{B}\,d\vec{S} = \int_S \mu_0 \vec{j} \cdot d\vec{S}$$

plocha natažená na smyčku

platí pro $\forall S$

$$\Rightarrow \boxed{\text{rot}\,\vec{B} = \mu_0 \vec{j}}$$ dif. tvar Ampér. z.

(stacionární případ !!!)



$\oint = 0$ ale $\Gamma \neq 0$ — Maxwellův proud

proud plochou

Maxwellovy rovnice stacionární vakuum

$\text{div}\,\vec{E} = \frac{\rho}{\varepsilon_0}$ $\quad$ $\text{rot}\,\vec{E} = 0$

$\text{rot}\,\vec{B} = \mu_0 \vec{j}$ $\quad$ $\text{div}\,\vec{B} = 0$

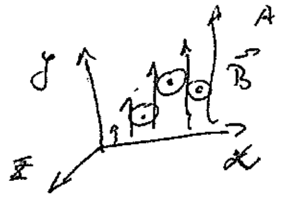
Vektorový potenciál

$\operatorname{div} \vec{B} = 0 \qquad \exists \vec{A} \quad \operatorname{div} \operatorname{rot} \vec{A} = 0$

$\vec{B} = \operatorname{rot} \vec{A} \qquad (E = -\operatorname{grad} \varphi)$

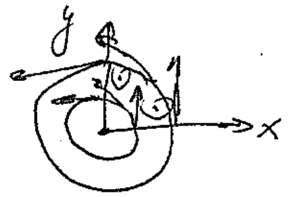
příklad: $\vec{B} = (0, 0, B_z) = \text{konst}$

1) $\vec{A} = (0, x B_z, 0)$



2) $\vec{A} = (-\frac{1}{2} y B_z, \frac{1}{2} x B_z, 0)$

3) $\vec{A} = (-y B_z, 0, 0)$



$\operatorname{rot} \vec{B} = \operatorname{rot} \operatorname{rot} \vec{A} = \operatorname{grad} \operatorname{div} \vec{A} - \Delta \vec{A} = \mu_0 \vec{j}$

hledáme spec. kalibraci, aby se dobře počítalo.

~~kdybychom~~ našli $\vec{B} = \operatorname{rot} \vec{A}$; $\operatorname{div} \vec{A} = 0$ potom máme $\Delta \vec{A} = -\mu_0 \vec{j}$

ale to umíme, mějme $\vec{A}$ tak, že $\operatorname{rot} \vec{A} = \vec{B}$

$\operatorname{div} \vec{A} = \partial_x A_x + \partial_y A_y + \partial_z A_z = f(x, y, z)$ - fce

a hledejme $\vec{A} = \vec{A}' + \vec{A}''$ tak, aby $\operatorname{div} \vec{A}' = 0$, $\operatorname{rot} \vec{A}' = \vec{B} = \operatorname{rot} \vec{A}$

tj chceme aby $\operatorname{rot} \vec{A}'' = 0$, $\operatorname{div} \vec{A}'' = -\operatorname{div} \vec{A} = -f(x, y, z)$

$\Rightarrow$ platí: $\operatorname{div} \operatorname{grad} \vec{A}'' = -\mu_0 \vec{j}$ $\quad$ ($\operatorname{div} \vec{E} = \frac{\rho}{\varepsilon_0}$ při řešení)

ale to je $\operatorname{div} \operatorname{grad} \varphi = -\frac{\rho}{\varepsilon_0}$

$$\Rightarrow \vec{A} = \frac{\mu_0}{4\pi} \int_{V'} \frac{\vec{j}(\vec{r}\,')}{R} \, dV' , \qquad R = |\vec{r} - \vec{r}\,'|$$

$$\begin{aligned} \Delta A_x &= -\mu_0 j_x \\ \Delta A_y &= -\mu_0 j_y \\ \Delta A_z &= -\mu_0 j_z \\ \Delta \varphi &= -\frac{\rho}{\varepsilon_0} \end{aligned}$$

pro drát $\vec{j} \parallel d\vec{l}$

$$\vec{A} = \frac{\mu_0}{4\pi} \int_{V'} \frac{\vec{j}(x',y',z')}{R}\, dV' = \frac{\mu_0}{4\pi} \int_l \int_S \frac{\vec{j}\cdot d\vec{S} \cdot d\vec{l}}{R}$$

$$= \frac{\mu_0}{4\pi} \int_l \frac{I\, d\vec{l}}{R} \qquad \vec{R} = (\vec{r} - \vec{r}\,')$$

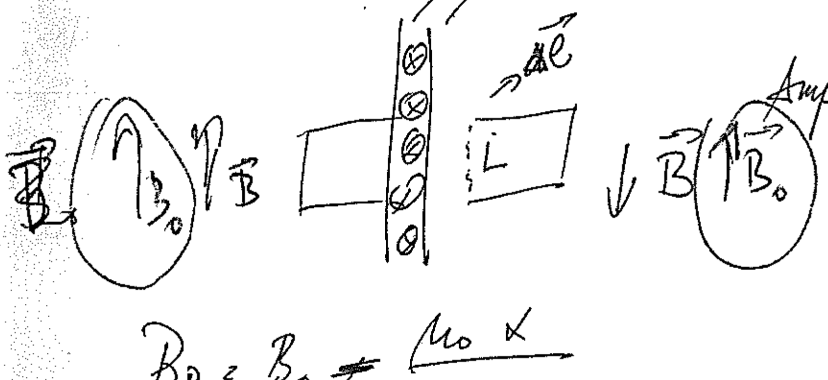
a z toho: $\vec{B} = \mathrm{rot}\,\vec{A} = \frac{\mu_0}{4\pi} \int_{V'} \mathrm{rot}\, \frac{\vec{j}(\vec{r}\,')}{R}\, dV' =$

$$= \frac{\mu_0}{4\pi} \int_{V'} \left(\mathrm{grad}\,\frac{1}{R} \times \vec{j}\right) dV' = \frac{\mu_0}{4\pi} \int_{V'} \frac{\vec{j} \times \vec{R}}{R^3}$$

$$= \frac{\mu_0}{4\pi}\, I \int_l \frac{d\vec{l} \times \vec{R}}{R^3} \quad \text{– Biot Savart}$$

<u>Plošné proudy</u> – síla na proud v plo

$$\vec{A} = \frac{\mu_0}{4\pi} \int_S \frac{\vec{\alpha}}{R}\, dS \quad \left(= \frac{\mu_0}{4\pi} I \int \frac{d\vec{l}}{R} \right. \quad \vec{j} \to \vec{\alpha}$$



Amper zákon

$$\Gamma = (B + B)L = \mu_0 \alpha$$

$$\Rightarrow B = \frac{\mu_0 \alpha}{2}$$

$$B_P = B_0 - \frac{\mu_0 \alpha}{2}$$

$$B_L = B_0 + \frac{\mu_0 \alpha}{2}$$

$$|\Delta F| = |I\, d\vec{l} \times \vec{B}_0| = \alpha \underbrace{l\, dl}_{dS}$$

$$p = \frac{dF}{dS} = \alpha B_0 = \rightarrow$$

$$= \underbrace{\frac{1}{\mu_0}(B_L - B_P)}_{\alpha} \underbrace{\left(\frac{B_P + B_L}{2}\right)}_{B_0} = \frac{1}{2\mu_0}\left(B_L^2 - B_P^2\right) \quad (1$$

