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Interakce nabitých částic s komplexem elektromagnetickým vln šířících se plazmatem v magnetickém poli Země

The interaction of charged particles with the electromagnetic wave complex propagating in plasma in the Earth's magnetic field

DIPLOMA THESIS

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Pred svazanim misto tehle stranky $\boxed{\text{vlozite zadani prace}}$ s podpisem dekana (bude to jediny oboustranny list ve Vasi praci) !!!!

Prohlášení

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V Praze dne ....

....
Bc. Hana Zemanová

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Bc. Hana Zemanová

Title:
The interaction of charged particles with the electromagnetic wave complex propagating in plasma in the Earth's magnetic field
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Abstract: In my research, I am concerned by unrelativistic motion of an electron in a storm cloud, which is represented by homogenous electric field, along with an element of friction, in this case, Chandrasekhar function. Next, I introduced into the equation of motion an element describing interaction between given electron and electromagnetic R-wave. The R wave is right-handed, just like moving electron in magnetic field, so the assumption can be made that these two phenomena strongly interact. From previous works I know that the electron in electric field will be accelerated to relativistic velocities, so I now use a relativistic approach. As motion equation for the given electron I chose relativistic Lorentz equation, along with Chandrasekhar function representing Coulomb scattering. The most important part of my diploma thesis is the numeric solution of this equation.

Key words: lightning, thunderstorm, runaway breakdown, R wave, special relativity

Název práce:
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Abstrakt: Ve svém výzkumu se věnuji nerelativistickému pohybu elektronu v bouřkovém oblaku spolu se třecím členem reprezentovaným Chandrasekharovou funkcí. Dále jsem přidala člen popisující interakci daného elektronu s elektromagnetickou R vlnou. R vlna je pravotočivá stejně jako elektron pohybující se v magnetickém poli, proto lze předpokládat, že tyto dva jevy budou interagovat. Z předchozích prací vím, že elektron v elektrickém poli bude urychlován na relativistické rychlosti, proto nyní používám relativistické rovnice. Jako pohybovou rovnici pro uvažovaný elektron jsem zvolila relativistickou Lorentzovu pohybovou rovnici s přidanou Chandrasekharovou funkcí reprezentující Coulombovské srážky. Nejdůležitější částí mé diplomové práce je numerické řešení této rovnice.

Klíčová slova: blesk, bouřka, urychlování elektronů, R vlna, speciální relativita

Contents

List of symbols

$\mathbf{A}$	vector quantity
A	norm of vector quantity
$\nabla_{\mathbf{x}}$	nabla from particle position
b_0	critical impact parameter
$\mathbf{B}$	magnetic field
$\mathbf{B}_0$	homogenous magnetic field
c	speed of light
c_α	speed of sound for particle type α
e	elementary charge
$\mathbf{D}$	electric displacement field
$\mathbf{E}_0$	homogenous electric field
E_{tresh}	norm of treshold value of electric field
f_α	probability density function of particle type α
g	norm of vector of relative velocity
$G_{\alpha\beta}$	second Rosenbluth potential
$H_{\alpha\beta}$	first Rosenbluth potential
i	imaginary unit
$\mathbf{j}$	current density
$\mathbf{k}$	wave's vector
k_B	Boltzmann constant
$\ln \Lambda_{\alpha\beta}$	Coulomb logarithm
m_α	mass of particles of type α
N	refractive index
n_α	concentration of particles of type α
Q_α	charge of particles of type α
R_{L}	Larmor radius
$\mathbf{S}_{\alpha\beta}$	Boltzmann scatteering integral
T_α	temperature of particles of type α
T_{R}	radiation period
$\mathbf{u}_{\mathrm{e}}$	velocity of electron fluid
$\mathbf{v}_{perp}$	velocity perpendicular to magnetic field
$\mathbf{v}_f$	phase velocity
Z	degree of plasma ionization

α	angle between homogenous electric field and Earth's magnetic field
	angle between wave vector and homogenous magnetic field
γ_α	polytropic coeficient fo particle type α
$\delta\mathbf{A}$	variation of vector quantity
$\delta(x)$	Dirac function
ε_0	vacuum permitivity
λ_D	Debye length
μ_0	vacuum permeability
$\phi(x)$	error function
$\psi(x)$	Chandrasekhar function
ω	angular frequency of wave
ω_c	cyclotron frequency
ω_D	angular frequency of Hertz dipole
$\omega_{\mathrm{p}\alpha}$	plasma frequency of particle type α

Introduction

In this work, I begin by summarizing current knowledge about types of lightning. I also describe lightning formation, along with the role of runaway breakdown in this process. The second part is concerned with R waves, which occur when high-frequency electromagnetic wave passes through the plasma with external magnetic field. Finally, I am concerned with an electron, originating from cosmic rays, and his eventual acceleration.

One of the main goals of my work is to study interaction between R wave and cosmic rays electron. But equally significant goal is to investigate whether the considered electron will be accelerated to relativistic velocities. For this purpose, I used relativistic Lorentz equation for motion of charged particle in external magnetic and electric fields. Also, in one part of this work, Coulomb scattering term was added to motion equation. This term is derived from nonrelativistic Fokker-Planck equation. Although the scattering term is nonrelativistic, it can be used because its influence on motion is insignificant. Despite two years of attempts, its relativistic extension had not been found yet. The required derivation is very difficult, if not impossible.

Chapter 1

Physical Situation

1.1 Storm Cloud and Types of Lightning

In this chapter I follow approach used in my previous work [10]. A storm is created in clouds named cumulonimbus (from Latin: rain column). A shape of these clouds can be described as an anvil or a mushroom. Their nearly flat bottom base is located in heights approximately 300 m above surface [11]. Their peak reaches to around 12 km. In cumulonimbus cloud, significant vertical streaming promotes charge separation, a phenomenon, which is described below.

Charge centres are created in cloud after charge separation. According to some theories [9], there are two charge centres, lower centre in bottom and middle parts of cloud, with negative charge, and larger, upper charge centre with positive charge. By some other theories [8] third charge centre exists on cloud base with small amount of positive charge.

Due to location of stroke, lightning is divided into two main categories. First category is lightning in or under cloud, which are drawn on Fig. 1.2 along with charge distribution in storm cloud. This category comprises several types [9]:

- intracloud – if the cloud moves at high velocity, individual strokes can not be distinguished and planar lightning can be seen,
- cloud to cloud – first two types comprise up to 75 % of all lightning strokes,
- cloud to air lightning,
- cloud to ground lightning - its polarity can be determined from branching and also from charge flow:
  - negative lightning (90–95 percent of cloud to ground lightning) – originates from negatively charged cloud base and propagates downwards Earth, this type is described in detail in following chapter,
  - positive lightning (5–10 percent of cloud to ground lightning) – lasts ten times longer than negative lightning and redistributed electric current

is also ten times larger than in the other type. Negative stepped leader channel rises from Earth, positive streamer proceeds from upper cloud layer, at first horizontally, and then it reaches down to Earth. Therefore, both streamers can connect and the lightning strike also in location without thunderstorm.

Second category, drawn on Fig. 1.3, are Transient Luminous Events. Their creation is still subject of intensive research [6] [7]:

- Blue jets – aim form upper cloud layers up to mesosphere. These blue discharges extend in upward direction.
- Sprites and sprite halos – occur in mesosphere in height about 80 km. These most frequent discharges are composed of positive and negative streamers, which are wider and longer than those found in lightning from the first category, due to lower air density. Main strokes are not present. Sprite halo preceding sprite is smaller, and located at lower altitudes, than sprite.
- Trolls (from: Transient Red Optical Luminous Lineament) – are similar to sprites. These discharges, which follow sprites, occupy space between sprites and top layers of cloud.
- Elves (from: Emissions of Light and Very Low Frequency Perturbations from Electromagnetic Pulse Sources) – are toroidal discharges around sprites.

1.2 Lightning Formation

In this whole subsection, I will study negative cloud to ground lightning (unless otherwise specified).

Basic theory of lightning formation was created by Heinz Kasemir [8], after which creation and formation of lightning takes place in several parts [9]:

1. Charge separation – charge is accumulated into layers[1] by little understood mechanism. The most probable mechanism is the following: particles of cloud are charged by Coulomb scattering along with gravity and upward streaming inside cumulonimbus. [6]
2. Creation of stepped leader channel, which is negatively charged streamer oriented from cloud to earth (in case of positive lightning, its direction is opposite). This channel is called stepped because it descends by individual steps lasting about 50 ms [9] – it divides in branches searching for connection with Earth, as shown on these videos [12] [13].
3. Creation of positive streamer – if there is an elevated conductive area on a surface of Earth.

[1]In this part creation of two charge centres is described.

4. Connection of both streamers forming conductive channel, named main stroke. In this step, a part of negative charge flows from cloud to Earth.

5. Return stroke – positive charge flows from Earth to cloud (electric current is approximately 3×10^4 A, the temperature is up to 3×10^4 K [9]). This is perceived as a lightning. A thunder is caused by an expansion of return stroke due to high pressure (5 atmospheres) in it.

After this sequence, another strokes (formed by leading stroke and positive streamer) can follow in the same conductive channel as first stroke. These restrikes, in most cases, are more feeble than first strike. Together, they form stroboscopic effect, which is occasionally seen. Between individual sequences, there is "rest period" lasting from 30 to 50 ms [9].

1.3 Conventional and Runaway Breakdown

To create stepped leader, an area of discharge has to exist in the cloud. So in this part I am concerned by creation of such area. One assumption to this process was conventional breakdown. First to study this problem was A. V. Gurevich [2] [3]. Fast electrons with energy 10 – 20 eV can ionize surrounding molecules and thus create new free electrons. Slower part of these new-born electrons are caught by air molecules and recombine with them. If electric field exceeds a treshold value E_{tresh}, electron creation exceeds recombination and so number of free electrons rises exponentially. This process is called conventional breakdown. Only few electrons loose energy by ionization and gamma ray bursts are not observed.

For conventional breakdown, treshold of electric field E_{tresh} is proportional to concentration of air molecules. In the air at standard conditions, it holds: $E_{\mathrm{tresh}} \approx 2{,}3\,\mathrm{MV/m}$ [3]. But in storm cloud electric field is not large enough, so an area of discharge has to be created by another mechanism. Runaway breakdown along with cosmic rays is a promising candidate to required mechanism.

Runaway breakdown, which means air breakdown by runaway electrons, was proposed by Alexandr V. Gurevich, Gennady M. Milikh and Robert Roussel-Dupre [2] in year 1992. At first, I will describe what runaway electrons are. Cosmic ray particles, which arrive mostly from the Sun, ionize air molecules in the course of passing through the atmosphere. If electric field overcomes energy losses due to scattering, given particle will be accelerated up to relativistic velocities. When using nonrelativistic approach, it can be derived that treshold value needed for electron acceleration in the ionosphere is approximately $70\,\mathrm{ms}^{-1}$ (for electron-electron collisions) [10].

Next, I will describe how air breakdown is caused by runaway electrons. If new born electron, caused by ionization, has a velocity greater than threshold, this electron is accelerated and ionizes air molecules along its path. Some of knocked out electrons can also be faster than treshold velocity and so they are also accelerated and also ionize molecules. By this process, an avalanche of runaway electrons is created, which is called the runaway breakdown. Gamma ray bursts are observed. Threshold electric field of runaway breakdown is one tenth of treshold electric value of conventional breakdown [3].

