Nambu sigma models and their algebraic structure

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Outline

Nambu sigma model

Notation basics

- Joint work with Branislav Jurčo and Peter Schupp.
- In whole talk $p \geq 1$ is fixed integer.
- We wish to cook a classical field theory. We have to create the environment where all objects live:
  1. Let Σ be a $(p+1)$-dimensional orientable compact manifold, possibly with boundary. Σ is called **worldvolume**, with local coordinates $(\sigma^0, \sigma^1, \dots, \sigma^p)$, where σ^0 is observed as time.
  2. Let M be a n-dimensional manifold, called **target manifold**, with local coordinates $(y^1, \dots, y^n)$.
- In whole talk small Latin letters denote components w.r.t. y^i coordinates.
- Capital Latin letters denote strictly ordered p-indices, $I = (i_1, \dots, i_p)$, $i_1 < \cdots < i_p$.
- Let $X : \Sigma \to M$ be a smooth map of manifolds. We denote $X^i = y^i(X)$ and $dX^I = dX^{i_1} \wedge \ldots \wedge dX^{i_p}$.
- Finally $\widetilde{\partial X}^I = (dX^I)_{1 \dots p}$, "spacelike" components of p-form dX^I on Σ.

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Nambu sigma model

Action

- Moreover, we introduce auxiliary fields η_i and $\widetilde{\eta}_I$, both in $C^\infty(\Sigma)$, well transforming according to their index structure.
- The action of Nambu sigma model is given as integral:

$$S[\eta, \widetilde{\eta}, X] := \int d^{p+1}\sigma \Big[-\frac{1}{2}(G^{-1})^{ij}\eta_i\eta_j + \frac{1}{2}(\widetilde{G}^{-1})^{IJ}\widetilde{\eta}_I\widetilde{\eta}_J + \eta_i \partial_0 X^i + \widetilde{\eta}_I \widetilde{\partial X}^I - \Pi^{iJ}\eta_i\widetilde{\eta}_J - B_{iJ}\partial_0 X^i \widetilde{\partial X}^J \Big], \quad (1)$$

where

- $(G^{-1})^{ij}$ is the inverse of Riemannian metric G on M,
- $(\widetilde{G}^{-1})^{IJ}$ is the inverse of fiberwise Riemannian metric $\widetilde{G}$ on $\Lambda^p TM$,
- Π is a $(p+1)$-vector field on M,
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- Note that generalized generalized geometry is a natural playground for NSM, i.e. the geometry of vector bundle $TM \oplus \Lambda^p T^*M$.

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Hamiltonian formulation

- We can try to naively construct a Hamiltonian corresponding to this Lagrangian.
- The canonical momenta has the form:

$$P_i = \eta_i - B_{iJ}\widetilde{\partial X}^J.$$

- We can thus express η_i using P and B. Define

$$H[X, P, \widetilde{\eta}] := \int d^p\sigma \dot{X}^m P_m - \mathcal{L}[X, P, \widetilde{\eta}].$$

- For G, $\widetilde{G}$ nonzero, one can express $\widetilde{\eta}$'s using their EQM, to get new Hamiltonian $H = H[X, P]$.
- For $G^{-1} = \widetilde{G}^{-1}$ we cannot do that $\Rightarrow$ topological Nambu sigma model.
- We obtain the Hamiltonian of X and P only, not loosing any dynamics.

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- Define the following currents:

$$K_i := \eta_i = P_i + B_{iK}\widetilde{\partial X}^K, \quad \widetilde{K}^I := \widetilde{\partial X}^I - \Pi^{mI} K_m.$$

- The resulting Hamiltonian is quadratic and has the form

$$H[X,P] = \frac{1}{2}\int d^p\sigma[(G^{-1})^{ij}K_iK_j + \widetilde{G}_{IJ}\widetilde{K}^I\widetilde{K}^J].$$

- Expanding the K and $\widetilde{K}$ we can express it as quadratic form in P, and $\widetilde{\partial X}$:

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- The matrix $\mathbf{H}$ can be written as following product:

$$\mathbf{H} = \begin{pmatrix} 1 & 0 \\ B^T & 1 \end{pmatrix}\begin{pmatrix} 1 & -\Pi \\ 0 & 1 \end{pmatrix}\begin{pmatrix} G^{-1} & 0 \\ 0 & \widetilde{G} \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -\Pi^T & 1 \end{pmatrix}\begin{pmatrix} 1 & B \\ 0 & 1 \end{pmatrix}.$$

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$$\mathbf{H} = \begin{pmatrix} 1 & 0 \\ B^T & 1 \end{pmatrix}\begin{pmatrix} 1 & -\Pi \\ 0 & 1 \end{pmatrix}\begin{pmatrix} G^{-1} & 0 \\ 0 & \widetilde{G} \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -\Pi^T & 1 \end{pmatrix}\begin{pmatrix} 1 & B \\ 0 & 1 \end{pmatrix}.$$

Nambu sigma model

Hamiltonian formulation

- Define the following currents:

$$K_i := \eta_i = P_i + B_{iK}\widetilde{\partial X}^K, \quad \widetilde{K}^I := \widetilde{\partial X}^I - \Pi^{mI} K_m.$$

- The resulting Hamiltonian is quadratic and has the form

$$H[X,P] = \frac{1}{2}\int d^p\sigma[(G^{-1})^{ij}K_iK_j + \widetilde{G}_{IJ}\widetilde{K}^I\widetilde{K}^J].$$

- Expanding the K and $\widetilde{K}$ we can express it as quadratic form in P, and $\widetilde{\partial X}$:

$$H[X,P] = \frac{1}{2}\int d^p\sigma[\mathbf{H}^{ij}P_iP_j + 2\mathbf{H}^i{}_J P_i\widetilde{\partial X}^J + \mathbf{H}_{IJ}\widetilde{\partial X}^I\widetilde{\partial X}^J].$$

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Higher brackets

Higher Dorfman bracket

- Vector field commutator on M can be viewed as skew-symmetric bracket on $\Gamma(TM)$, satisfying
  1. $[e_1, [e_2, e_3]] = [[e_1, e_2], e_3] + [e_2, [e_1, e_3]]$ (*Jacobi identity*),
  2. $[e_1, fe_2] = (\rho(e_1).f)e_2 + f[e_1, e_2]$ (*Leibniz rule*), for all $e_1, e_2, e_3 \in \Gamma(TM)$ and $f \in C^\infty(M)$, where $\rho = Id_{TM}$
- Replace now TM with $E = TM \oplus \Lambda^p T^*M$. Is there a bracket with similar properties?
- Answer = **higher Dorfman bracket**. Define

$$[V + \xi, W + \eta]_D = [V, W] + \mathcal{L}_V \eta - i_W d\xi,$$

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- This bracket has many interesting properties. For instance, define $\langle \cdot, \cdot \rangle : \Gamma(E) \times \Gamma(E) \to \Omega^{p-1}(M)$ as

$$\langle V + \xi, W + \eta \rangle := i_V \eta + i_W \xi,$$

for all $(V + \xi), (W + \eta) \in \Gamma(E)$. This is a non-degenerate pairing, that is $\Gamma(E)^{\perp} = \{0\}$.

- This pairing is "invariant" under higher Dorfman bracket, there holds:

$$\mathcal{L}_{\rho(e)} \langle e_1, e_2 \rangle = \langle [e, e_1]_D, e_2 \rangle + \langle e_1, [e, e_2]_D \rangle,$$

for all $e, e_2, e_2 \in \Gamma(E)$.

- Let $\mathcal{D} = j \circ d$, where $j : \Omega^p(M) \to \Gamma(E)$ is an inclusion. Then we have

$$[e, e]_D = \frac{1}{2} \mathcal{D} \langle e, e \rangle,$$

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Higher brackets

Twisting the brackets

- We may ask how to modify the bracket, not spoiling the good properties.
- Let H be a closed $(p+2)$-form. Define **H-twisted higher Dorfman bracket**:

$$[V+\xi, W+\eta]_D^{(H)} = [V+\xi, W+\eta]_D + i_W i_V H.$$

- Let $\Pi^{\#} : \Omega^p(M) \to \Gamma(E)$ be a $C^\infty(M)$-linear map of sections. Define new anchor map ρ as

$$\rho(V+\xi) := V - \Pi^{\#}(\xi),$$

twisted inclusion $j : \Omega^p(M) \to \Gamma(E)$ as

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- Next, we have to modify the pairing, denote it as $\langle\cdot,\cdot\rangle_R$:

$$\langle e_1, e_2\rangle_R := i_{\rho(e_1)} pr_2(e_2) + i_{\rho(e_2)} pr_2(e_1),$$

for all $e_1, e_2 \in \Gamma(E)$, where $pr_2 : \Gamma(E) \to \Omega^p(M)$.