Rozdíl B_P a B_L je způsoben proudem v povrchu a mění se skokem

$B = 0$ | $B \downarrow$ | $B = 0$

tlak na rozhraní desku

$$p = \frac{B^2}{2\mu_0}$$

energie $= F \cdot l$

energie na jednotku plochy $= \frac{F \cdot l}{S} = p \cdot l$

hustota energie $\frac{F \cdot l}{S \cdot l} = p = w = \frac{B^2}{2\mu_0}$

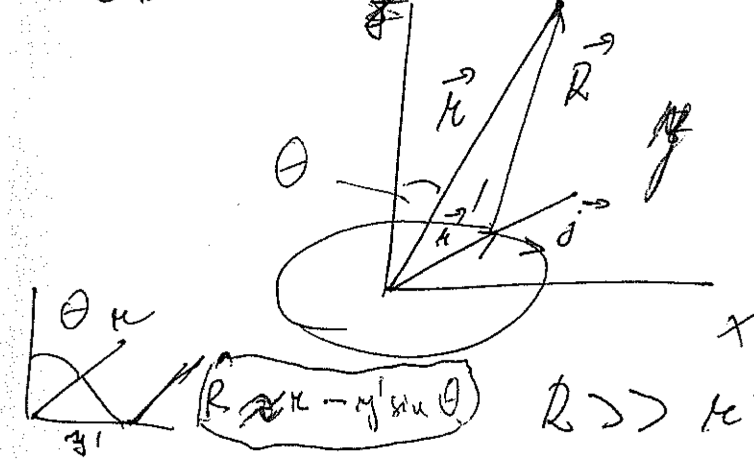
$$\left(\frac{\varepsilon_0 E^2}{2}\right)$$

$$\boxed{w_{elmg} = \frac{\varepsilon_0 E^2}{2} + \frac{B^2}{2\mu_0}}$$

---

mg. pole (malé smyčky) $\vec{A}(0, y, z) = (A_x, 0, 0)$

- kruhová:



$$A_x = \frac{\mu_0}{4\pi} \oint_{l'} \frac{I_x}{R} dl' =$$

$$= \frac{\mu_0}{4\pi} I \oint_{l'} \frac{dx'}{R} =$$

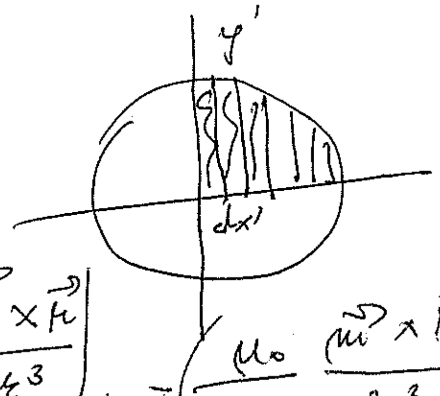
$(R \approx r - y' \sin\theta)$ $R \gg r'$

$$\frac{1}{R} \approx \frac{1}{r}\left(1 + \frac{y' \sin\theta}{r}\right)$$

16

$$\approx \frac{\mu_0}{4\pi} I \oint_{l'} \frac{\sin\theta}{r^2}\, y'\, dx' \qquad \text{protože } \oint_{l'} dx' = 0$$

$$= \frac{\mu_0}{4\pi} I \frac{\sin\theta}{r^2} \oint_{l'} y'\, dx'$$



$S_{smyčky}$

$$= \frac{\mu_0}{4\pi} I \frac{\sin\theta}{r^2} S = \left(\frac{\mu_0}{4\pi} I \frac{\vec S\times\vec r}{r^3}\right)_x = \left(\frac{\mu_0}{4\pi} \frac{\vec m\times\vec r}{r^3}\right)_x$$

$\vec m = I\cdot\vec S$ – Ampérův mg. dipól. moment (pravotočivě)

Obecně pro malou rozměr smyčky identita

$$\vec A = \frac{\mu_0}{4\pi} I \oint_{l'} \frac{d\vec l'}{R} = \frac{\mu_0}{4\pi} I \int_S -\nabla'\left(\frac{1}{R}\right)\times d\vec S =$$

(identita $\oint_{l'} f\, d\vec l' = -\int_S \operatorname{grad} f \times d\vec S$)

$$= \frac{\mu_0}{4\pi} I \int_S \frac{d\vec S\times\vec R}{R^3} \approx \frac{\mu_0}{4\pi}\frac{I\vec S\times\vec r}{r^3} = \qquad \boxed{\vec R = \vec r - \vec r'}$$

$$= \frac{\mu_0}{4\pi}\frac{\vec m\times\vec r}{r^3} \quad \leftrightarrow \quad \left(\varphi = \frac{1}{4\pi\varepsilon_0}\frac{\vec p\cdot}{r}\right.$$

$$\vec B = \operatorname{rot}\vec A = -\frac{\mu_0}{4\pi}\operatorname{rot}\left(\vec m\times\operatorname{grad}\frac{1}{r}\right) = -\frac{\mu_0}{4\pi}\operatorname{grad}\frac{\vec m\cdot\vec r}{r^3}$$

$(\vec E = -\operatorname{grad}\varphi$

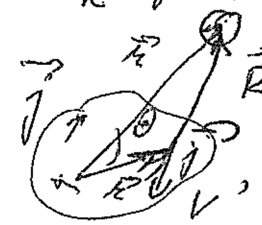
Multipólový rozvoj   viz. A

$$\vec A = \frac{\mu_0}{4\pi}\int_{V'}\frac{\vec j\, dV'}{R} \dots$$

$$\vec B = \frac{\mu_0}{4\pi}\left[\frac{3(\vec m\cdot\vec r)\vec r}{r^5} - \frac{\vec m}{r^3}\right]$$

Multipólový rozvoj (netriviální – moc identit) (A)

$R = \sqrt{r^2 + r'^2 - 2rr'\cos\theta}$



$$\vec A = \frac{\mu_0}{4\pi}\int_{V'} \frac{\vec j(\vec r')\cdot dV'}{R} = \text{Taylor}$$

$$= \frac{\mu_0}{4\pi}\int_{V'} \vec j(\vec r')\left(\frac{1}{r} + \frac{\vec r'\cdot\vec r}{r^3} + \dots\right)dV' =$$

identita: 1) $\vec j\cdot(\vec r'\cdot\vec r) = \frac{1}{2}\left[(\vec r\times\vec j)\times\vec r'\right] + \frac{1}{2}\left[\vec j(\vec r\cdot\vec r') + \vec r'(\vec j\cdot\vec r')\right]$

2) pro libovolný vektor $\vec a$

$$(\vec a\cdot\vec j)(\vec r'\cdot\vec r) + (\vec a\cdot\vec r)\cdot(\vec j\cdot\vec r') =$$

$$= \vec j\cdot \mathrm{grad}'\left[(\vec a\cdot\vec r')(\vec r'\cdot\vec r)\right] =$$

$$= \mathrm{div}'\left[\vec j(\vec a\cdot\vec r')(\vec r'\cdot\vec r)\right] - (\vec a\cdot\vec r')(\vec r'\cdot\vec r)\underbrace{\mathrm{div}'\vec j}_{=0}$$

$$= \underbrace{\frac{\mu_0}{4\pi r}\int_{V'}\vec j(\vec r')\cdot dV'}_{=0} + \frac{\mu_0}{4\pi r^3}\int_{V'}\left[\frac{1}{2}(\vec r\times\vec j)\times\vec r'\right]dV' +$$

$$+ \underbrace{\frac{\mu_0}{4\pi r^3}\int_{V'}\mathrm{div}'\left[\vec j(\vec a\cdot\vec r')(\vec r'\cdot\vec r)\right]dV'}_{} + \dots$$

$$\text{gauss } \underbrace{= \frac{\mu_0}{4\pi r^3}\oint_{S'}\vec j\cdot(\vec a\cdot\vec r')(\vec r\cdot\vec r)\cdot d\vec S'}_{=0}, \text{ ale } \vec j\perp d\vec S' \text{ na povrchu}$$

$$\vec A \approx \frac{\mu_0}{4\pi r^3}\int_{V'}\frac{1}{2}\left[(\vec r\times\vec j)\times\vec r'\right]dV' = \frac{\mu_0}{4\pi r^3}\left[\int_{V'}\frac{1}{2}(\vec r'\times\vec j)\,dV'\right]\times\vec r$$

$$= \frac{\mu_0}{4\pi r^3}(\vec m\times\vec r)$$

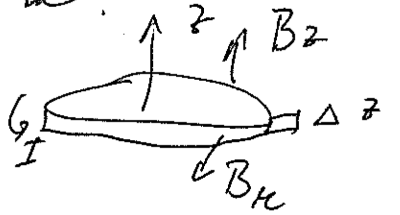
$$\vec m = \frac{1}{2}\int_{V'}(\vec r\times\vec j)\,dV'$$