It is not clear how this avalanche of runaway electrons, which is split up in space, creates narrow corridor of ionized air, what stepped leader and main stroke are (main stroke is about 5 cm wide [9]). One possible theory is following [6]. Runaway particle causes runaway breakdown in narrow area of ionized air surrounded by nonionized air, which can be called streamers. The charge of these conductive lines is concentrated on their tips, where electric field is enhanced. When electric field is strong enough, then conventional breakdown starts around the tip of streamer. So runaway breakdown area expands and moves downward Earth. Also charge is accumulated around streamer's tip. By this process, stepped leader is created.

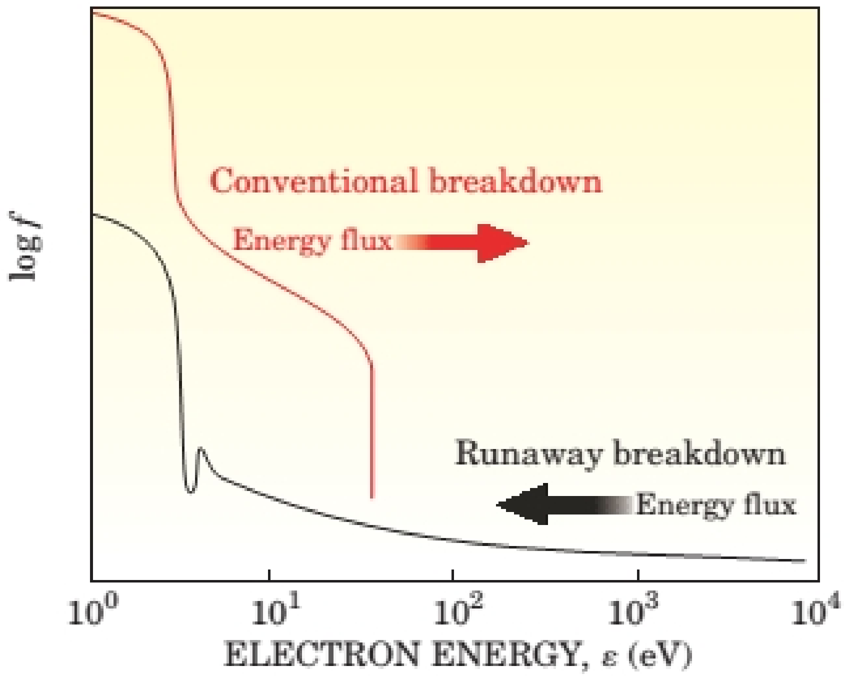

Figure 1.1: Comparison of conventional and classical breakdown. Picture describes electron distribution function f for both types of breakdown. Arrows signal the direction of energy flow. Runaway breakdown is composed by faster particles than conventional breakdown. [3].

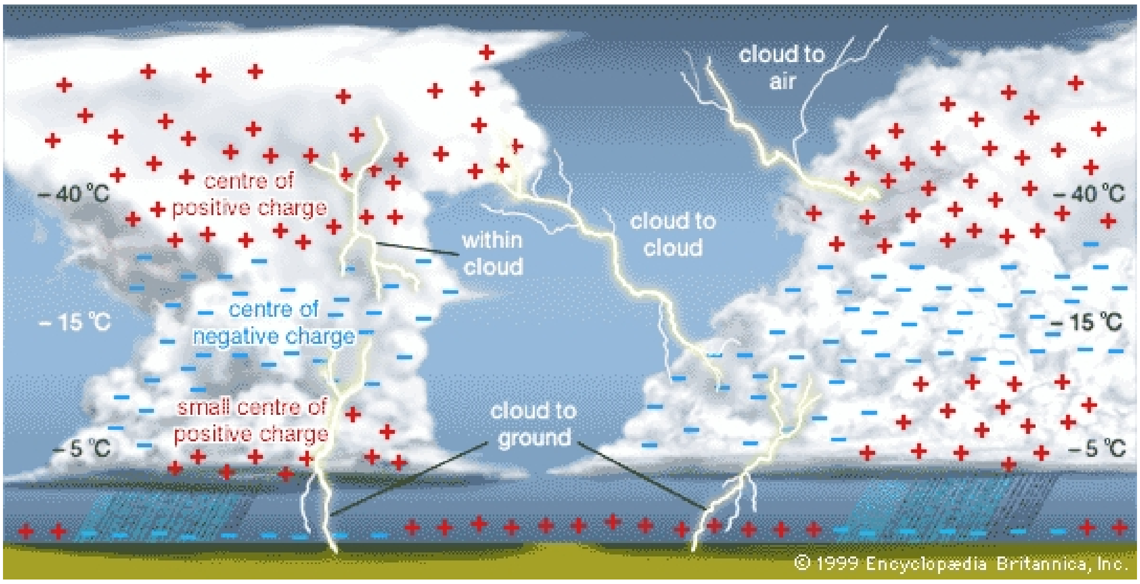

Figure 1.2: Charge density in storm cloud [14].

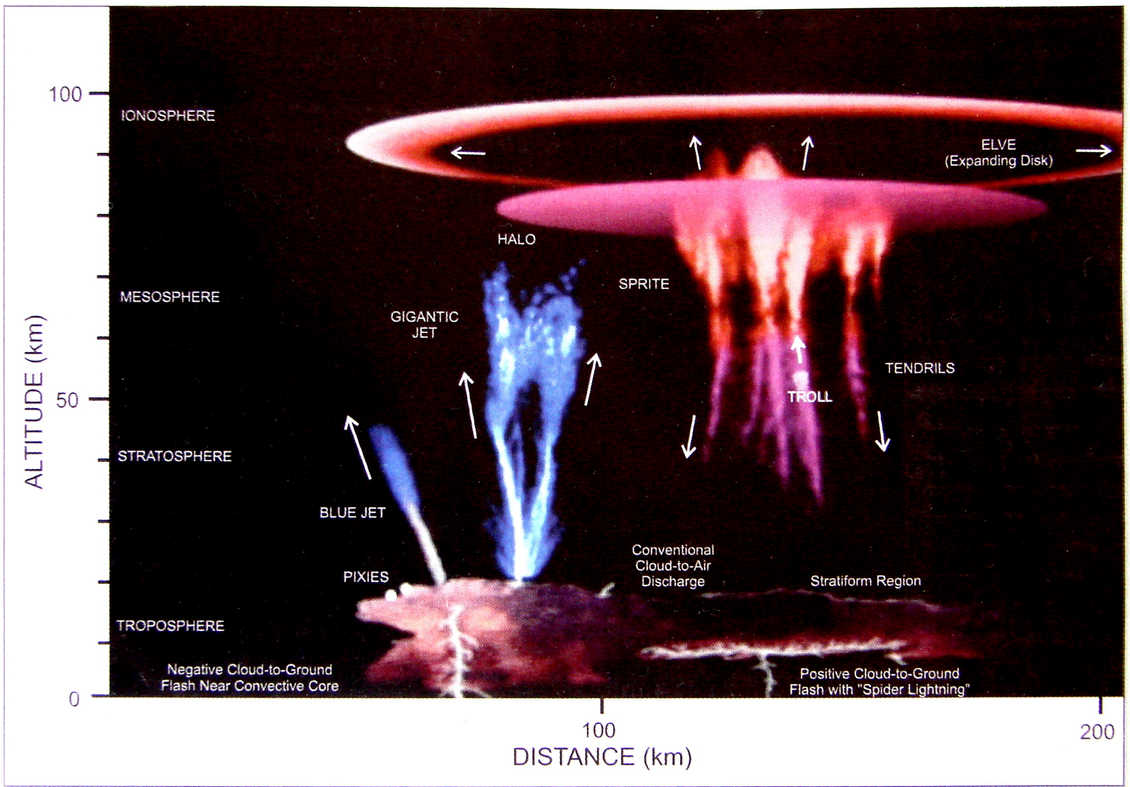

Figure 1.3: Transistent luminous events above thunderstorm [6].

Chapter 2

Plasma and Linear Waves

2.1 Plasma

Plasma is quasineutral area of gas discharge containing unbounded charged particles with collective behaviour [1]. Plasma is divided by several criteria into categories, for example by degree of ionization or if electron-positron pairs are created. Most often plasma is classified by electron concentration and the temperature.

2.2 Cyclotron and Plasma Frequency

Cyclotron frequency ω_c is related to motion of charged particle in homogenous magnetic field $\mathbf{B}$. If particle with charge Q and mass m moves perpendicularly to magnetic field with velocity v_{perp}, it rotates in a plane perpendicular to magnetic field along circle with Larmor radius R_L and cyclotron frequency ω_c:

$$R_L \equiv \frac{mv_\perp}{QB}, \quad \omega_c \equiv \frac{QB}{m}, \tag{2.1}$$

where B is absolute value of magnetic field vector: $B = |\mathbf{B}|$. When charged particle has also velocity component in magnetic field's direction (z axis), it moves along helix, because motion is combination of described Larmor rotation (gyration) and uniform linear motion along magnetic field's direction.

Plasma frequency is related to plasma oscillations. Let us have plasma without magnetic field, but with electric field $\mathbf{E}$. Wave propagating through plasma causes three modes in it: plasma oscillations and plasma and iont waves. As follows from Maxwell equation ($rot\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} = 0$), these waves are longitudinal, i. e. $\mathbf{k} \parallel \delta\mathbf{E}$. On external stimulus, electron field moves first, because electron has small mass. Then ions move in reaction and so electric field is created. Due to different masses electrons and ions oscillate on different frequencies and thus two-fluid model has to be used. In this case we have dispersion relation:

$$\left[\left(\omega^2 - \omega_{\text{pe}}^2\right)\left(\omega^2 - \omega_{\text{pi}}^2\right) - \omega_{\text{pe}}^2\omega_{\text{pi}}^2\right]^2 \left[\left(\omega^2 - \omega_{\text{pe}}^2 - c_{\text{e}}^2 k^2\right)\left(\omega^2 - \omega_{\text{pi}}^2 - c_{\text{i}}^2 k^2\right) - \omega_{\text{pe}}^2\omega_{\text{pi}}^2\right] = 0, \tag{2.2}$$

where ω is wave's angular frequency, $\mathbf{k}$ its wave's vector, c_{e}^2 and c_{i}^2, respectively, are "speed of sound" of electrons and ions done by relations:

$$c_{\text{e}}^2 \equiv \frac{\gamma_{\text{e}} k_{\text{B}} T_{\text{e}}}{m_{\text{e}}}, \text{ resp. } c_{\text{i}}^2 \equiv \frac{\gamma_{\text{i}} k_{\text{B}} T_{\text{i}}}{m_{\text{i}}}.$$

In these relations γ_{e} and γ_{i} is polytrophic coefficient, respectively, for electrons and ions, k_{B} denotes Boltzmann constant, T_{e} and T_{i}, respectively, is temperature of electrons and ions, m_{e} and m_{i}, respectively, are masses of electrons and ions.

Plasma frequency of electrons ω_{pe} and ions ω_{pi} also appears in dispersion relation. They are given by relations:

$$\omega_{\text{pe}}^2 \equiv \frac{n_{\text{e0}}\, e^2}{m_{\text{e}}\varepsilon_0}, \text{ resp. } \omega_{\text{pi}}^2 \equiv \frac{n_{\text{i0}}\, e^2}{m_{\text{i}}\varepsilon_0}, \tag{2.3}$$

where n_{e0} and n_{i0}, respectively, are concentrations of electrons and ions, e is elementary charge, m_{e} and m_{i}, respectively, are masses of electron and ion, ε_0 is vacuum permittivity.

