02 - Characteristic properties of detectors

Introduction

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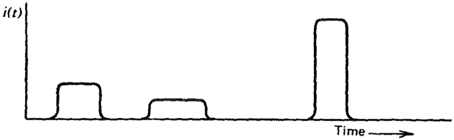
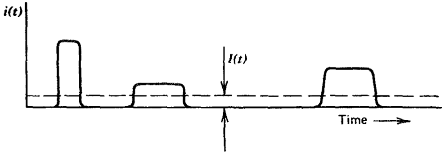
Modes of operation of the detectors

- Pulse counting (and) pulse height spectra

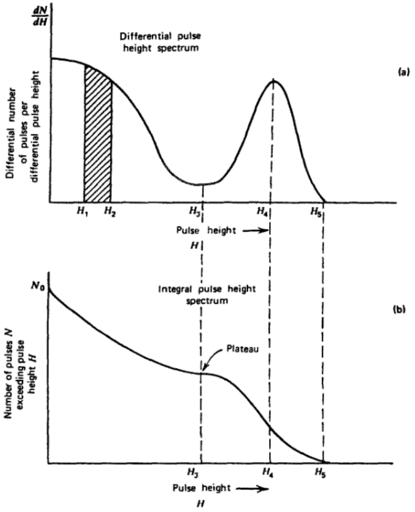


- Current mode

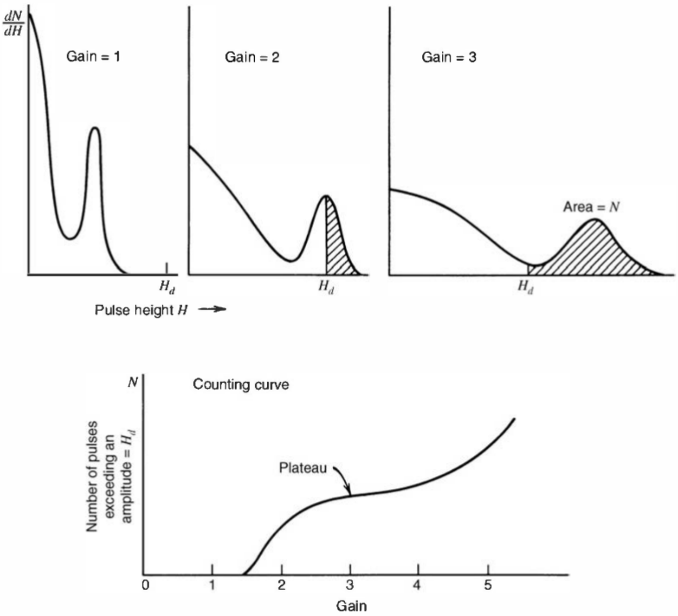


Pulse height spectra

- Differential spectrum as a number of pulses with amplitude between H_1 and H_2 dN/dH
- Integral spectrum as a number of pulses above height H



Counting plateau by varying the gain



Resolution of the measured quantity

- Expectation value when $z = z_{meas} - z_0$, where z_{meas} is measured value and z_0 is true value, $D(z)$ is probability density function

$$\langle z \rangle = \int \mathrm{d}z \cdot z D(z) \Big/ \int \mathrm{d}z \cdot D(z) \tag{1}$$

- Variance of the measured quantity

$$\sigma_z^2 = \int \mathrm{d}z \, (z - \langle z \rangle)^2 \, D(z) \Big/ \int \mathrm{d}z \cdot D(z) \tag{2}$$

Example: multi-wire proportional chamber (MWPC)

- Position along the z-coordinate with wire spacing δz

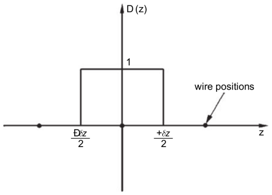


Figure : MWPC, variance of the rectangular distribution

$$\langle z \rangle = \int_{-\delta z/2}^{+\delta z/2} \mathrm{d}z \cdot z \cdot 1 \Bigg/ \int_{-\delta z/2}^{+\delta z/2} \mathrm{d}z = 0 \tag{3}$$

$$\sigma_z^2 = \int_{-\delta z/2}^{+\delta z/2} \mathrm{d}z \cdot (z-0)^2 \cdot 1 \Bigg/ \delta z = \frac{(\delta z)^2}{12} \tag{4}$$

Energy resolution

- Radiation spectroscopy = measurement of the energy distribution of the incident radiation
- Response of the detector to the monoenergetic radiation

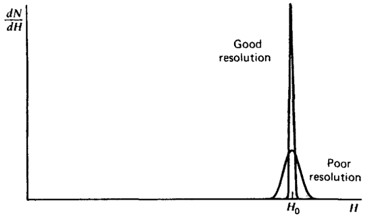


Figure : Response function of the detector

- The width rises with the fluctuations in the detected pulses

Full width at half maximum, FWHM

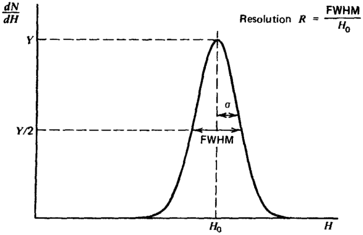


Figure : Definition of detector resolution

- Derived from Gaussian function, FWHM = 2.35σ

$$G(H) = \frac{A}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(H-H_0)^2}{2\sigma^2}\right) \qquad (5)$$