Síla na dipól . . – zase smyčka (18)

nádhera z Φ a zobecnit

nehomogenní pole

Plošná smyčka: kruhová



$F_{rad} = B_z \cdot \Delta z \cdot I$

síla do středu zúžení – deformace

Δ $F_{axial} = 2\pi r \, I B_r$ – Lorentzova

$$\Phi_{podst + podst} = \pi r^2 \left[-B(z) + B(z+\Delta z) \right] = \pi r^2 \frac{\partial B_z}{\partial z} \Delta z$$

$\Phi_{podst + podst} + \Phi_{plášť} = 0 \Rightarrow \Phi_{plášť} = -\Phi_{podst + podst}$

$$\Phi_{plášť} = 2\pi r \cdot \Delta z \cdot B_r = -\pi r^2 \frac{\partial B_z}{\partial z} \Delta z$$

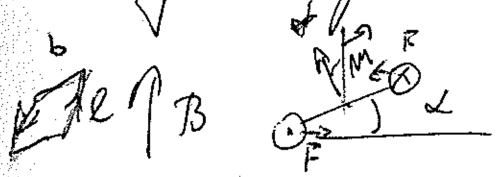
$$B_r = -\frac{r}{2} \frac{\partial B_z}{\partial z}$$

$$|F_z| \overset{2\pi r I B_r}{=} \pi r^2 I \frac{\partial B_z}{\partial z} = \mu_z \cdot \frac{\partial}{\partial z} B_z$$

F_x, F_y

$$\vec{F} = (\vec{\mu} \cdot \vec{\nabla}) \vec{B}$$

Homogenní pole $\frac{\partial B}{\partial x_i} = 0 \Rightarrow \vec{F} = 0$

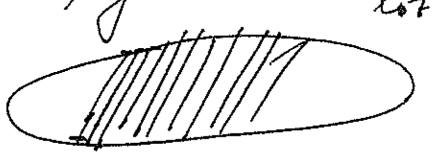


$\vec{N} = I l B b \sin\alpha \, \vec{e}_z = \vec{\mu} \times \vec{B}$

energie $W = \int_0^\alpha N \, d\alpha = -I l B b \cos\alpha + konst = -\vec{\mu} \cdot \vec{B}$

Obecná smyčka (19)

rotace ~~na~~ malé smyčky



Gyromagnetický poměr nabité částice v homogenním mg. poli

(kruhový pohyb)

$$m = I \cdot S = \frac{qv}{2\pi r} \cdot \pi r^2 = \frac{1}{2} qrv = \frac{1}{2}\frac{q}{m_0}(m_0 r v)$$

mg. moment

při mnoha nábojích

pro nabitá tělesa $\gamma = \frac{|\vec{m}|}{|\vec{L}|}$ – gyromag. poměr

Indukované momenty $\vec{m} = \gamma \cdot \vec{L}$

moment síly $\vec{N} = \frac{d\vec{L}}{dt} = \vec{m} \times \vec{B}$ $\left(\vec{F} = m\vec{a} = \frac{d\vec{p}}{dt}\right)$

z. z. momentu hybnosti $\frac{d\vec{m}}{dt} = \gamma(\vec{m} \times \vec{B}) = -\vec{m} \times \vec{\omega}_L$

Larmorova precese

$$\vec{\omega}_L = -\gamma \cdot \vec{B}$$

úhlová rychlost s níž se otáčí $\vec{m}$ kolem $\vec{B}$

$$\vec{m}_{vyšl} = \vec{m} - \vec{m}_{ind};\ \vec{m}_{ind} = \gamma \cdot \underset{\uparrow}{\Delta \vec{L}_L} = -\beta \vec{B}$$

změna díky precesi

$$\Delta L_L = m_e \langle \rho^2 \rangle \omega_L$$
I ω

$m_{ind} = \gamma \Delta L_L$

$\beta B = (\gamma \cdot m\langle\rho^2\rangle \cdot \gamma) B$

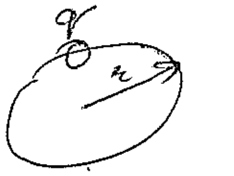
$\beta = q^2 m \langle \rho^2 \rangle$

$$\vec{p} = \alpha \vec{E}$$

$$\Delta L = m \langle \rho^2 \rangle \omega_L$$
I ω

gyromagnetism...

Bodově nabitá částice v homog. mag. poli

kruhový pohyb  $m \doteq \mu$

$$\mu = I \cdot S = \frac{qv}{2\pi r} \cdot \pi r^2 = \frac{1}{2} q r v = \frac{1}{2} \frac{q}{m_0} \underbrace{(m_0 r v)}_{L}$$

$$= \gamma \cdot L$$

gyromagnetický poměr

$$\begin{cases} \vec{\mu} = \gamma \cdot \vec{L} \\ \vec{N} = \dfrac{d\vec{L}}{dt} = \vec{\mu} \times \vec{B} \end{cases} \quad \leftarrow \text{Larmorova precese}$$

$$\frac{d\vec{\mu}}{dt} = \gamma \cdot \frac{d\vec{L}}{dt} = \gamma (\vec{\mu} \times \vec{B}) \Rightarrow \frac{d\vec{\mu}}{dt} = \gamma (\vec{\mu} \times \vec{B}) = -\vec{\mu} \times \vec{\omega}_L \quad ; \quad \vec{\omega}_L = -\gamma \cdot \vec{B}$$

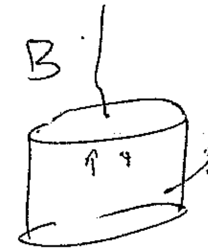
$$\vec{\mu}_{gyr} = \vec{\mu} - \vec{\mu}_{ind}$$

$$\vec{\mu}_{ind} = \gamma \cdot \Delta \vec{L} = -\beta \vec{B} \qquad (p = \alpha \vec{E})$$

změna momentu hybnosti díky precesi

Rotující nabité tělísko – skripta

Einstein – de Haas 1915 $\uparrow B$ flip $\downarrow B$ otočení!



spin $L_{elektronu} = \frac{1}{2}\hbar$; $\gamma = \frac{\mu}{L} = \frac{e}{m_e}$

$\hbar = 1{,}054 \cdot 10^{-34}$ Js; $\mu_B = 9{,}273 \cdot 10^{-24}$ Am² ← měření přibližně vyšlo

μ_p ; μ_n ≈ velice složitá

$q = 0$

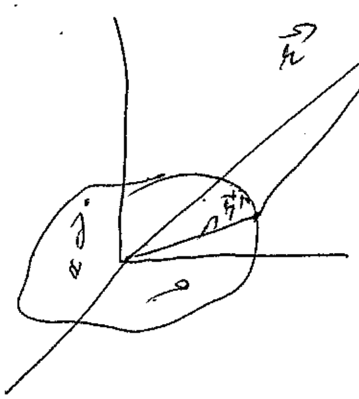
Vektor magnetizace

jako polarizace

$M = N \cdot \vec{m}_1$

hustota magn. momentu

hustota dipólů



$\vec{R} = (\vec{r} - \vec{r}')$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int_{V'} \frac{\vec{M}(\vec{r}') \times \vec{R}}{R^3} \, dV' =$$

$$= \frac{\mu_0}{4\pi} \left[- \int_{V'} \mathrm{rot}' \left(\frac{\vec{M}}{R} \right) dV' + \int_{V'} \frac{\mathrm{rot}' \vec{M}(\vec{r}')}{R} dV' \right] =$$