As follows from (2.2), plasma without external magnetic field has two modes:

- plasma oscillations of electrons and ions on frequency $\omega_{\text{pe}}^2 + \omega_{\text{pi}}^2$ – described by first bracket (wave vector is not present). Because mass of electrons is much smaller than mass of ions, second term is insignificant. In limit case of infinite ion mass, plasma oscillates on plasma frequency of electrons and ions remain motionless,
- plasma waves – described by second bracket (wave vector is present). As is common, the system behaves differently for low and high frequencies:
  - high frequencies (limit of infinite ion mass): only electrons move,
  - low frequencies (limit of massless electrons): ions move on negative electron background because electrons react to stimulus immediately and move, where they are needed.

2.3 Electromagnetic Waves

Let us have plasma composed from ions and electrons with concentration n_{e} with homogenous magnetic field $\mathbf{B}_0$. Through plasma, electromagnetic waves begin to propagate. In this section, only high frequency waves are taken into account and so only electrons can interact with it. Thus, for easy computation, limit of high frequencies holds, which means that ions have infinite mass (motionless ions) and we have no pressure (thermal/chaotic motion is insignificant). We say that we have cold electron plasma. Due to Maxwell equations it holds: $\frac{\partial \mathbf{D}}{\partial t} \neq 0$, so electric field can not be neglected.

Let us have a function f depending on location and time, for example a velocity of electron fluid. In plasma, total derivative of that function with respect to time can be decomposed into explicit change (held on given place) and convection change (flowed in):

$$\frac{\mathrm{d}\, f}{\mathrm{d}\, t} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x^k}\frac{\mathrm{d}\, x^k}{\mathrm{d}\, t} = \frac{\partial f}{\partial t} + (\mathbf{u} \cdot \nabla)\, f.$$

With these assumptions initial set of equations can be written as follows:

$$\begin{aligned} \frac{\partial n_{\mathrm{e}}}{\partial t} + \mathrm{div}(n_{\mathrm{e}}\mathbf{u}_{\mathrm{e}}) &= 0, \\ m_{\mathrm{e}} n_{\mathrm{e}} \frac{\partial \mathbf{u}_{\mathrm{e}}}{\partial t} + m_{\mathrm{e}} n_{\mathrm{e}} (\mathbf{u}_{\mathrm{e}} \cdot \nabla)\mathbf{u}_{\mathrm{e}} &= -e n_{\mathrm{e}} \mathbf{E} + \mathbf{j} \times \mathbf{B}, \\ \frac{\partial \mathbf{B}}{\partial t} &= -\mathrm{rot}\mathbf{E}, \\ \frac{\partial \mathbf{E}}{\partial t} &= \frac{1}{\varepsilon_0 \mu_0}\mathrm{rot}\mathbf{B} - \frac{\mathbf{j}}{\varepsilon_0}, \\ \mathbf{j} &= -e n_{\mathrm{e}} \mathbf{u}_{\mathrm{e}}, \end{aligned} \tag{2.4}$$

where m_{e} is electron mass, $\mathbf{u}_{\mathrm{e}}$ the velocity of electron fluid. $\mathbf{E}$ and $\mathbf{B}$, respectively, are electric and magnetic field, $\mathbf{j}$ is current density. ε_0 is vacuum permittivity and μ_0 is vacuum permeability. First equation is called continuity equation for electron density, second equation called velocity equation represents equation of motion for electron fluid element. Last two equations are Maxwell equations for electric and magnetic fields.

These equations are non-linear, so computations start with perturbation theory:

$$n_e = n_0 + \delta n_e, \quad \mathbf{u}_e = \delta \mathbf{u}_e, \quad \mathbf{B} = \mathbf{B}_0 + \delta \mathbf{B}, \quad \mathbf{E} = \delta \mathbf{E}. \tag{2.5}$$

Because now given set of equations is linear, we can assume superposition theorem which is equivalent with Fourier transformation. This means, that waves are decomposed to plane waves, for example of electric field: $\mathbf{E} = \mathbf{E}_1 \exp(\mathrm{i}(\mathbf{kx} - \omega t))$, where $\mathbf{E}_1$ is wave's amplitude, $\mathbf{k}$ its wave vector and ω its angular frequency, $\mathbf{x}$, t are its coordinates.

After these and some other adjustments, following set of equations, e. g. for perturbation of electric field $\delta\mathbf{E}$, can be obtained:

$$\mathbb{M}_{\mathbf{E}} \cdot \delta \mathbf{E} = 0. \tag{2.6}$$

Coordinate system is chosen in way that vector of magnetic filed $\mathbf{B}_0$ points in direction of axis z and wave's vector lies in plane xz, i. e.

$$\mathbf{B}_0 = \begin{pmatrix} 0 \\ 0 \\ B_0 \end{pmatrix}, \mathbf{k} = \begin{pmatrix} k \sin \alpha \\ 0 \\ k \cos \alpha \end{pmatrix}, \tag{2.7}$$

Then matrix $\mathbb{M}_{\mathbf{E}}$ of set of equations has following form:

$$\mathbb{M}_{\mathbf{E}} = \begin{pmatrix} \omega^2 - \omega_{\mathrm{p}}^2 - c^2k^2\cos^2\alpha, & \mathrm{i}\frac{\omega_{\mathrm{c}}}{\omega}\left(\omega^2 - c^2k^2\right), & c^2k^2\cos\alpha\sin\alpha, \\ -\mathrm{i}\frac{\omega_{\mathrm{c}}}{\omega}\left(\omega^2 - c^2k^2\cos^2\alpha\right), & \omega^2 - \omega_{\mathrm{p}}^2 - c^2k^2, & -\mathrm{i}\frac{\omega_{\mathrm{c}}}{\omega}c^2k^2\cos\alpha\sin\alpha, \\ c^2k^2\cos\alpha\sin\alpha, & 0, & \omega^2 - \omega_{\mathrm{p}}^2 - c^2k^2\sin^2\alpha. \end{pmatrix}, \tag{2.8}$$

where c is speed of light in vacuum, ω_{c} is cyclotron frequency, ω_{p} is plasma frequency of electrons, α is angle between magnetic field and direction of wave propagation.

Finally, from condition on nontrivial solution follows that determinant of matrix $\mathbb{M}_{\mathbf{E}}$ has to be zero. From this condition dispersion relation of electromagnetic complex can be obtained:

$$-\left[c^4k^4\, cos^2\alpha\, sin^2\alpha\right]\cdot\left[\left(\omega^2 - \omega_{\mathrm{p}}^2 - c^2k^2\right) - \left(\frac{\omega_{\mathrm{c}}}{\omega}\right)^2\left(\omega^2 - c^2k^2\right)\right] + \left[\omega^2 - \omega_{\mathrm{p}}^2 - c^2k^2\sin^2\alpha\right]\cdot$$
$$\cdot\left[\left(\omega^2 - \omega_{\mathrm{p}}^2 - c^2k^2\cos^2\alpha\right)\cdot\left(\omega^2 - \omega_{\mathrm{p}}^2 - c^2k^2\right) - \left(\frac{\omega_{\mathrm{c}}}{\omega}\right)^2\left(\omega^2 - c^2k^2\right)\left(\omega^2 - c^2k^2\cos^2\alpha\right)\right] = 0. \tag{2.9}$$

When electromagnetic wave moves perpendicularly to magnetic field ($\alpha = 90°$), ordinary and extraordinary waves are created which are know from optics. But this work is concentrated on R and L waves created when electromagnetic wave moves along magnetic field, i. e. $\alpha = 0$. In this case dispersion relation has form:

$$\left[\omega^2 - \omega_{\mathrm{p}}^2\right]\cdot\left[\left(\omega^2 - \omega_{\mathrm{p}}^2 - c^2k^2\right)^2 - \left(\frac{\omega_{\mathrm{c}}}{\omega}\right)^2\left(\omega^2 - c^2k^2\right)^2\right] = 0. \tag{2.10}$$

First bracket describes plasma oscillations on plasma frequency of electrons ω_{p}. Second bracket is related to two other modes, R and L waves.

2.3.1 R and L Waves

For R and L wave, following dispersion relation holds, which is derived from (2.10):

$$\left(\omega^2 - \omega_{\mathrm{p}}^2 - c^2k^2\right)^2 - \left(\frac{\omega_{\mathrm{c}}}{\omega}\right)^2\left(\omega^2 - c^2k^2\right)^2 = 0. \tag{2.11}$$

This dispersion relation can be easily solved with respect to ck:

$$c^2k^2 = \omega^2 - \frac{\omega_{\mathrm{p}}^2}{1 \mp \omega_{\mathrm{c}}/\omega}. \tag{2.12}$$

Top sign means left-handed (L) polarized wave and bottom sign means right-handed (R) polarized wave. The handedness of wave is justified below.

New quantity, refractive index is introduced by relation

$$N(\omega) \equiv \frac{ck(\omega)}{\omega}. \tag{2.13}$$

Refractive index depends on concentration (due to plasma frequency) and on magnetic field (due to cyclotron frequency), see (2.1) and (2.3). In this case, for refractive index holds:

$$N^2 = 1 - \frac{\left(\omega_{\mathrm{p}}/\omega\right)^2}{1 \mp \omega_{\mathrm{c}}/\omega}. \tag{2.14}$$

If N^2 is negative, then refractive index is imaginary and the wave is absorbed by plasma. For wave propagation, refractive index has to have values between zero and infinity. In border points, plasma behaves strangely. If refractive index tends to infinity, i. e. phase velocity $v_{\mathrm{f}} \to 0$, so-called cut-off occurs, which denotes boundary between propagating and non-propagating of wave. Right and left cut-off frequency can be easily derived from (2.14):

$$\omega_{\mathrm{R,L}} \equiv \mp\frac{\omega_{\mathrm{c}}}{2} + \frac{1}{2}\sqrt{\omega_{\mathrm{c}}^2 + 4\omega_{\mathrm{p}}^2}. \tag{2.15}$$

In case that refractive index tends to zero, i. e. phase velocity $v_{\mathrm{f}} \to \infty$, resonance occurs. This condition can be fulfilled only for R wave on cyclotron frequency $\omega = \omega_{\mathrm{c}}$. As described later in this work, polarization plane of these waves rotates right-handed along wave vector $\mathbf{k}$, same as fast electron. Thus large amounts of energy are transfered between electron and R wave. So assumption that R wave accelerates electron can be made, which is motivation for this work.

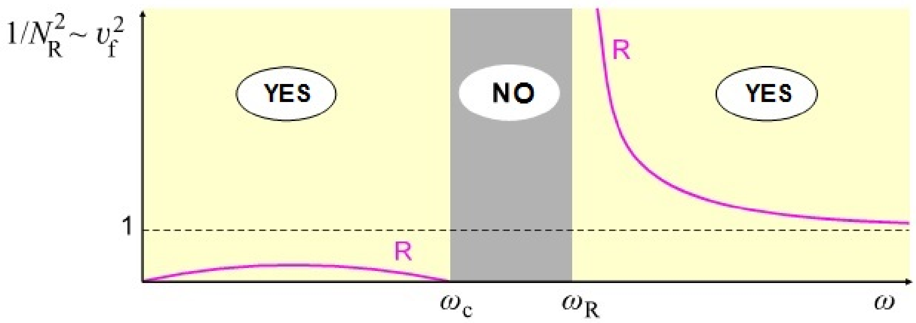

Figure 2.1: Dispersion relation of R wave. Wave propagates only in light strips. [1]

To obtain wave polarizatons, direction analysis has to be done. Matrix (2.8) of equation for perturbation of electric field has this form for $\alpha = 0$:

$$\mathbb{M}_{\mathbf{E}} = \begin{pmatrix} \omega^2 - \omega_{\mathrm{p}}^2 - c^2k^2, & \mathrm{i}\frac{\omega_{\mathrm{c}}}{\omega}\left(\omega^2 - c^2k^2\right), & 0, \\ -\mathrm{i}\frac{\omega_{\mathrm{c}}}{\omega}\left(\omega^2 - c^2k^2\right), & \omega^2 - \omega_{\mathrm{p}}^2 - c^2k^2, & 0, \\ 0, & 0, & \omega^2 - \omega_{\mathrm{p}}^2. \end{pmatrix}.$$

After substitution from (2.12), we obtain this relation between pertubations of electric field:

$$\delta E_{\mathrm{y}} = \pm\mathrm{i}\delta E_{\mathrm{x}}, \qquad \delta E_{\mathrm{z}} = 0.$$

This means that x and y compounds oscillate with phase shift, because i is related to phase delay $\frac{\pi}{2}$. So wave is circularly polarized electromagnetic wave, the tip of perturbation vector of electric field moves along circle. Thus we have left-handed (L, top sign) and right-handed (R, bottom sign) polarized wave.