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- This bracket is isomorphic to $[\cdot,\cdot]_D^{(H)}$.
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$$[e_1, e_2]_R := [\rho(e_1), \rho(e_2)] + j\big(\mathcal{L}_{\rho(e_1)} pr_2(e_2) - i_{\rho(e_2)} d(pr_2(e_1)) + i_{\rho(e_2)} i_{\rho(e_1)} H\big),$$

  for all $e_1, e_2 \in \Gamma(E)$. We call this bracket **higher Roytenberg bracket**.
- This bracket is isomorphic to $[\cdot,\cdot]_D^{(H)}$.
- $[\cdot,\cdot]_D^{(H)}$ is isomorphic to $[\cdot,\cdot]_D^{(H')}$ iff $[H] = [H']$ in $H^{p+2}_{dR}(M)$.

Higher brackets

Higher Roytenberg bracket

- Next, we have to modify the pairing, denote it as $\langle \cdot, \cdot \rangle_R$:

$$\langle e_1, e_2 \rangle_R := i_{\rho(e_1)} pr_2(e_2) + i_{\rho(e_2)} pr_2(e_1),$$

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Charge algebra

Generalized charges

- Dynamics of the Nambu sigma model is governed by a canonical Poisson bracket:

$$\{X^m(\sigma), P_n(\sigma')\} = \delta^m_n \delta^p(\sigma - \sigma'),$$

where σ, σ' are p-tuples of spacelike coordinates on Σ.

- To any two functionals F, G of $[X, P]$, we can assign new field functional:

$$\{F, G\}[X, P] = \int d^p\sigma \sum_{m=1}^{n} \frac{\delta F[X,P]}{\delta X^m(\sigma)} \frac{\delta G[X,P]}{\delta P_m(\sigma)} - \frac{\delta G[X,P]}{\delta X^m(\sigma)} \frac{\delta F[X,P]}{\delta P_m(\sigma)}.$$

- Let f be a test function on Σ. Define generalized charges as:

$$Q_f(V + \xi) = \int d^p\sigma [V^m(X) K_m + \xi_I(X) \widetilde{K}^I](\sigma) f(\sigma).$$

- For many computational purposes, we were interested in Poisson bracket of two such charges $\{Q_f(V + \xi), Q_g(W + \eta)\}$.

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- The resulting Poisson bracket of the charges is

$$\{Q_f(V+\xi), Q_g(W+\eta)\} = -Q_{fg}([V+\xi, W+\eta]_R) - \int d^p\sigma g(\sigma)(df \wedge X^*(\langle V+\xi, W+\eta\rangle_R))_{1\dots p}\,, \quad (2)$$

  for all $(V+\xi), (W+\eta) \in \Gamma(E)$.
- Note that restriction of charges onto an isotropic subbundle of E closes to Poisson algebra.
- Also setting $f = 1$ implies the vanishing of the "anomalous term".
- The right-hand side is skew-symmetric in both inputs, despite the fact $[\cdot,\cdot]_R$ is not.