Stokes

$\boxed{- \text{grad}'(r - r')}$ $\boxed{\frac{\partial}{\partial x'}}$

$$= \frac{\mu_0}{4\pi} \left[\oint_{S'} \frac{\vec{M}(\vec{r}') \times d\vec{S}'}{R} + \int_{V'} \frac{\mathrm{rot}' \vec{M}(\vec{r}')}{R} dV' \right] =$$

$$= \frac{\mu_0}{4\pi} \left[\oint_{S'} \frac{\vec{\alpha}_m(\vec{r}') \, dS'}{R} + \int_{V'} \frac{\vec{j}_m(\vec{r}')}{R} dV' \right]$$

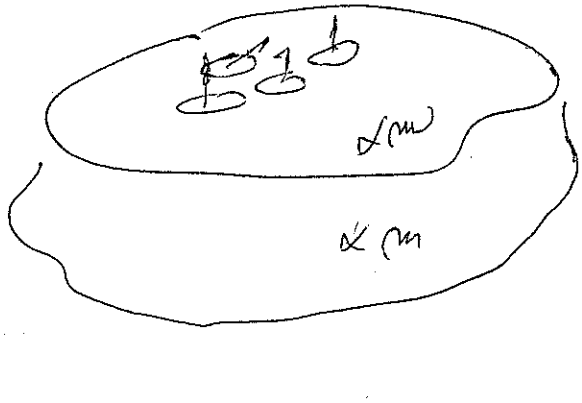
$d\vec{S} = \vec{e}_n \cdot dS, \quad \vec{\alpha}_m = \vec{M} \times \vec{e}_n$

$\vec{j}_m = \mathrm{rot}$

$\boxed{\rho_V = - \mathrm{div}}$

$\boxed{\text{magnetizační proudy}}$

ať to vzniklo jakkoli



$$\vec{A} = \frac{\mu_0}{4\pi} \int \frac{\vec{\alpha}}{R} dS$$

pro plošné proudy

Magnetika v mg. poli

$$\text{rot}\,\vec{B} = \mu_0(\vec{j} + \vec{j}_m) = \mu_0(\vec{j} + \text{rot}\,\vec{M})$$

def: Intenzita mg. pole $\quad (\text{div}\,\vec{B} = 0)$

$$\vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M} \quad \Rightarrow \text{rot}\,\vec{H} = \vec{j}$$

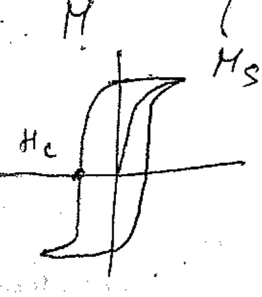
pole které by vytvořily volné proudy

$$\vec{M} = \varkappa \cdot \vec{H}$$

pro dia a para

mg. susceptibilita

$$\vec{B} = \mu_0 \vec{H} + \varkappa\mu_0\vec{H} = \mu_0(1+\varkappa)\vec{H} = \mu_0\mu_r\vec{H}$$



$H = 0$; $B \neq 0 = -\mu_0 M$

	Bi ($\varkappa =$)	Al	Fe
$\varkappa$:	$-176 \cdot 10^{-6}$	$23 \cdot 10^{-6}$	10^4

Solenoid s jádrem

$$\vec{B}_m = \mu_0\vec{M}$$

bez jádra $\quad B_0 = \mu_0 I n$ — počet závitů na 1 m

$$\vec{M} = (\mu_r - 1)\vec{H}$$

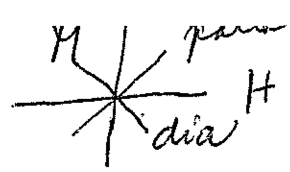
$$\vec{B} = \vec{B}_0 + \mu_0\vec{M} = \vec{B}_0 + \mu_0\mu_r\vec{H} - \underbrace{\mu_0\vec{H}}_{\vec{B}_0} \ldots$$

$$\cancel{\frac{B}{B_0} = \left(1 + \frac{\mu_0 H}{B_0}\right)}$$

$$= \vec{B}_0 + \mu_0\varkappa\vec{H} = \vec{B}_0 + \mu_0(\mu_r - 1)\vec{H} = B_0 + \mu_0\mu_r H - \overset{B_0}{\mu_0 H} = \mu_0\mu_r \ldots$$

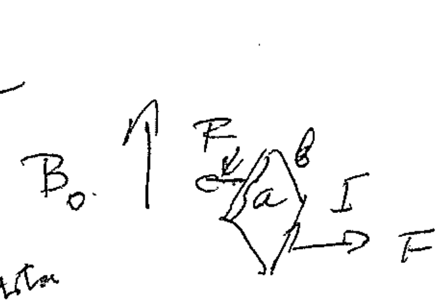
$$= \mu_r B_0$$

Rozdíl para a dia



1) Para

$F = q(\vec{u} \times \vec{B}) = I(d\vec{l} \times \vec{B}_0) = I a B_0$



$\vec{N} = \vec{m} \times B_0$

hustota

$\vec{B}_m = \mu_0(\vec{m} \cdot N) = \mu_0 \vec{M}$ | ~~$B_m = \mu_0(\vec{m} \cdot N) = \mu_0 M$ sesílení~~

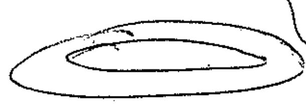
hustota

dáť pevnou se nemění! na bytí

obecně pro systém

nejde vysvětlit ani zjistit

Model



2) Dia lze vysvětlit nebo zjistit

má být na zrychlení - hmotnost m_0

$F = \frac{m_0 v^2}{r}$ $\quad \frac{\Delta F}{\Delta v} = \frac{2 m_0 v}{r}$

(FARADAYŮV ZÁKON TO VYSVĚTLÍ)

$\Delta F = q v B \Rightarrow \Delta v = \frac{q r B}{2 m_0}$

$m = \frac{q v}{2\pi r} \pi r^2 = \frac{1}{2} q v r$

$\Delta m = \frac{1}{2} q r \Delta v = \frac{q^2 r^2}{4 m_0} B_0$

$B_m = \mu_0 N \Delta m$ - záporné znaménko

MATIKA

$\sum q_i (d\vec{l} \times \vec{R}) = -\sum q_i \vec{B}$

Pohyb v el. a mg. poli

a) $\vec{B} = 0$; $\vec{E} = (0, E_y, 0)$ $\quad m \cdot \frac{d\vec{v}}{dt} = q(\vec{E} + \vec{v} \times \vec{B})$

$$\vec{r} = \frac{1}{2m} q \vec{E} t^2 + \vec{v}_0 t + r_0$$

b) $\vec{B} = (0, 0, B_z)$, $\vec{E} = 0$

Lorentz 1) $\frac{dv_x}{dt} = \frac{q}{m} v_y B_z$; 2) $i \cdot \left| \frac{dv_y}{dt} = -\frac{q}{m} v_x B_z \right.$

$$\mu = v_x + i v_y$$

$$\frac{d\mu}{dt} + i \underbrace{\frac{qB_z}{m}}_{\omega_c} \mu = 0$$

cyklotronová resonance

$$\mu = A \cdot e^{-i\omega_c t}$$

$$q v B = m \frac{v^2}{R}$$

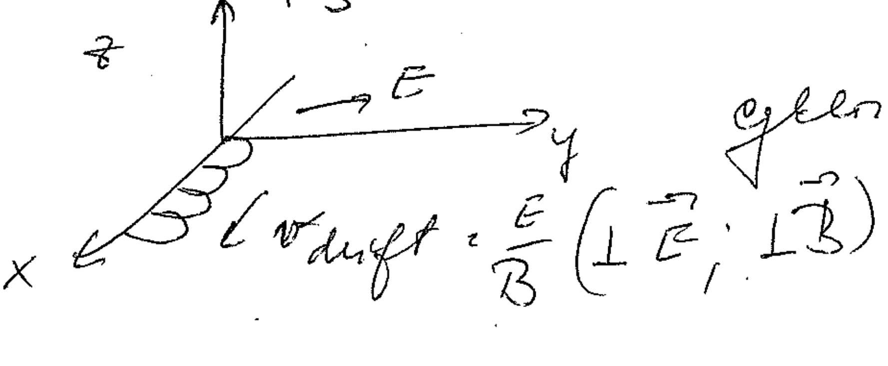
$$\boxed{\frac{v}{R} = \frac{qB}{m}}$$

c) obě pole – předchozí + přidáme do 2. členu $q\frac{E}{m}$

$$\frac{d\mu}{dt} + i \frac{qB}{m} \mu = i \frac{qE}{m}$$

řešení – homogenní + 1. řešení nehomogenní $\cancel{i\frac{E}{B}}$ $\mu = \frac{E}{B}$ je reál. je ve směru x