From equations also follows that perturbation of electric field is perpendicular to homogenous magnetic field: $\delta\mathbf{E} \perp \mathbf{B}_0$. So wave propagates in z-axis direction: $\delta E_x = A\exp(\mathrm{i}(kz - \omega t))$. Real parts of electric perturbations are equal to:

$$\delta\mathbf{E} = E_1 \begin{pmatrix} \cos(\mathbf{k}\cdot\mathbf{x} - \omega t) \\ \mp\sin(\mathbf{k}\cdot\mathbf{x} - \omega t) \\ 0 \end{pmatrix}. \tag{2.16}$$

We can also compute magnetic perturbation:

$$\delta\mathbf{B} = B_1 \begin{pmatrix} \mp\sin(\mathbf{k}\cdot\mathbf{x} - \omega t) \\ \cos(\mathbf{k}\cdot\mathbf{x} - \omega t) \\ 0 \end{pmatrix}.$$

So whole magnetic field of R wave is equal to (see (2.5)):

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \\ B_0 \end{pmatrix} + B_1 \begin{pmatrix} \sin(\mathbf{k}\cdot\mathbf{x} - \omega t) \\ \cos(\mathbf{k}\cdot\mathbf{x} - \omega t) \\ 0 \end{pmatrix}. \tag{2.17}$$

2.4 Radiation of Charged Particle

In this work, I study motion of electron trough both electric and magnetic fields. So spectral analysis of emitted electric field [5] has to be done. When charged particle changes its velocity, it emits electromagnetic waves. This radiation is classified by several criteria into following categories:

- for nonrelativistic velocities:
  - cyclotron radiation – moving charged particle changes the direction of its velocity, occurring for example in external magnetic field,
  - bremsstrahlung – charged particle changes the magnitude of its velocity, occurring for example when it collides into a barrier,
- synchrotron radiation – for ultrarelativistic velocities.

Because, as can be seen in Chapter 4, studied electron gains ultra relativistic velocities very quickly, synchrotron radiation is dominant. In areas, far removed from the source of radiation, electromagnetic flux emitted by moving charged particle can be expanded into a series of power basis $\frac{1}{r}$, where r is the distance from the source of radiation. For electric and magnetic fields $\mathbf{E}, \mathbf{B}$ depending on distance as $\frac{1}{r}$ corresponding Poynting vector wanes as $\frac{1}{r^2}$. So, when the total emitted power on distant sphere is computed, the part of fields **E**, **B** depending on a distance as $\frac{1}{r}$ only remain. To this part, which is then independent on distance from the source of radiation, I will refer as a "radiation power".

Total radiation power P emitted by moving electron is given by Lienard's formula:

$$P = \frac{e^2}{6\pi\varepsilon_0 c}\frac{\dot{\beta}^2 - (\beta \times \dot{\beta})^2}{(1-\beta^2)^3}. \tag{2.18}$$

The maximum of radiation power lies on frequency $0.29\omega_c$. [5]

2.4.1 Electric Dipole

In previous section, emitted power was described for general multipole. In first approximation, the emitted electric field of studied electron can be modeled as electric dipole. Total power I^{E1} radiated by dipole field [4] can be derived as:

$$I^{E1} = \frac{1}{6\pi\varepsilon_0 c^3}\ddot{\mathbf{p}}^2,$$

where **p** is electric dipole moment. Because electric dipole moment for one particle can be written as $\mathbf{p} = Q\mathbf{r}$, where **r** is position vector, total power radiated by one electron can be derived as:

$$I^{E1} = \frac{e^2}{6\pi\varepsilon_0 c^3}\ddot{\mathbf{r}}^2. \tag{2.19}$$

Radiating atom can be modeled as Hertz dipole (harmonic oscillator) [4]:

$$\mathbf{p} = \mathbf{p}_0 \sin(\omega_D t),$$

where $\mathbf{p}_0$ is basic electric dipole moment and ω_D is angular frequency of Hertz dipole. Hertz dipole rotates along axis z. Then spectral analysis of radiation can be done. When emitted energy is replenished by external force, total power radiated as a function of dipole angular frequency ω_D can be derived as:

$$I^{E1} = \frac{\omega_D^4 p_0^2}{6\pi\varepsilon_0 c^3}\sin^2(\omega_D t) \tag{2.20}$$

When putting together equations (2.19) and (2.20), a formula for spectral analysis, which I will use later, can be derived:

$$\frac{e^2}{6\pi\varepsilon_0 c^3}\ddot{\mathbf{r}}^2 = I^{EI} = \frac{\omega_D^4 p_0^2}{6\pi\varepsilon_0 c^3}\sin^2(\omega_D t). \tag{2.21}$$

Chapter 3

Derivation of Equation of Motion

3.1 Nonrelativistic Approach

In my previous work [10], I studied the behaviour of an electron accelerated by electric field. The electron was moving through plasma, so it was slowed down by Coulomb scattering, which was represented by Chandrasekhar function (A.2). For small velocities the electron was slowed down to certain equilibrium velocity. But for velocities high enough (greater than roughly $70\,\mathrm{ms}^{-1}$ [10]), the electron was accelerated. Because I used unrelativistic approach, the electron soon reached velocity greater than speed of light, which is in contradiction to physical laws.

At first, I would like to derive nonrelativistic equation of motion for an electron in plasma with electric field. This equation, which is special variation of Fokker-Planck equation, follows from kinetic theory.

3.1.1 Kinetic Theory

Let us have plasma close to thermodynamic equilibrium: target beam of particles of type β, into which monochromatic beam of particles of type α enters. The interaction of both beams is described by Coulomb scattering of chosen particles of incoming and target beam, respectively, with mass m_α and m_β, with charge Q_α and Q_β.

Considering that, probability density function of monochromatic beam α, whose particles have the same velocity and concentration, is given by Dirac distribution, and probability density function of target is given by Maxwell distribution, it holds that:

$$f_\alpha = n_\alpha \delta\left(\mathbf{v}_\alpha - \mathbf{v}(t)\right), \quad f_\beta = n_\beta \left(\frac{m_\beta}{2\pi k_B T_\beta}\right)^{\frac{3}{2}} \exp\left(-\frac{m_\beta v_\beta^2}{2k_B T_\beta}\right),$$

where T_β is a temperature of β beam, k_B is Boltzmann constant.

Initial equation of kinetic theory is Boltzmann equation [1]. In kinetic theory of

Coulomb scattering several assumptions are made. The most important assumption for this work is as follows. Interaction is considered only with particles in its Debye sphere (see Appendix 1), which chosen particle perceives as point particles. The potential in particle position is a superposition of potentials of particles within the Debye sphere. When this condition is met along with other conditions, Boltzmann equation becomes Fokker-Planck equation [1]:

$$\frac{\partial}{\partial t} f_\alpha + (\mathbf{v}_\alpha \cdot \nabla_\mathbf{x}) f_\alpha + \frac{1}{m_\alpha} (\mathbf{F}_\alpha \cdot \nabla_\mathbf{v}) f_\alpha = S_{\alpha\beta},$$

$$S_{\alpha\beta} = K_{\alpha\beta} \ln \Lambda_{\alpha\beta} \left[-\frac{m_\alpha + m_\beta}{m_\beta} \nabla_\mathbf{v} \cdot (f_\alpha \nabla_\mathbf{v} H_{\alpha\beta}) + \frac{1}{2} (\nabla_\mathbf{v} \nabla_\mathbf{v}) : (f_\alpha \nabla_\mathbf{v} \nabla_\mathbf{v} G_{\alpha\beta}) \right], \tag{3.1}$$

where $f_\alpha = f_\alpha(t, \mathbf{x}, \mathbf{v}_\alpha)$ is searched probability density function of α particles and

$$K_{\alpha\beta} \equiv 4\pi \left(\frac{Q_\alpha Q_\beta}{4\pi\varepsilon_0 m_\alpha} \right)^2, \qquad \ln \Lambda_{\alpha\beta} \equiv \ln \left(\frac{\lambda_\mathrm{D}}{b_0} \right) \tag{3.2}$$

and

$$H_{\alpha\beta}(\mathbf{v}_\alpha) \equiv \int \frac{1}{g} f_\beta \, \mathrm{d}^3 \, \mathbf{v}_\beta, \qquad G_{\alpha\beta}(\mathbf{v}_\alpha) \equiv \int g f_\beta \, \mathrm{d}^3 \, \mathbf{v}_\beta. \tag{3.3}$$

$\ln \Lambda_{\alpha\beta}$ is called Coulomb logarithm, $:$ is double scalar product, λ_D is Debye length (A.1) and b_0 is critical impact parameter of Coulomb scattering, $H_{\alpha\beta}$ and $G_{\alpha\beta}$ are Rosenbluth potentials. g is absolute value of relative velocity. Because monochromatic beam is equivalent to one particle, it does not diffuse and value of Rosenbluth potential $G_{\alpha\beta}$ is zero.

Under these terms, first moment of Fokker-Planck equation is as follows:

$$\frac{\partial \mathbf{v}(t)}{\partial t} - \frac{Q_\alpha \mathbf{E}}{m_\alpha} = -\frac{m_\alpha + m_\beta}{m_\beta} K_{\alpha\beta} \ln \Lambda_{\alpha\beta} 2 n_\beta v_{0\beta}^{-2} \, \psi \left(\frac{v}{v_{0\beta}} \right) \frac{\mathbf{v}}{v}, \tag{3.4}$$

where

$$\begin{aligned} v_{0\beta} &= \sqrt{\frac{2 k_\mathrm{B} T_\beta}{m_\beta}}, \\ C &= 2 n_\beta \frac{m_\alpha + m_\beta}{m_\beta} K_{\alpha\beta} \ln \Lambda_{\alpha\beta} \, v_{0\beta}^{-2}. \end{aligned} \tag{3.5}$$

This equation is nonrelativistic equation of motion for electron accelerated in plasma by homogenous electric field:

$$\ddot{\mathbf{x}} = \frac{Q\mathbf{E}}{m_\alpha} - C \cdot \psi \left(\frac{v}{v_{0\beta}} \right) \frac{\mathbf{v}}{v}. \tag{3.6}$$

For detailed kinetic theory, please see [1] or [10].

3.2 Relativistic Approach

In this work I use relativistic approach to electron acceleration in plasma. Because described kinetic theory is nonrelativistic, its relativistic extension has to be found. Scattering term of Fokker-Planck equation in special relativity is very difficult to find. As can be seen from kinetic theory, the origin of scattering term is in Coulomb scattering within Debye sphere. But for particles with high velocities, other particles briefly pass through its Debye sphere and so they influence its motion only slightly. Thus, for high velocities of electron, scattering is insignificant and can be omitted, which I have done in the first part of my computations. In second part of computations, I used nonrelativistic scattering term. This inconsistent approach can be made, because Chandrasekhar function tends to zero for large arguments (A.3). From this we can also conclude, that nonrelativistic scattering term also vanishes for relativistic velocities.