- The energy resolution measured as FWHM, expressed in %
- Semiconductor diodes for alpha spectroscopy have typically <1%, scintillation detectors for gamma-rays have 5-10%

Limiting resolution R due to statistical fluctuations

- A counting experiment follows the Poisson statistics with variance $\sqrt{N}$ for N detected counts
- Amplitude of the response function proportional to the number of counts N by factor K: $H_0 = KN$
- The Gaussian standard deviation is then $\sigma = K\sqrt{N}$
- Limiting resolution from 5 is

$$R|_{Poisson\ limit} \equiv \frac{\mathrm{FWHM}}{H_0} = \frac{2.35}{\sqrt{N}} \tag{6}$$

- Number of successfully registered pulses by the detector should be >55 000 for resolution better than 1%
- Semiconductor detectors have high resolution for high efficiency in collecting the deposition (a lot of carriers generated per unit of deposited energy)

Correction to Poisson statistics

- Processes leading to generation of the charge carriers are correlated, preventing the use of Poisson statistics
- Limiting resolution lower than expectation from 6
- The Fano factor is the correction to the Poisson distribution

$$F \equiv \frac{\text{observed variance in } N}{\text{Poisson predicted variance } = N} \tag{7}$$

$$R|_{\text{Statistical limit}} = \frac{2.35K\sqrt{N}\sqrt{F}}{KN} = 2.35\frac{F}{N} \tag{8}$$

- $F \ll 1$ for semiconductors and proportional counters, $F \approx 1$ for scintillators

Detection efficiency, definition

- Probability p to detect the incident radiation
- An experiment with two possible outcomes (detected / not detected) follows the binomial statistics
- With r successes in n trials and $p = r/n$ and $q = 1 - p$, the binomial distribution is

$$
\begin{aligned}
f(n, r, p) &= \frac{n!}{r!(n-r)!} p^r q^{n-r} \\
\langle r \rangle &= n \cdot p \\
\sigma^2 &= n \cdot p \cdot q
\end{aligned}
\tag{9}
$$

Detection efficiency, absolute, intrinsic and peak

$$\epsilon_{abs} = \frac{\#\text{ of recorded}}{\#\text{ of emitted}} \tag{10}$$

$$\epsilon_{int} = \frac{\#\text{ of recorded}}{\#\text{ of incident}} \tag{11}$$

- The absolute efficiency depends on the detector geometry
- Intrinsic efficiency depends on the detector material and construction and interaction properties
- Peak efficiency: accepted only interactions with full energy deposition

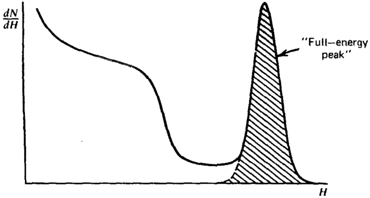


Figure : Full-energy peak in differential spectrum

Efficiency measurement

- Two detectors 1 and 2 with efficiencies $\epsilon_{1,2}$ in coincidence with the detector of unknown efficiency ϵ

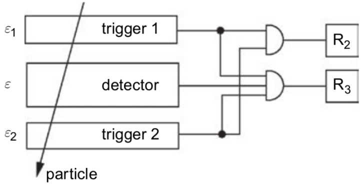


Figure : Efficiency by the coincidence measurement

- Coincidence measurement carried with n true events in the detectors
- Twofold coincidence rate $R_2 = \epsilon_1 \epsilon_2 n$
- Threefold coincidence rate $R_3 = \epsilon_1 \epsilon_2 \epsilon n$
- The efficiency is then

$$\epsilon = \frac{R_3}{R_2} \tag{12}$$

Measurement of the absolute activity

- Needed to know intrinsic peak efficiency of the detector ϵ_{ip}
- The number of radioactive decays in the source S is given by N recorded events in the solid angle Ω covered by the detector

$$S = N \frac{4\pi}{\epsilon_{\text{ip}} \Omega} \tag{13}$$

- In the geometry of point-size isotropic source, Ω is

$$\Omega = 2\pi \left(1 - \frac{d}{\sqrt{d^2 + a^2}} \right) \tag{14}$$

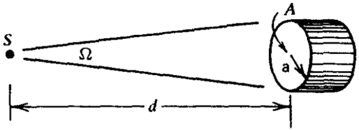


Figure : Solid angle covered by a cylindrical detector

Calibration of the detector

- Relation between the measured value $\langle z \rangle$ and the true value z_0
- In the case of the linear response, there are constants c and d

$$\langle z \rangle = c \cdot z_0 + d \tag{15}$$

- The non-linear case requires the response function

$$\langle z \rangle = c(z_0) \cdot z_0 + d \tag{16}$$

- The calibration may correct for some systematical shift in the measured values
- And/or provide the relation from the electronics output like analog-digital converter to physics quantities like time or energy