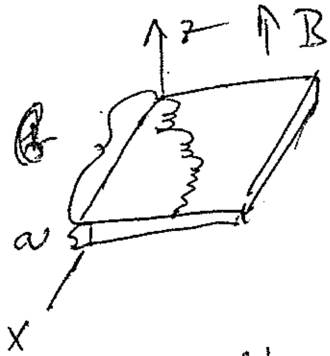
$$\mu_{výsled} = A \cdot e^{-i\omega_c t} + \frac{E}{B} \cancel{x}$$



cykloida

Hallův jev

v polovodiči $\vec{v}$ B a E



$F_x = q\, u_y B$

E_{ef}

$U_x = E_{ef} \cdot b = u_y B b =$

$j_y = nq u_y$

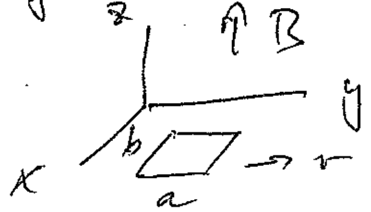
$= \frac{j_y}{nq} B b \frac{la}{la} = \frac{IB}{nqa} = K \cdot \frac{IB}{a}$

$\Rightarrow$ měření B pomocí U_x

Kvazistacionární pole

1831 – Faraday

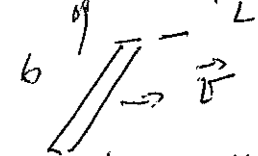
Elmg. indukce



$b << a$

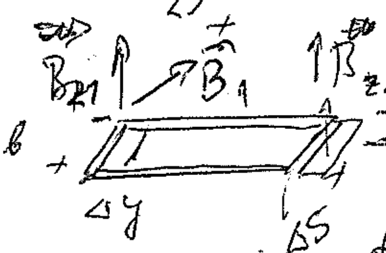
b malé

$\vec{B}$ $\quad \vec{F}_L = q\, \vec{v} \times \vec{B}$



$\vec{F}_L + q\, \vec{E}_{ind} = 0$

$\vec{E}_{ind} = -\vec{v} \times \vec{B}$



$\vec{E}_{ind1} = -\vec{v}_1 \times \vec{B}_1$ $\quad \vec{E}_{ind2} = -\vec{v}_2 \times \vec{B}_2$

$\vec{v} \perp \vec{S}$

$v = \frac{\Delta y}{\Delta t}$

$\oint_l \vec{E}\, d\vec{l} = \vec{v} \times (\vec{B}_1 - \vec{B}_2) \cdot b =$

$b\, (\cancel{\vec{v}_1 \times \vec{B}_1} - \cancel{\vec{v}_2 \times \vec{B}_2})$

$= \vec{v} \cdot (B_{1z} - B_{2z}) \cdot b = \frac{B_{z1} \cdot \Delta S - B_{z2}}{\Delta t}$

$\vec{v} \perp \vec{S}$

$\cancel{\frac{B_1 \Delta S \cos\alpha + B_2 \Delta S \cos\alpha}{\Delta t}} = -\frac{\Delta \Phi}{\Delta t}$

$$\mathcal{E}^{ind} = \oint \vec{E} \cdot d\vec{l} = -\frac{d\phi}{dt}$$

pokud se pohybuje B – relativita $\gamma \approx 1$

$$\oint_l \vec{E} d\vec{l} = -\frac{d}{dt} \int_S \vec{B} \cdot d\vec{S}$$ platí pro $\forall$ smyčku

Stokes

$$\boxed{\mathrm{rot}\, \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

a pro $\forall \frac{d\phi}{dt}$

s výjimkou

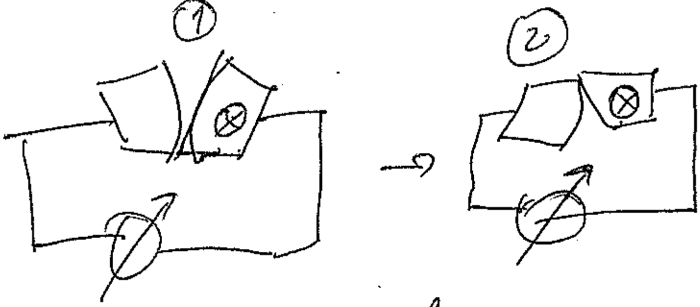
Výjimky z Faradaye

1) Barlowovo kolečko

$\Delta\phi = 0$

$\mathcal{E}_{ind} \neq 0$

2)

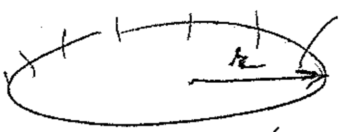


$\Delta\phi \neq 0$

$\mathcal{E}_{ind} = 0$

Pravidlo se aplikuje pouze na obvody, kde se nemění materiál.

Betatron – Bez vodičů



$B_{obr.}$

$\langle B \rangle_{stř}$ – ale ho budou ta[ke] měnit

$$\mathcal{E}_{ind} = 2\pi r E = -\frac{d}{dt}(\langle B \rangle \pi r^2);$$

tečné

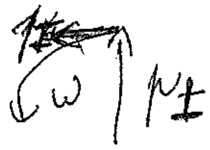
$$E = -\frac{r}{2} \frac{d\langle B \rangle_{stř}}{dt}$$

$$\frac{dp_T}{dt} = qE = \frac{qr}{2} \frac{dB_{stř}}{dt}$$

$$p_T = p_{T0} + \frac{qr}{2} \Delta \langle B \rangle_{stř}$$

$F_{\text{částí}} = q\vec{v} \times \vec{B}_{\text{obr}}$; $q v B_{\text{obr}} = \frac{dp_\perp}{dt}$

$\langle B \rangle_{\text{stř}}$



$$q r B_{\text{obr}} = p_{T_0} + \frac{q r}{2} \Delta \langle B \rangle_{\text{stř}}$$

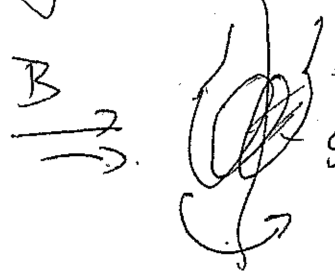
$\frac{dp_\perp}{dt} = \omega$

$$2 \Delta B_{ob} = \Delta \langle B \rangle_{\text{stř}}$$

$\omega = \frac{v}{r}$

udrží na kruhové dráze
a urychluje.