3.2.1 Equation of Motion

Let us have electron flying into homogenous isotropic plasma. There are external electric and magnetic homogenous fields (from cloud and Earth, respectively). Also, R wave enters into the studied area of plasma. It can arrive from various distant areas, moving along Earth's magnetic field lines. Equation of motion for considered electron is chosen as relativistic Lorentz equation for motion of charged particle:

$$m_0 \frac{\mathrm{d}}{\mathrm{d}\, t} \frac{\mathbf{v}}{\sqrt{1-\mathbf{v}^2/c^2}} = Q(\mathbf{E} + \mathbf{v} \times \mathbf{B}), \tag{3.7}$$

where magnetic field is given in (2.17), electric field is a sum of homogenous field in one direction (specified later) and electric field perturbation, see (2.16) (with bottom sign). Q, m_0 and $\mathbf{v}$ are charge, invariant mass, and velocity of given electron, respectively, c is the speed of light.

In a part of computations, scattering term was added to equation of motion. Equation of motion follows:

$$\frac{\mathrm{d}}{\mathrm{d}\, t} \frac{\mathbf{v}}{\sqrt{1-\mathbf{v}^2/c^2}} = \frac{Q(\mathbf{E} + \mathbf{v} \times \mathbf{B})}{m_0} - C \cdot \psi\left(\frac{v}{v_{0\beta}}\right)\frac{\mathbf{v}}{v}, \tag{3.8}$$

where ψ is Chandrasekhar function (A.2) (for other terms see kinetic theory section). The last term representing Coulomb scattering can be written in nonrelativistic form because Coulomb scattering is insignificant for relativistic velocities (see previous section).

3.3 Initial Values and Geophysical Parameters

Most interesting location for electron acceleration to take place is D-layer of ionosphere in height about 80 km (during the day it lies in heights between 60 km and

90 km) [15] [16]. At this height the air has temperature approximately 200 K [16]. In the ionosphere air molecules are ionized (in accordance with its name), the concentration of electrons is approximately $10^4\,\mathrm{cm}^{-3}$, the concentration of NO^+ ions is approximately $10^3\,\mathrm{cm}^{-3}$ and of O_2^+ ions approximately $5 \times 10^2\,\mathrm{cm}^{-3}$ [15]. Also, the concentration of neutral particles is larger than that of ions and electrons, so the ionosphere has properties of weakly ionized plasma, which is first reason for choice of ionosphere. The second reason is that the D-layer of ionosphere is relatively close to the Earth, so an electron has a short path to travel to storm clouds.

The crucial part of runaway breakdown is presence of relativistic electrons. So determining of the lowest velocity value to accelerate electron in given electric and magnetic fields is the focal point of my work. As can be seen from my previous work [10], the lowest value of velocity needed for electron acceleration grows for smaller electric fields. Also, as can be seen from the form of equation of motion (3.7), with smaller electric fields the electron will be less accelerated. Because electric field in the atmosphere does not reach large intensities, it is better to build the model of runaway breakdown on smaller electric fields. In accordance with this conclusion, every computation was done for electric field intensity $100\,\mathrm{Vm}^{-1}$ and for small initial velocities. When only homogenous electric field of Earth (with value $100\,\mathrm{Vm}^{-1}$) is taken into account, the lowest value of initial velocity to electron acceleration is about $70\,\mathrm{ms}^{-1}$ for electron-electron scattering [10].

Homogenous magnetic field appearing in the equation (3.7) is local approximation of Earth's magnetic field. Because magnetic field of dipole decreases with distance from dipole, Earth's magnetic field also decreases with the height. The value of homogenous magnetic field on the surface of Earth varies between $(22 - 66)\,\mu\mathrm{T}$ [17], on 45 degrees of latitude, magnetic field on Earth surface is about $50\,\mu\mathrm{T}$, so intensity of homogenous magnetic field can be chosen as $3 \times 10^{-5}\,\mathrm{T}$. For a point on the equator, magnetic field is horizontal, pointing northwards. When moving northwards, Earth's magnetic field lines point more and more downwards, until they point to Earth's centre on North Magnetic Pole. On southern hemisphere the magnetic field points upwards, until it points upwards in purely radial direction at South Magnetic Pole. [18]

The amplitude of electric field of R wave can be derived from its magnetic field, so only R wave's magnetic field amplitude has to be chosen. This amplitude varies depending on the type of R wave and also on the distance and direction from the point of creation of the considered R wave. So magnetic field of the R wave is a parameter of my system, and more of its values have to be chosen. Because I want to test whether homogenous magnetic field or R wave is dominant, I chose the amplitude of R wave's magnetic field greater than Earth's magnetic field, at first. So for most computations, I chose the amplitude of R wave's magnetic field as $B_1 = 10^{-4}\,\mathrm{T}$. This is greater than homogenous magnetic field. After that magnetic field amplitude was reduced by one order to $B_1 = 10^{-5}\,\mathrm{T}$. In this case Earth's magnetic field is greater, so it should have the greatest influence on the motion.

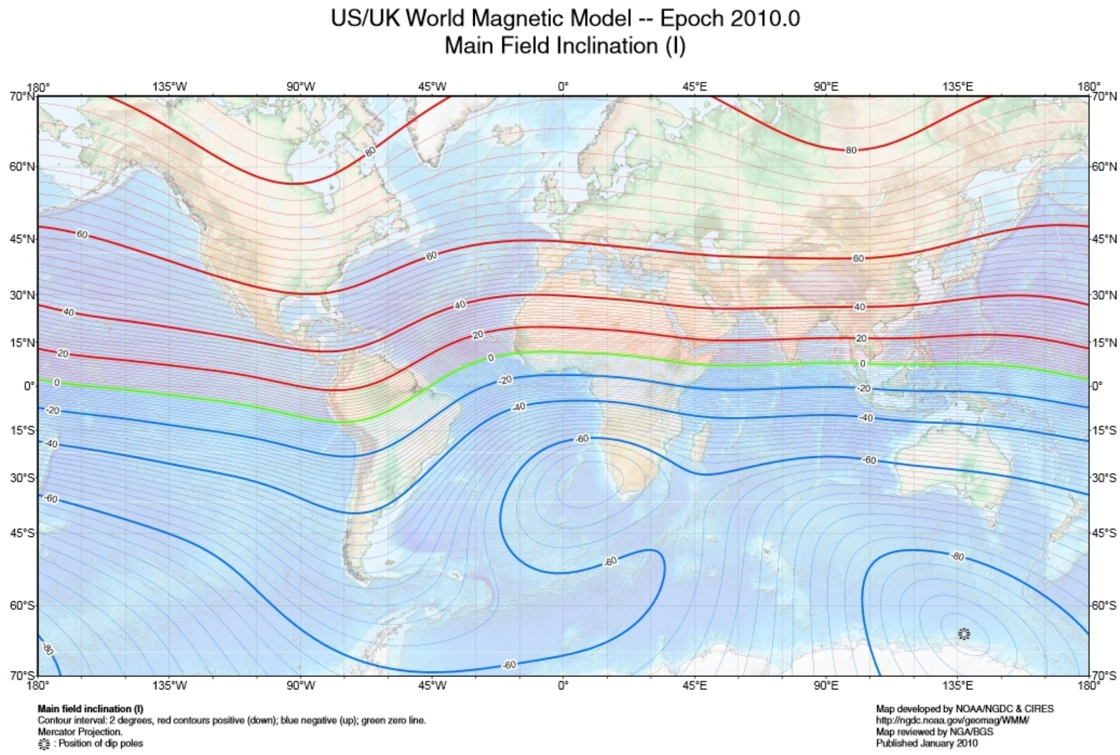

Figure 3.1: The inclination of Earth's main magnetic field. [18]

3.4 Coordinate System

Axes of coordinate system needed for computation are chosen in directions of particular fields. z axis is chosen by propagation of R wave. R wave travels along Earth's magnetic field lines. Magnetic field in every point on Earth is tangential to magnetic field line in that point. So homogenous magnetic field points also in z axis, which is in accordance with coordinate system in chapter 2 (2.7). As can be easily seen from description of Earth's magnetic field, for a point on the equator, R wave propagates horizontally in south-north direction. But when moving to greater latitudes, direction of R wave propagation rotates until Magnetic Poles, where R wave can propagate only in radial direction. So axis z of needed coordinate system, which is chosen in direction of R wave's propagation, is horizontal and pointing northwards only on the equator. On other latitudes z axis turns along with Earth's magnetic field.

Homogenous electric field determines the orientation of another axis. Because homogenous electric field is created between storm cloud and surface of Earth or the ionosphere, its direction is vertical, i. e. perpendicular to the surface. Because electron from cosmic rays comes from the space, it moves downward the surface of Earth. So electric field in every point can be chosen as vertical and pointing downwards. Because z axis rotates with the latitude, x axis can not be chosen in direction of homogenous electric field in Cartesian coordinate system. So x axis is chosen perpendicular to z axis in a plane defined by polar axis and z axis. On Magnetic Poles x axis is horizontal. So, in every point, homogenous electric field is in xz plane and the angle α between it and axis z varies from $0°$ and $180°$ in direction from north to south. Thus, homogenous electric field has components:

$$\mathbf{E}_0 = -E_0 \begin{pmatrix} \sin\alpha \\ 0 \\ \cos\alpha \end{pmatrix}. \tag{3.9}$$

Minus sign follows from opposite orientations of homogenous electric field and axis x. Homogenous electric field points upwards above clouds, because cloud upper layer has positive charge (see Fig. 1.2) and the centre of ionosphere layer can be modelled as plane with zero charge.

Along with (2.16), for whole electric field in the system, which is composed by homogenous electric field and electric field of R wave, holds following equation:

$$\mathbf{E} = -E_0 \begin{pmatrix} \sin\alpha \\ 0 \\ \cos\alpha \end{pmatrix} + E_1 \begin{pmatrix} \cos(\mathbf{k}\cdot\mathbf{x} - \omega t) \\ \sin(\mathbf{k}\cdot\mathbf{x} - \omega t) \\ 0 \end{pmatrix}. \tag{3.10}$$

Magnetic field in the system is given by equation (2.17).

The last, y coordinate to determine is perpendicular to both x and z axes creating right-handed Cartesian coordinate system. So y axis is horizontal and pointing west.

3.5 Equation of Motion Solutions Expected from Theory

On Magnetic Poles homogenous magnetic and electric fields are parallel, so there should be gyration. It means that the electron should move along helix, e. g. it should do Larmor rotation in xy plane and move uniformly in z direction. Also, because intensities of electric and magnetic fields change with respect to position and time, the centre and radius of Larmor rotation can change. Because homogenous electric field points from Earth, considered electron should also gyrate radially from Earth.

Contrary to this, on the equator, homogenous magnetic and electric fields are crossed, so studied electron should drift in perpendicular direction to both fields. It means that the electron should move along the trochoid, or along the cycloid, in case of zero initial velocity. The trochoid also can be modified by changing intensities of both homogenous fields.