Dead time

- Minimum amount of time required to distinguish two separate pulses
- Can be 1 ns for Cherencov counters up to 1 ms in Geiger-Muller tubes
- When a time τ is needed to follow each true event (live period), there can be paralyzable and nonparalyzable response:

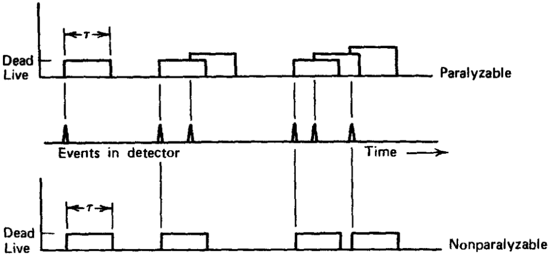


Figure : Dead time

- Nonparalyzable: 4 events recorded out of 6 physical
- Paralyzable: dead time extended by τ after each physical event, only 3 counts out of 6

Correction of dead time

- The true interaction rate n is obtained from recorded count rate m and dead time τ
- In nonparalyzable detector the rate is corrected by the total dead state $m \cdot \tau$

$$n = \frac{m}{1 - m \cdot \tau} \tag{17}$$

- With paralyzable detector the distribution of intervals between events is needed

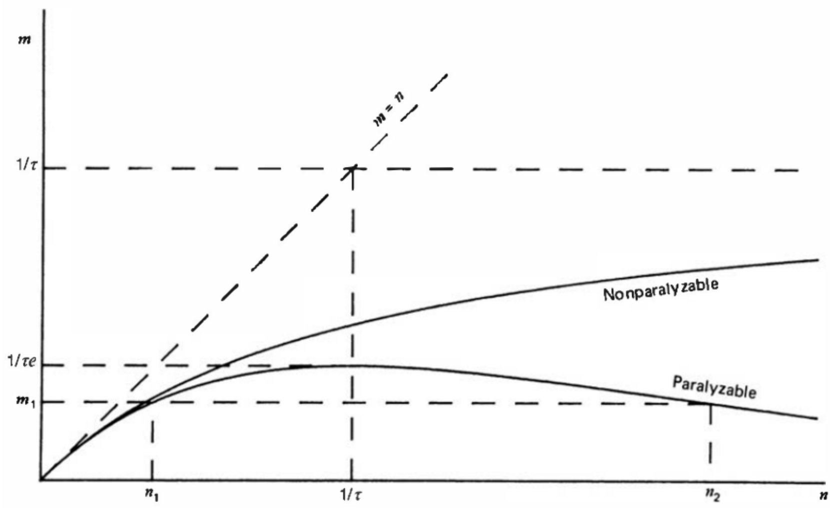
$$P_1(T)\mathrm{d}T = ne^{-nT}\mathrm{d}T \tag{18}$$

$$P_2(\tau) = \int_{\tau}^{\infty} \mathrm{d}T P_1(T) = e^{-n \cdot \tau} \tag{19}$$

- Recorded (observed rate m) is correction to the true rate n:

$$m = ne^{-n \cdot \tau} \tag{20}$$

Observed rate vs. true rate



Dead time measurement by two-source method

- Rate from two sources 1 and 2 measured individually and together
- Assuming negligible background and nonparalyzable model, dead time is given by the observed rates

$$\tau = \frac{m_1 m_2 - [m_1 m_2 (m_{12} - m_1)(m_{12} - m_2)]^{1/2}}{m_1 m_2 m_{12}} \tag{21}$$

Dead time measurement by the decaying source method

- Count rate from a short-lived radioisotope
- Observation of departure from the known exponential decay
- Measurement of the counting rate m as a function of time t

$$\lambda t + \ln m = -n_0 \tau e^{-\lambda \tau} + \ln n_0 \tag{22}$$

- Provides value of the dead time and also tests the validity of the model
- If $m(t)$ does not follow equation 22, the nonparalyzable model is not applicable

Dead time losses from pulsed sources

- Interactions from the pulsed beam at frequency f with pulse length T

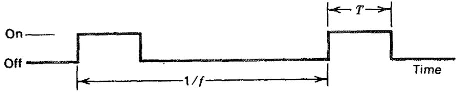


Figure : Pulsed source

- Interest of average number of true events per source pulse n/f
- In the case of dead time longer than T but shorter than the pulse spacing, $T < \tau < (1/f - T)$, probability of recording rate per source pulse m/f is given by Poisson statistics

$$\frac{m}{f} = P(> 0) = 1 - e^{-n/f} \tag{23}$$

- The true rate n is then

$$n = f \ln\left(\frac{f}{f - m}\right) \tag{24}$$

Other characteristic times

- Recovery time: period following the dead time when pulses are recorded but with reduced amplitude

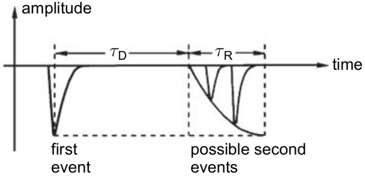


Figure : Recovery time for GM detector

- Sensitive time: time window after the trigger signal
- Readout time: period needed to register the event (electronics of film)