Generátor ~ proudu



$$\mathcal{E}^{ind} = -\frac{d\phi}{dt}$$

$$\phi = n B S \cos\varphi \qquad \varphi = \omega t$$

$$\mathcal{E}^{ind} = -n B S \omega \sin(\omega t)$$

Energie $\underset{\text{driftová}}{\vec{v}_{dr}} \parallel d\vec{l}$ — ve vodiči $\Delta P_{\Delta V} = \vec{F}_1 \cdot \vec{v} \cdot N A$

průřez

$$P = \int_l \vec{F}_1 \cdot d\vec{l}\, \underset{\text{hustota}}{N} v S = \int_l \frac{\vec{F}_1}{q} d\vec{l}\, \underbrace{q v N S}_{j S}$$

$$= \mathcal{E}^{ind} \cdot I$$

Maxwell. r. ve vakuu

$$\operatorname{div}\vec E = \frac{\varrho}{\varepsilon_0} \qquad \operatorname{rot}\vec E = -\frac{\partial \vec B}{\partial t}$$

$$\operatorname{div}\vec B = 0 \qquad \operatorname{rot}\vec B = \mu_0 \vec j$$

obecné pro ∀ proudy a náboje

Maxwell si všiml, že div rot = 0 ⇒ div $\vec j$ by bylo 0

ale $\operatorname{div}\vec j = -\frac{\partial \varrho}{\partial t}$ = z kontinuity

$$= -\frac{\partial}{\partial t}\varepsilon_0 \operatorname{div}\vec E = -\varepsilon_0 \operatorname{div}\frac{\partial \vec E}{\partial t}$$

$(x, y, z) \neq f(t)$

$$0 = \mu_0 \operatorname{div}\left(\vec j + \varepsilon_0 \frac{\partial \vec E}{\partial t}\right)$$

$$\varepsilon_0\mu_0 = \frac{1}{c^2}$$

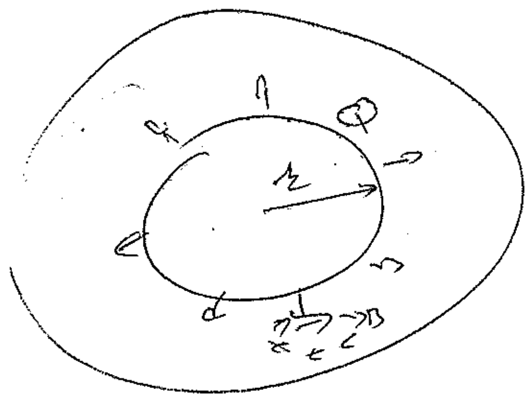
$$\boxed{\operatorname{rot}\vec B = \mu_0 \vec j + \frac{1}{c^2}\frac{\partial \vec E}{\partial t}}$$

Maxwellův posuvný proud

$$\vec j_{pol} = \frac{\partial \vec P}{\partial t}$$

vysvětlení

1) nabitá koule a náboj odchází zevnitř



$\vec j \neq 0$ ale B se vyruší

nevznikne

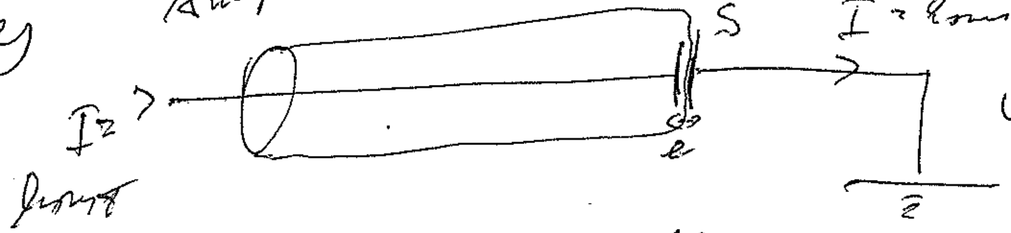
$$\frac{dQ(t)}{dt} = -4\pi r^2 j(r) \quad (\neq 0)$$

$$E(r) = \frac{Q(r)}{4\pi r^2 \varepsilon_0} \qquad \operatorname{rot}\vec B =$$

$$\vec j(r) = -\frac{1}{4\pi r^2}\frac{dQ}{dt} = -\varepsilon_0 \frac{\partial E(r)}{\partial t} \Rightarrow \operatorname{rot}\vec B = 0$$

protože $B = 0$ všude

2) Amperov za



$$\oint_l \vec{B}\cdot d\vec{l} = \mu_0 I = \mu_0 \frac{dQ}{dt} = \mu_0 \frac{d}{dt}(CU) =$$

$\vec{E}S$

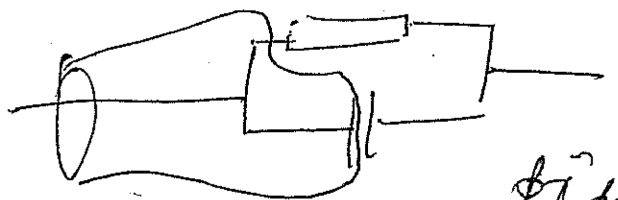
$$= \mu_0 \frac{d}{dt}\left(\varepsilon_0 \frac{S}{e}\cdot E\cdot e\right) = \mu_0 \varepsilon_0 \underbrace{\frac{dE}{dt}\cdot S}_{} =$$

$\int_S \vec{E}\cdot d\vec{S}$

$$= \int_S \frac{\partial \vec{E}}{\partial t}\cdot d\vec{S}$$

$$= \mu_0 \varepsilon_0 \int_S \frac{\partial \vec{E}}{\partial t}\, d\vec{S}$$

$$\Rightarrow \operatorname{rot} \vec{B} = \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}$$



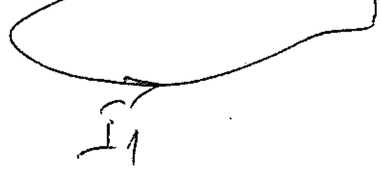
$$\oint_l \vec{B}\, d\vec{l} = \mu_0 \int_S \vec{j}\cdot d\vec{S} + \mu_0 \varepsilon_0 \int_S \frac{\partial \vec{E}}{\partial t} d\vec{S}$$

$$= \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_{el}}{dt}$$

$$\boxed{\operatorname{rot} \vec{B} = \mu_0 \vec{j} + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}}$$

$\Phi \sim I$

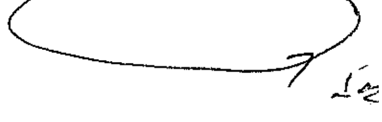
Induktivnost



$$\Phi_1 = L_{11} I_1 + L_{12} I_2$$

z B.S. - zakonom

$$\Phi_2 = L_{21} I_1 + L_{22} I_2$$



L – indukčnost vlastní L (34)
a vzájemná M_{ij}

Že je symetrická $\Phi_{2(1)} = \int_{S_2} \vec{B}_{2(1)} \cdot d\vec{S}_2 =$

$$= \int_{S_2} \text{rot}\, \vec{A}_{2(1)}\, d\vec{S}_2 = \oint_{l_2} \vec{A}_{2(1)}\, d\vec{l}_2 = \frac{\mu_0 I_1}{4\pi} \oint_{l_1}\oint_{l_2} \frac{d\vec{l}_1 \cdot d\vec{l}_2}{R}$$

$=$ pro $I_1 = I_2$
$\Phi_{1(2)}$

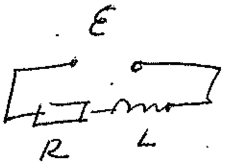
$$\frac{\Phi_{2(1)}}{I_1} = \frac{\Phi_{1(2)}}{I_2}$$

$$\Phi_{1(2)} = \frac{\mu_0}{4\pi} I_2 \oint_{l_1}\oint_{l_2} \frac{d\vec{l}_2 \cdot d\vec{l}_1}{R}$$

$$\mathcal{E}^{ind} = -\frac{d\Phi}{dt} = -L\frac{dI}{dt}$$ Faraday (Dynamická definice indukčnosti)

$\boxed{\text{Energie cívky}}$

Vezmeme obvod s $L, R, \mathcal{E}$
časově proměnné



$$\mathcal{E} + \mathcal{E}^{ind} = RI$$

$$I \cdot / \quad \mathcal{E} - L\frac{dI}{dt} = RI$$

Joule: $\underbrace{\mathcal{E}I}_{\text{výkon zdroje}} - \underbrace{LI\frac{dI}{dt}}_{\text{vytvoření } B} = \underbrace{RI^2}_{\text{Ohm}}$ výkon!!!