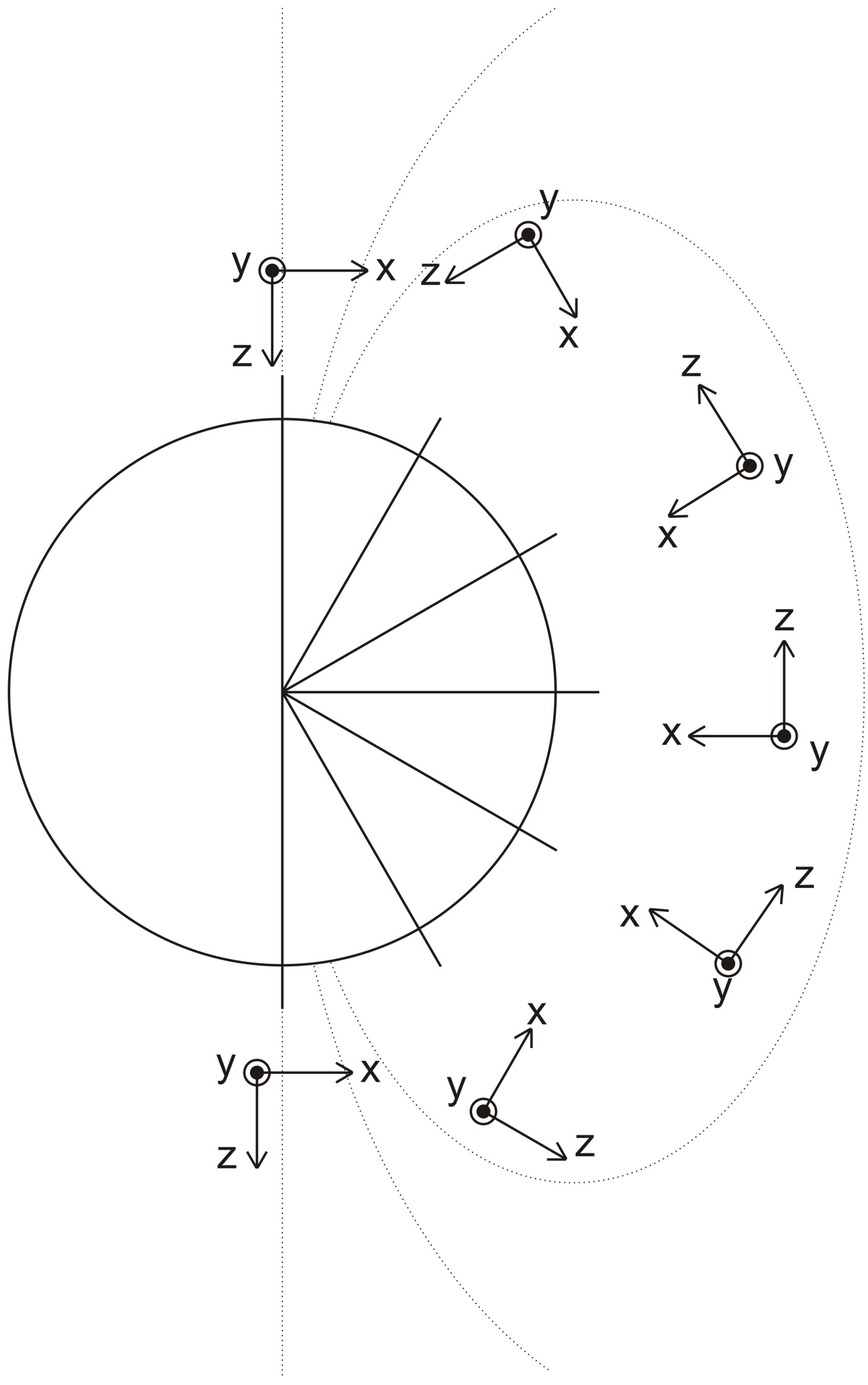

Figure 3.2: Coordinate system on different latitudes. y axis points upwards on Figure, to west on Earth.

Chapter 4

Computed Solutions of Equation of Motion

Equation of motion (3.7) was solved numerically by relativistic version of Boris-Buneman scheme (see Appendix B), written in Fortran 90 language.

According to my computations, the biggest part of acceleration takes place on the very beginning. It means, that chosen time interval for computations can be very small, for example as 10^{-5} s. For solutions of (3.7), there are two significant periodic processes: Larmor rotation with angular frequency ω_c and R wave. To follow both of them, time step has to be chosen as at least one tenth of smaller period.

4.1 Equation of Motion without Coulomb Scattering Term

4.1.1 Small Frequencies of R Wave

I began with computations for small frequencies of R wave, which lie in left, light strip on Fig. 2.1.

For small frequencies on all latitudes, stabilized state is established independently on initial velocity, when x and y velocity components oscillate more and more slowly, approximately about values $2.3 \times 10^8\,\mathrm{ms}^{-1}$ and $1.6 \times 10^8\,\mathrm{ms}^{-1}$, respectively (see Figures 4.1, 4.2 and 4.3). The component of velocity in axis z steadies about value approximately $1.2 \times 10^8\,\mathrm{ms}^{-1}$. The motion is held on a curved line, which straightens (see Fig. 4.5). The electron is relativistic, which is in accordance with expectations. The anisotropy in the axis z is also expected, because in this axis magnetic field acts, creating Larmor rotations on perpendicular plane, which now have very small radius.

For even smaller frequencies on all latitudes, stabilized state is also established independently on initial velocity, when x and y velocity components gain value approximately about 2×10^8 ms^{-1}. z velocity component oscillates approximately about value 1.2×10^7 ms^{-1}. The electron has relativistic velocity.

For small frequencies on all latitudes, kinetic energy grows, but after some time it oscillates with greater and greater difference between its maximum and minimum (see Fig. 4.4). The gain of kinetic energy is also faster with greater frequency of R wave.

When magnetic and electric field of R wave is smaller, electron begins to gyrate with growing Larmor radius on Magnetic Poles. At first, these gyrations are elliptical and wavy, but after some time they become circular. The initial behaviour can be explained by compounding two rotations with different centre and radius, one of these rotations becomes smaller and insignificant. Electron motion does not depend on initial velocity (exception being the very short time after the beginning).

When homogenous electric field is disabled, electron motion is affected only a little. The same conclusion holds for changing or disabling of homogenous magnetic field. The following conclusion can be made, that for small frequencies of R wave, dominant phenomenon influencing electron motion is the R wave, which accelerates electron to an stabilized state on relativistic velocities. And because there are no drifts and the electron is accelerated, it seems that electric field of the R wave is more dominant.

But for frequencies close to cyclotron frequency ω_c the situation is quite different. On Magnetic Poles homogenous magnetic and electric fields are parallel, so there should be gyration. In z axis, i. e. in direction of both fields, studied electron moves nearly uniformly in direction from Earth. But in xy axis two rotations are composed. On North Magnetic Pole two Larmor gyrations nearly cancel out creating curved line with few very eccentric ellipses. On South Magnetic Pole two rotations with same velocity in z axis are composed.

On the equator for frequencies close to cyclotron frequency ω_c, the electron drifts in crossed fields southwards from Earth. For zero initial velocity, the motion is held on cycloid. When considered electron has initial velocity, it moves on trochoid. Unlike classical drifts, the maximum velocity magnitude grows to relativistic velocities, which is caused by acceleration by both electric fields acting in xy plane.

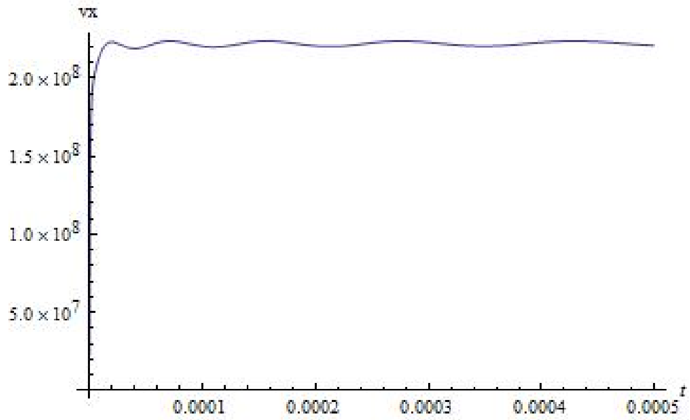

Figure 4.1: Dependence of x component of velocity $vx[\mathrm{ms}^{-1}]$ on time t[s] for $\omega = \omega_c/100$, $\mathbf{v}_0 = (0, 100, 0)\,\mathrm{ms}^{-1}$ and on the equator.

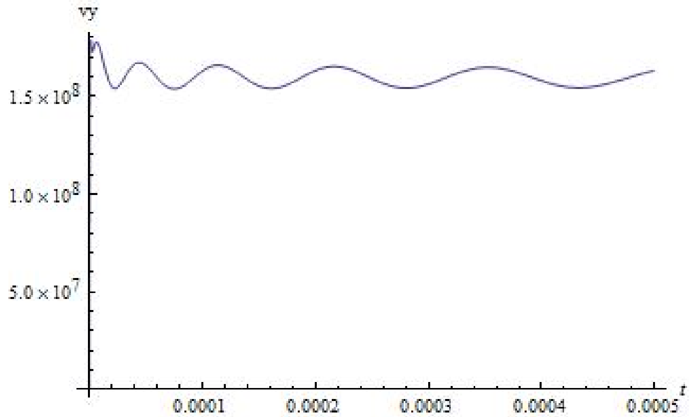

Figure 4.2: Dependence of y component of velocity $vy[\mathrm{ms}^{-1}]$ on time t[s] for $\omega = \omega_c/100$, $\mathbf{v}_0 = (0, 100, 0)\,\mathrm{ms}^{-1}$ and on the equator.

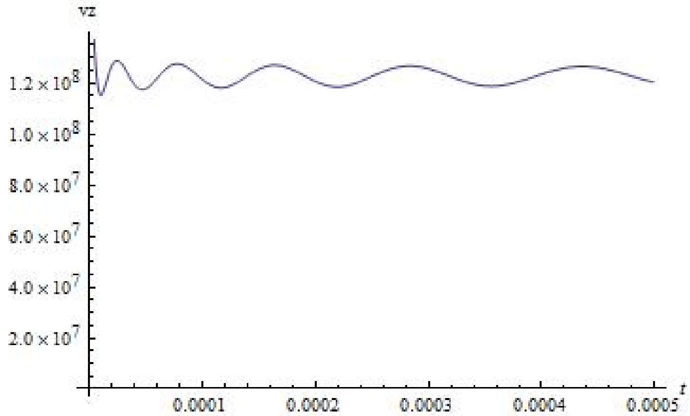

Figure 4.3: Dependence of z component of velocity $vz[\mathrm{ms}^{-1}]$ on time t[s] for $\omega = \omega_c/100$, $\mathbf{v}_0 = (0, 100, 0)\,\mathrm{ms}^{-1}$ and on the equator.

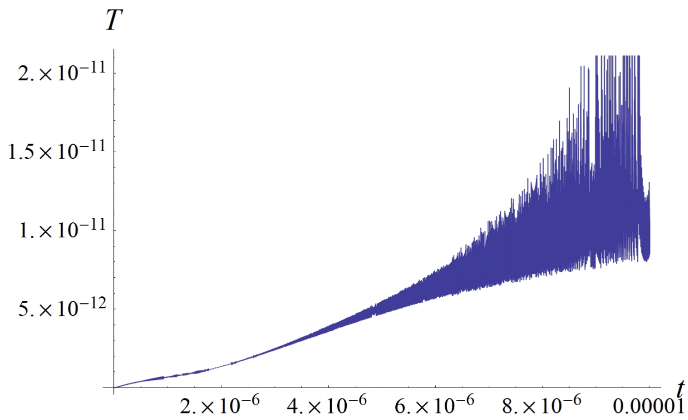

Figure 4.4: Dependence of kinetic energy T[J] on time t[s] for $\omega = \omega_c/100$, $\mathbf{v}_0 = (0, 100, 0)\,\mathrm{ms}^{-1}$ and on the equator.