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Charges conservation

- One may naturally ask when do the generalized charges $Q_f(V+\xi)$ conserve under time evolution.
- For simplicity, we have assumed only $Q(V+\xi) = Q_1(V+\xi)$.
- Thus we have to find sufficient conditions to solve $\{Q(V+\xi), H\} = 0$.
- Using the result above, one arrives to the following set of equations:

$$\mathcal{L}_W(G)_{ij} = G_{in}\Pi^{nL}(W^m dB_{mjL} - (d\xi)_{jL}) + (i \leftrightarrow j),$$

$$\mathcal{L}_W(\widetilde{G})_{IJ} = \widetilde{G}_{IL}\Pi^{nL}(W^m dB_{mnJ} - (d\xi)_{nJ}) + (I \leftrightarrow J),$$

$$\mathcal{L}_W(\Pi)^{kI} = (\Pi^{kJ}\Pi^{nI} - (\widetilde{G}^{-1})^{IJ}(G^{-1})^{kn})(W^m dB_{mnJ} - (d\xi)_{nJ}),$$

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- There exists a nice geometrical interpretation of these conditions. Define a fiberwise metric $(\cdot,\cdot)$ on $TM \oplus \Lambda^p T^*M$ as

$$(V+\xi, W+\eta) := \begin{pmatrix} V \\ \xi \end{pmatrix}^T \begin{pmatrix} G & 0 \\ 0 & \widetilde{G}^{-1} \end{pmatrix} \begin{pmatrix} W \\ \eta \end{pmatrix},$$

for all $(V+\xi), (W+\eta) \in \Gamma(E)$.

- The conditions on the previous slide are then equivalent to the "Killing equations" for $V+\xi$ and $(\cdot,\cdot)$:

$$\rho(V+\xi).(e_1,e_2) = ([V+\xi, e_1]_R, e_2) + (e_1, [V+\xi, e_2]_R),$$

for all $e_1, e_2 \in \Gamma(E)$.

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Topological model

Action

- Starting all over with the action $S[X, \eta, \widetilde{\eta}]$, we may set $G^{-1} = \widetilde{G}^{-1} = 0$. We call this a **topological Nambu sigma model**.
- One comes to the new Hamiltonian

$$H[X, \widetilde{\eta}, P] = - \int d^p \sigma \widetilde{\eta}_I \widetilde{K}^I \,. \tag{3}$$

- One of the original EQM is $\widetilde{K}^I = 0$, which can be considered as constraint, with $\widetilde{\eta}_I$ as a corresponding Lagrange multiplier.
- Without $G, \widetilde{G}$ Nambu sigma model becomes a constrained system. To have consistent system, one has to check if

$$\{H, \widetilde{K}^I(\sigma)\} \approx 0,$$

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Consistency of constraints

- Calculation may be carried out using the Poisson bracket of generalized charges.
- The result has the following form:

$$\{\widetilde{K}^I(\sigma), \widetilde{K}^J(\sigma')\} = -\delta(\sigma - \sigma')(R^{IJk} K_k + S^{IJ}_K \widetilde{K}^K)(\sigma') \\ - \big(d(\delta(\sigma - \cdot)) \wedge X^*(\langle dy^I, dy^J \rangle_R)\big)_{1\dots p}(\sigma'), \quad (4)$$

where R^{IJk} are (one of) structure functions of $[\cdot, \cdot]_R$.

- One may demand R^{IJk} to vanish. For $p > 1$, this is exactly the differential part of the equation

$$(\mathcal{L}_{\Pi^\#(\xi)}(\Pi))^\#(\eta) = -\Pi^\#(i_{\Pi^\#(\eta)}(d\xi)),$$

which says that Π is a Nambu-Poisson structure. For $p = 1$ this is exactly Jacobi identity for Poisson bracket induced by Π.

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Consistency of constraints

- Calculation may be carried out using the Poisson bracket of generalized charges.
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- The vanishing of anomalous term is a little bit problematic, since $\langle dy^I, dy^J \rangle_R$ can in general vanish only for $\Pi = 0$.
- The key is to add a set of secondary constraints:

$$\chi_q^{IJ} \equiv (X^* \langle dy^I, dy^J \rangle_R)_{1 \dots \hat{q} \dots p} = 0 \,.$$

- Again, one has to check if they are consistent with a time evolution.
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Conclusions

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- Higher Roytenberg bracket was rederived using worldsheet algebra of this model.
- Consistency of topological model equations can be assured by introducing Nambu-Poisson structures.
- **Future efforts:**
  1. Understand the generalized generalized geometry, especially the generalized generalized metric.
  2. Understand the higher Courant algebroids, find their axiomatiozation.
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Thank you for your attention!