Práce: vybití · čas

$$W = \int_0^t \Big(LI \underbrace{\frac{dI}{dt}\Big) dt}_{dI} = \int_0^{I_t^*} LI\,dI = \frac{1}{2} L I_t^2 = \frac{1}{2} I \Phi$$

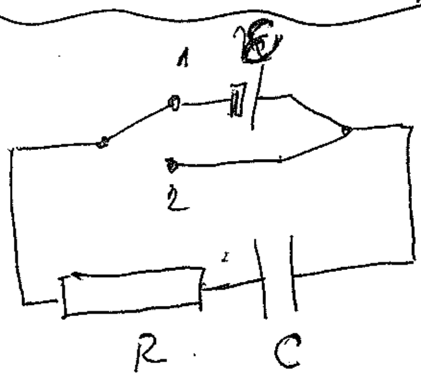
$I: 0 \to I_t$, $0 \to t$

Přechodové stavy v el. obvodech

RC:

zapnu ① → ②

v $t=0$ $Q=Q_0$



① – $Q_0 = C \cdot \mathcal{E}$

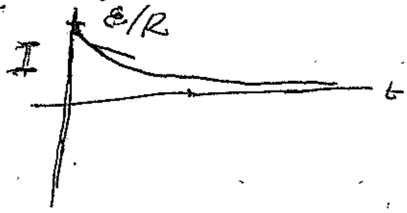
② – $Q = C \cdot U$; $\mathcal{E} = 0$

$$I = \frac{dQ}{dt}$$

obecně ($\mathcal{E} \neq 0$)

$$\mathcal{E} - U = RI$$

$$\frac{\mathcal{E}}{R} = \frac{Q}{RC} + \frac{dQ}{dt}$$



$$Q = konst\ e^{-\frac{t}{RC}} + \mathcal{E} \cdot C$$

RL:

$$\mathcal{E} - L\frac{dI}{dt} = RI \qquad \dot{I} + \frac{R}{L} I = \frac{\mathcal{E}}{L}$$

řešení

$$I = \overset{I_0}{konst} \cdot e^{-\frac{R}{L}t} + \frac{\mathcal{E}}{L}$$

$$U = -L \cdot \frac{dI}{dt} = -RI_0 e^{-\frac{R}{L}t}$$

LC:

$$\mathcal{E} - L\frac{dI}{dt} - \frac{Q}{C} = 0$$

$$LC \frac{d^2U}{dt^2} + U = \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

$$L\frac{d^2Q}{dt^2} + \frac{Q}{C} = \mathcal{E}$$

$$Q = konst\ e^{-i\frac{t}{\sqrt{LC}}} + \mathcal{E}C \quad \text{„}Q_0\text{"}$$

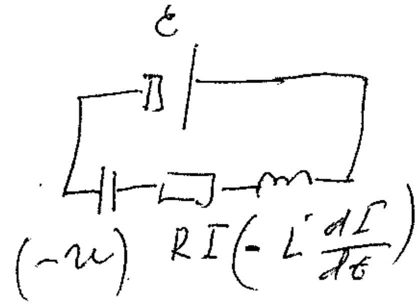
! $\mathcal{E} = 0$

$t = 0$ pro Q_0 na ...

harmonický oscilátor

$$U = U_0\, e^{-\frac{t}{\sqrt{LC}}}$$

RLC



$$I = \frac{dQ}{dt} = C \cdot \frac{du}{dt}$$

$$\mathcal{E} = LC\frac{d^2u}{dt^2} + RC\frac{du}{dt} + u$$

tlumený harm. oscilátor

$$\ddot{u} + \underbrace{\frac{R}{L}}_{2\delta}\dot{u} + \underbrace{\frac{1}{LC}}_{\omega_0^2}u = \frac{\mathcal{E}}{LC}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \; ; \; \omega = \sqrt{\omega_0^2 - \delta^2}$$

$$u = u_0\, e^{-\delta t}\sin(\omega t + \varphi_0) + \mathcal{E}$$

$$I = C \cdot \frac{du}{dt} = C u_0\, e^{-\delta t}\left[-\delta\sin(\omega t + \varphi_0) + \omega\cos(\omega t + \varphi_0)\right]$$

$$-\delta \equiv (\cos\alpha)\cdot K \; ; \; \omega = (\sin\alpha)\, K$$

$$= C u_0\, e^{-\delta t} K\left[\cos\alpha\sin(\quad) + \sin\alpha\cos(\quad)\right]$$

$$= C u_0\, e^{-\delta t} K \sin(\omega t + \varphi_0 + \alpha)$$

$$K \cdot \operatorname{tg}\alpha = -\frac{\omega}{\delta}$$

Impedance: $\quad \frac{Q}{C} \quad \mathcal{E} = \mathcal{E}_0\cos\omega t$ $\quad$ $\boxed{\text{RLC} + \mathcal{E} = \mathcal{E}_0\cos}$

$$L\frac{dI}{dt} + RI + U_c = \mathcal{E}_0\cos\omega t$$

$$L\frac{d^2I}{dt^2} + R\frac{dI}{dt} + \frac{I}{C} = \cancel{I_0\cos(\omega t + \varphi_0)} \quad \frac{d\mathcal{E}}{dt} \equiv \mathcal{E}_0\omega\sin\omega t$$

hledám řešení $\quad I = I_0\cos(\omega t + \varphi_0)$

$$I_0\left[-\omega^2 L\cos(\omega t+\varphi_0)+\frac{1}{C}\cos(\omega t+\varphi_0)-R\omega\sin(\omega t+\varphi_0)\right]=-\omega\mathcal{E}_0\sin\omega t$$

$$I_0\Big[\left(\frac{1}{\omega C}-\omega L\right)(\cos\omega t\cos\varphi_0-\sin\omega t\sin\varphi_0)-$$
$$-R(\sin\omega t\cos\varphi_0+\cos\omega t\sin\varphi_0)\Big]=-\mathcal{E}_0\sin\omega t$$

rozdělí sin ωt a cos ωt

$$I_0\cos\omega t\underbrace{\left[\left(\frac{1}{\omega C}-\omega L\right)\cos\varphi_0-R\sin\varphi_0\right]}+$$
$$\sin\omega t\underbrace{\left[I_0\left(\omega L-\frac{1}{\omega C}\right)\sin\varphi_0-RI_0\cos\varphi_0+\mathcal{E}_0\right]}=0$$

platí pro ∀ t

$t=0$ $\Rightarrow =0$ $\quad\left(\frac{1}{\omega C}-\omega L\right)\cos\varphi_0=R\sin\varphi_0\Rightarrow$

$$\operatorname{tg}\varphi_0=\frac{\left(\frac{1}{\omega C}-\omega L\right)}{R},\quad \operatorname{tg}^2\varphi_0=\frac{1-\cos^2\varphi_0}{\cos^2\varphi_0}=$$

$t=\frac{\pi}{2}$

$$\Rightarrow \cos\varphi_0=\frac{R}{\sqrt{R^2+\left(\frac{1}{\omega C}-\omega L\right)^2}}$$

$\Rightarrow =0$

$$I_0=\frac{-\mathcal{E}_0}{\underbrace{\left(\omega L-\frac{1}{\omega C}\right)}\sin\varphi_0-R\cos\varphi_0}=$$

$$\left(-R\cdot\frac{\sin\varphi_0}{\cos\varphi_0}\right)$$

$$=\frac{\mathcal{E}_0}{R\left(\frac{\sin^2\varphi_0+\cos^2\varphi_0}{\cos\varphi_0}\right)}=\frac{\mathcal{E}_0}{\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}}$$