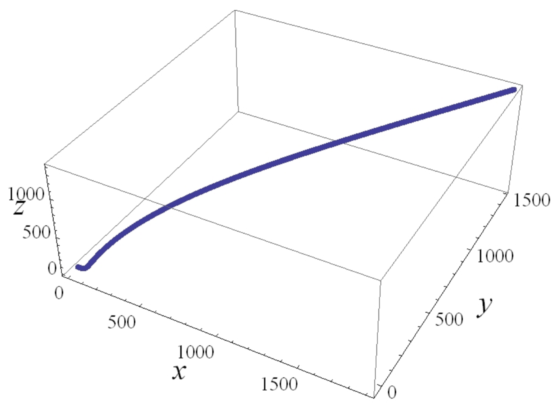

Figure 4.5: Electron configuration space for $\omega = \omega_c/100$, $\mathbf{v}_0 = (0, 100, 0)\,\mathrm{ms}^{-1}$ and on the equator. Units on axes are metres. Time interval is 10^{-5} s.

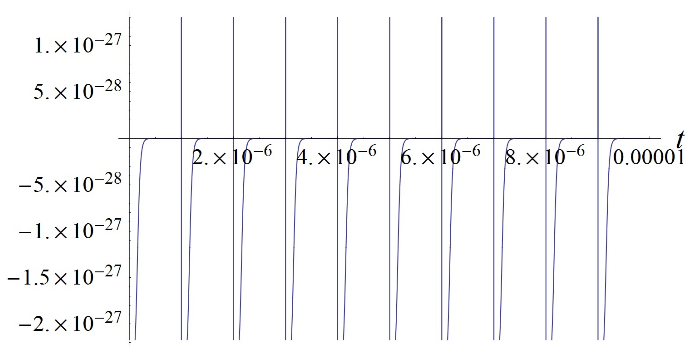

Figure 4.6: Dependence of dipole component of radiation power emitted by moving electron (y axis, in Wm^{-2}) on time t[s] for $\omega = \omega_c/100$, $\mathbf{v}_0 = (0, 0, 0)\,\mathrm{ms}^{-1}$ and on the equator.

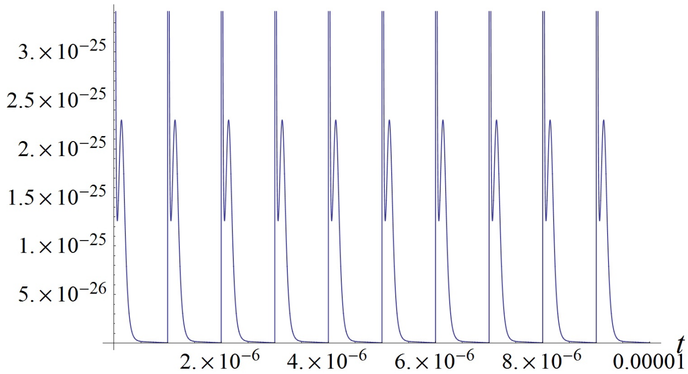

Figure 4.7: Dependence of radiation power emitted by moving electron (y axis, in Wm^{-2}) on time t[s] for $\omega = \omega_c/100$, $\mathbf{v}_0 = (0, 0, 0)\,\mathrm{ms}^{-1}$ and on the equator.

4.1.2 Large Frequencies of R Wave

I continued with computations for larger frequencies of R wave, which lie in right light strip on Fig. 2.1.

For large frequencies, the amplitude of electric and magnetic fields in R wave changes with so small period, that it only influences electron velocity. On electron position level, these changes in amplitude are centered. On Magnetic Poles the electron gyrates in magnetic field (see Fig. 4.10), so it moves along helix, radially from Earth. The period of Larmor rotation is increasing with respect to time. This increase cancels out with the decrease in the magnitude of velocity in xy plane, so Larmor radius remains constant. Electric field accelerates the electron in direction of R wave propagation. Absolute value of projection of velocity to the axis z such as kinetic energy (see Fig. 4.11) grows with respect to time, which is in accordance with the theory. Larmor radius is the same on both Magnetic Poles and is decreasing with growing frequencies of R wave. When the intensity of R wave is decreasing, Larmor radius of rotations is also decreasing.

On the equator, the electron drifts westwards in crossed fields (see Fig. 4.8). For zero initial velocity, the motion is held on cycloid. When considered electron has initial velocity, it moves on trochoid. Unlike in the situation with homogenous fields only, the plane, in which trochoid lies, precesses from xy plane to xz plane. This precession is greater with smaller electromagnetic field of R wave and larger R wave frequencies. Absolute value of velocity on the equator changes periodically from small to large values (in this section we have no friction), in case of $\omega = 100\omega_R$ maximum velocity is $10^7\,\mathrm{ms}^{-1}$. This maximum decreases with respect to increasing frequency of R wave. Kinetic energy in this situation oscillates between small values and $10^{-17}\,\mathrm{J}$ (see Fig. 4.9).

When homogenous electric field is disabled, the electron also gyrates as in the case on Magnetic Poles, but now with uniform motion in z axis. This means that the influences from homogenous electric field and from the R wave are similar in size. Next, when homogenous magnetic field is disabled, electron does not gyrate or drift. After some time the electron motion is almost uniform linear. However, velocity magnitude is the same as in case with all fields present. This means, that homogenous electric field changes the direction of velocity, not the absolute value of the velocity, which is in accordance with the theory.

Therefore, a conclusion can be made that, when parallel to Earth's magnetic field, homogenous electric field along with electric field of R wave accelerates studied electron to relativistic velocities, so that it becomes runaway. When perpendicular to Earth's magnetic field, both electric fields cause electron gyration and do not accelerate it. The presence of R wave causes precession of the plane, in which the electron gyrates. Also, Earth's magnetic field only curves electron's trajectory and does not change velocity magnitude.

For frequencies close to cut-off frequency, the situation is similar with respect to Earth on all latitudes. Considered electron gyrates in xy plane. Specifically, the motion in xy plane is composed of two Larmor rotations with different radii and centres, but with identical frequencies. Absolute value of velocity changes periodically. Kinetic energy also oscillates but its maxima and minima grow with respect to time. On Magnetic Poles, the minimum velocity is increasing to values close to the speed of light, as the electron becomes more relativistic. On the equator, the maximum velocity is approximately about $2.5 \times 10^8\ \mathrm{ms}^{-1}$. On both Magnetic Poles, studied electron gyrates radially from Earth. The difference between latitudes is in the motion in z axis. On both Magnetic Poles considered electron moves quadratically from Earth on z axis, while on the equator it moves linearly in northward direction. This difference is caused by the presence or absence of homogenous electric field in z axis.

When the Earth's magnetic field is disabled, electron's trajectory is similar to situation with all terms, but Larmor rotations are slower. This is in accordance with the theory, because total magnetic field is smaller and thus cyclotron frequency is also smaller. The situation when electromagnetic field is smaller is very similar to situation when homogenous electric field is disabled. In those cases the electron also gyrates, but the centre of gyration moves along helix. The difference lies in motion along the z axis. In case of disabled homogenous electric field, the motion is the same as in the situation with all terms.

So, following conclusion can be made, that in case of frequencies close to cut-off frequency ω_{R}, magnetic fields cause rotation of the considered electron. On Magnetic Poles, their influence is too small to prevent electron acceleration. Also, homogenous electric field captures the electron, so that it moves radially from Earth on Magnetic Poles and northwards on the equator.

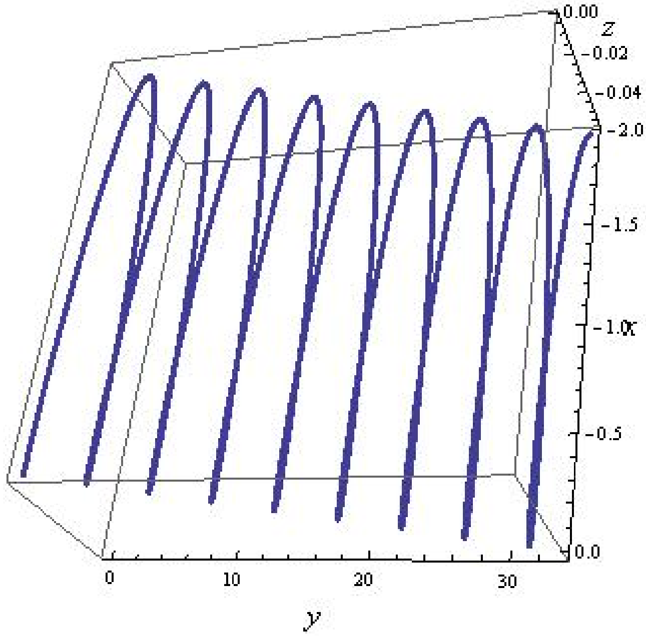

Figure 4.8: Electron configuration space for $\omega = 100\omega_R$, $\mathbf{v}_0 = (0, 0, 0)\,\mathrm{ms}^{-1}$ and on the equator. Units on axes are metres. Time interval is 10^{-5} s.

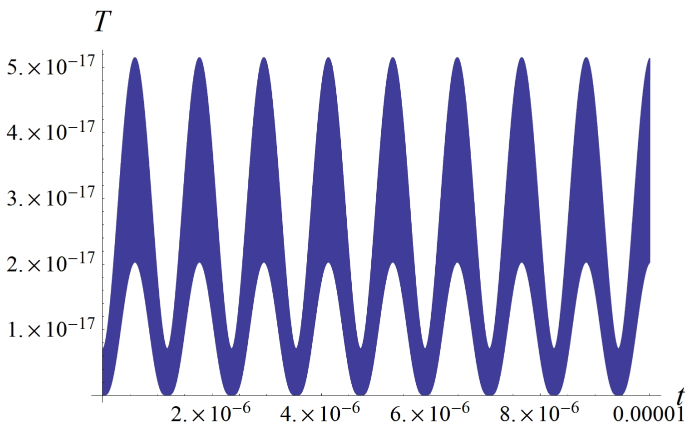

Figure 4.9: Dependence of kinetic energy T[J] on time t[s] for$\omega = 100\omega_R$, $\mathbf{v}_0 = (0, 0, 0)\,\mathrm{ms}^{-1}$ and on the equator.

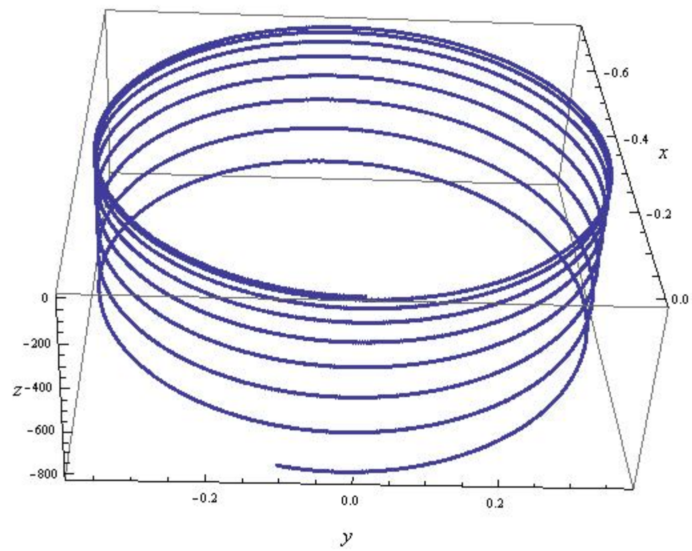

Figure 4.10: Electron configuration space for $\omega = 100\omega_{\mathrm{R}}$, $\mathbf{v}_0 = (0, 0, 0)\,\mathrm{ms}^{-1}$ and on North Magnetic Pole. Units on axes are metres. Time interval is 10^{-5} s.