Definujeme impedanci Z

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \quad ; \quad Z \text{ min pro } \frac{1}{\omega C} = \omega L$$

$$\boxed{Z = R + i\omega L - \frac{i}{\omega C}} \quad \text{Def.}$$

potom $I_{0\,max} = \frac{\mathcal{E}_0}{R}$; $\omega_0 = \frac{1}{\sqrt{LC}}$

fázové posuny

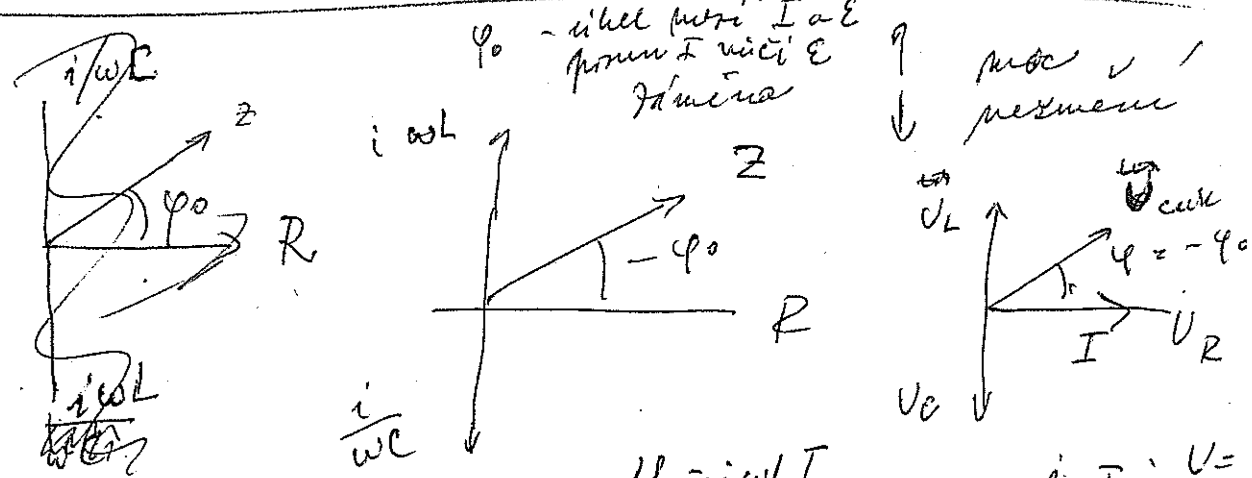
$R \to \infty$ $\left(\frac{1}{\omega C} = \omega L\right)$ $\to \operatorname{tg} \varphi_0 = 0$; $\varphi_0 = 0$

$L \to \infty$ $\operatorname{tg} \varphi_0 = -\infty$; $\varphi_0 = -\frac{\pi}{2}$

$\frac{1}{C} \to \infty$ $(C \to 0)$ $\operatorname{tg} \varphi_0 = +\infty$; $\varphi_0 = \frac{\pi}{2}$

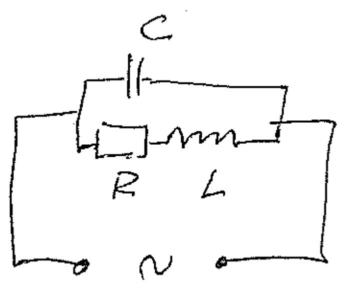
φ_0 úhel mezi I a $\mathcal{E}$ – posun I vůči $\mathcal{E}$

fázový posun dle impedance $\boxed{U = Z I}$



$U_L = i\omega L I$; $U_C = -\frac{i}{\omega C} I$; $U =$

Paralelní RLC obvody



$$Y = Y_1 + Y_2 = \frac{i}{\frac{1}{\omega C}} + \frac{1}{R + i\omega L}$$

$$= i\omega C + \frac{R - i\omega L}{R^2 + \omega^2 L^2} =$$

$$= \frac{R + i\left[\omega C(R^2 - \omega^2 L^2) - \omega L\right]}{R^2 - \omega^2 L^2}$$

$Y = \max$ pro $\omega C(R^2 - \omega^2 L^2) - \omega L = 0$, tj. pro $\omega = \sqrt{\frac{R^2}{L^2} - \frac{1}{LC}}$

atd.

Maxwell ve vakuu

$\operatorname{div}\vec{E} = \frac{\rho}{\varepsilon_0}$ $\qquad$ $\operatorname{div}\vec{B} = 0$ $\qquad$ v prostředí $\rho = \rho_{\mathrm{vol}} + \rho_{\mathrm{pol}}$, $\vec{j} = \vec{j} + \vec{j}_m + \vec{j}_P$

$\operatorname{rot}\vec{E} = -\frac{\partial\vec{B}}{\partial t}$ $\qquad$ $\operatorname{rot}\vec{B} = \mu_0\vec{j} + \frac{1}{c^2}\frac{\partial\vec{E}}{\partial t}$

$\boxed{\operatorname{rot}\vec{H} = \vec{j} + \frac{\partial\vec{D}}{\partial t}}$ $\quad \vec{D} = \varepsilon_0\vec{E} + \vec{P}$

záměna $\frac{\partial}{\partial t} \leftrightarrow \frac{\partial}{\partial x}$ pokud $\exists$

Jak s φ a $\vec{A}$

$$\operatorname{rot}\vec{E} = -\frac{\partial}{\partial t}\operatorname{rot}\vec{A} = -\operatorname{rot}\frac{\partial\vec{A}}{\partial t} \Rightarrow$$

$$\operatorname{rot}\left(\vec{E} + \frac{\partial\vec{A}}{\partial t}\right) = 0 \Rightarrow \exists\varphi;\ \vec{E} + \frac{\partial\vec{A}}{\partial t} = -\operatorname{grad}\varphi$$

$$\operatorname{div}\vec{E} = \frac{\rho}{\varepsilon_0} = -\operatorname{div}\left(\operatorname{grad}\varphi + \frac{\partial\vec{A}}{\partial t}\right)$$

$$\Delta\varphi = -\frac{\rho}{\varepsilon_0} - \frac{\partial}{\partial t}\operatorname{div}\vec{A}$$

$$\operatorname{rot}\vec{B} = \operatorname{rot}\operatorname{rot}\vec{A} = \mu_0\vec{j} - \varepsilon_0\mu_0\frac{\partial}{\partial t}\left(\operatorname{grad}\varphi + \frac{\partial\vec{A}}{\partial t}\right)$$

rot rot = grad div − div grad

$$\operatorname{div}\operatorname{grad}\vec{A} - \frac{1}{c^2}\frac{\partial^2 A}{\partial t^2} = -\mu_0\vec{j} + \operatorname{grad}\left(\operatorname{div}\vec{A} + \varepsilon_0\mu_0\frac{\partial\varphi}{\partial t}\right)$$

$\vec{A}'' = \vec{A}' + \vec{A};\ \operatorname{rot}\vec{A}' = 0;\ \cancel{\operatorname{div}(\vec{A}' + \vec{A}) = 0}$

$$\operatorname{div}\vec{A}' = \underbrace{-\frac{1}{c^2}\frac{\partial\varphi}{\partial t} - \operatorname{div}\vec{A}}_{\text{fce}} \quad \exists \text{ takové } \vec{A}'$$

$\vec{A} \to \vec{A}''$

$$\boxed{\operatorname{div}\vec{A}'' + \frac{1}{c^2}\frac{\partial\varphi}{\partial t} = 0}$$

Lorentzova kalibrační podmínka

$\Rightarrow \Delta\vec{A} - \frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2} = -\mu_0\vec{j}$

MR

$$\operatorname{div}\vec{A} + \frac{1}{c^2}\frac{\partial\varphi}{\partial t} = 0$$

$$\Delta\vec{A} - \frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2} = -\mu_0\vec{j}$$

$$\Delta\varphi - \frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2} = -\frac{\rho}{\varepsilon_0}$$

Minkowski

$j^\mu = (c\rho, \vec{j});\ A^\mu = \left(\frac{\varphi}{c}, \vec{A}\right)$

$$\boxed{\Box A^\mu = -\mu_0 j^\mu}$$