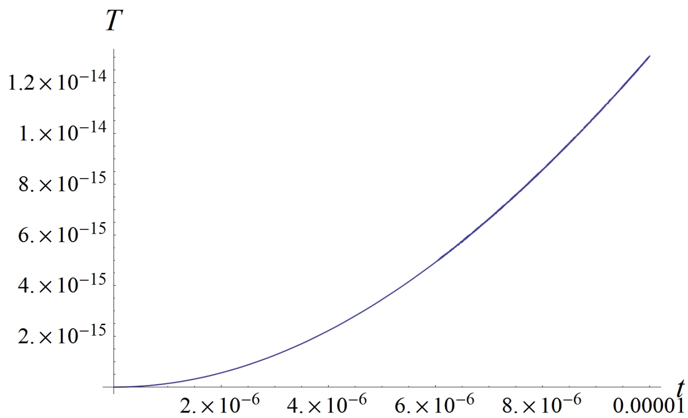

Figure 4.11: Dependence of kinetic energy T[J] on time t[s] for $\omega = 100\omega_{\mathrm{R}}$, $\mathbf{v}_0 = (0, 0, 0)\,\mathrm{ms}^{-1}$ and on North Magnetic Pole.

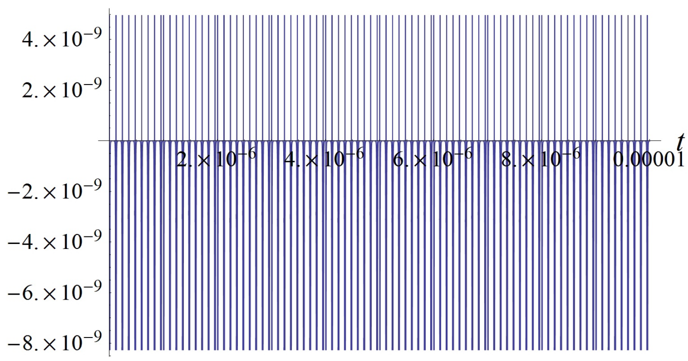

Figure 4.12: Dependence of radiation power emitted by moving electron (y axis, in Wm^{-2}) on time t[s] for $\omega = 100\omega_{\mathrm{R}}$, $\mathbf{v}_0 = (0, 0, 0)\,\mathrm{ms}^{-1}$ and on the equator.

4.2 Equation of Motion with Coulomb Scattering Term

I did the study of equation of motion with Coulomb scattering term for the case of electron-electron scattering. The results are presented in following paragraphs:

Coulomb scattering does not influence the course of motion. But for frequencies in right light strip in Fig. 2.1, the scattering affects motion's parameters (see Fig. 4.13 and 4.14). Larmor radius is approximately three times larger than in the case without the scattering. Also, angular frequency is slightly smaller and decreases with respect to time. This decrease cancels out with the decrease in magnitude of velocity in xy plane, so Larmor radius remains constant. In cases when the considered electron drifts, e. g. on the equator for large frequencies, Coulomb scattering causes greater precession of drift plane and also causes nonzero "initial" velocity in drift plane, so the motion holds on trochoid instead on cycloid.

The increase in the period of Larmor rotation can be caused by the Coulomb friction. Kinetic energy is slightly smaller than in the case without friction and so the absolute value of velocity. Because the velocity in axes x and y are smaller than in axis z, Chandrasekhar function acts more in these two axes. Thus, the influence of Coulomb scattering is greater in xy plane, which is in accordance with decrease in cyclotron frequency.

For frequencies close to cutt-off frequency, the motion with Chandrasekhar function is slower and with greater Larmor radius, and also z velocity component of the motion is composed of original and sinusoidal motion. From the aforementioned properties, it seems the presence of Coulomb scattering slightly decreases cyclotron frequency, which induces a hypothesis, that the scattering efficiently increases electron mass.

For small frequencies, Coulomb scattering has no visible effect. In these frequencies, there is no periodic movement and the electron soon reaches relativistic velocities. This means that it passes trough Debye spheres of plasma particles very quickly, which proves, that the Coulomb friction is insignificant to the motion.

4.3 Energies

Now, I will deal with spectral analysis using formulas (2.18) and (2.21). First observation is that considered electron does not radiate as dipole, as can be seen for example from the Fig. 4.7 and the Fig. 4.6. It means that some higher components of multipole expansion, such as quadrupole and octupole moment, have to be taken into account as well.

Radiation power emitted by moving electron for smaller frequencies of the R wave is very similar to the situation in the Fig. 4.7 and in the Fig. 4.6. When assuming Hertz dipole, its angular frequency is approximately about $\omega_{\mathrm{D}} = 2\pi \times 10^6\,\mathrm{s}^{-1}$, which lies in high radio frequencies. Although, it seems that for almost all situations the period of radiation is the same: $T_{\mathrm{R}} = 10^{-6}\,\mathrm{s}$, the radiation power is

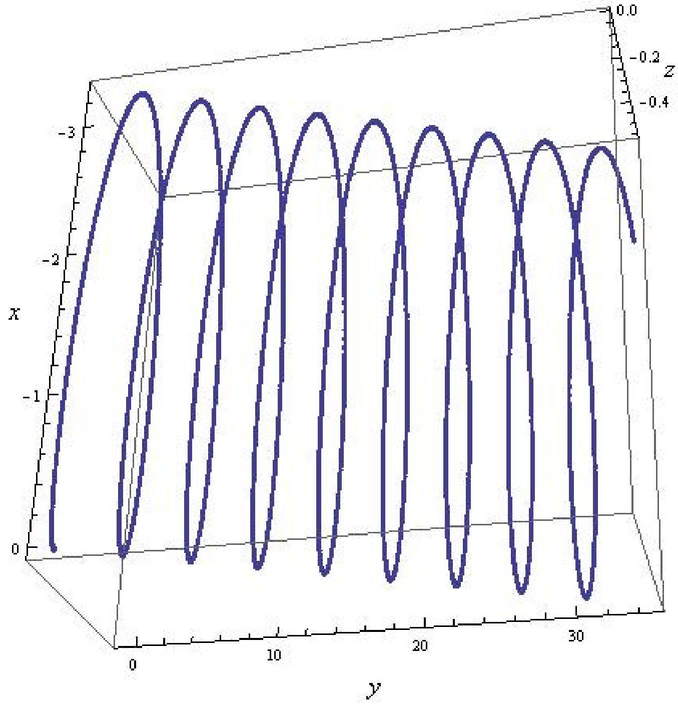

Figure 4.13: Electron configuration space for $\omega = 100\omega_{\mathrm{R}}$, $\mathbf{v}_0 = (0, 0, 0)\,\mathrm{ms}^{-1}$ and on the equator. Coulomb scattering is included. Units on axes are metres. Time interval is 10^{-5} s.

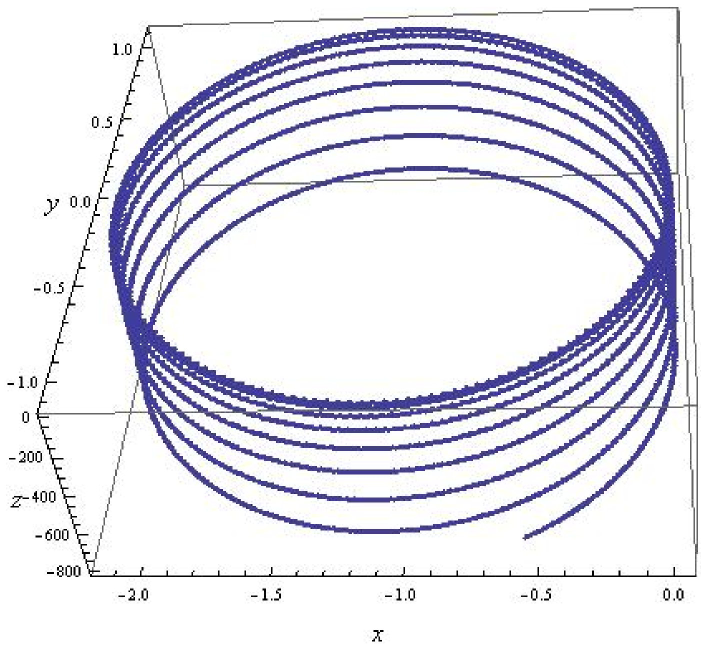

Figure 4.14: Electron configuration space for $\omega = 100\omega_{\mathrm{R}}$, $\mathbf{v}_0 = (0, 0, 0)\,\mathrm{ms}^{-1}$ and on North Magnetic Pole. Coulomb scattering is included. Units on axes are metres. Time interval is 10^{-5} s.

several order smaller that electron kinetic energy. Because studied electron gains kinetic energy approximately of order 10^{-14} J, this means, that radiation power of one particle is very small.

One of the exceptions from the behaviour described above is the behaviour for large frequencies and on the equator (see 4.12). In these cases, the difference between maximum and minimum dipole component of radiation power swiftly decreases with growing R wave frequency, so, for larger frequencies, the emitted intensity is almost constant on a value approximately fire orders smaller than kinetic energy. The radiation power seems to have two periodic processes. Slower one has an angular frequency approximately about $\omega_{\mathrm{D}} = 2\pi \times 10^{6}\,\mathrm{s}^{-1}$ again. Faster oscillations are not truly periodical, but in most instances their period is approximately about one ninth of the period of slower ones.

Conclusion

I established that the electron in ionosphere reacts not only to the homogenous electric field, created by storm cloud, but also to the presence of electromagnetic R wave. In most cases, the considered electron is accelerated to the speed of light. One exception from this rule is on the equator for large frequencies, lying in right light strip on Fig. 2.1 and for all considered intensities of R wave's magnetic and electric field. Maximum electron velocity decreases with frequency in the same manner as phase velocity of the R wave. These two velocities can only be in linear dependence, but they can not be identical, because phase velocity is greater than the speed of light. Also, according to my computations, mainly radio waves are created from the interaction between the considered electron and the electromagnetic R wave.

Also, I computed that Coulomb scattering has a visible effect on electron motion. This effect includes the increase in Larmor period and radius. This means that cyclotron frequency is slightly decreased due to the presence of Coulomb scattering, which induces a hypothesis, that the scattering efficiently increases electron mass.

In my future research efforts, I would like to explore two possible extensions of this work. First of them is a global approach. The motion of considered electron was solved locally, although considered electron travels long distances. This means that amplitudes of all electric and magnetic fields should change in every step of computations. The second extension is taking into account whole electromagnetic complex, not only the R wave.

Bibliography

[1] KULHÁNEK, Petr. *Úvod do teorie plazmatu.* First Edition, Praha: AGA, 2011, 375 s. ISBN: 978-80-904582-2-2. Available at: <`http://www.aldebaran.cz/studium/fpla.pdf`>.

[2] GUREVICH, A.V., G.M. MILIKH and R. ROUSSEL-DUPRE. *Runaway electron mechanism of air breakdown and preconditioning during a thunderstorm.* Physics Letters A. 1992, year 165, 5-6, p. 463-468. ISSN 03759601. DOI: 10.1016/0375-9601(92)90348-P. Available at: <`http://www.ees.lanl.gov/ees2/pdfs/Gurevich_RRD_1992.pdf`>.

[3] GUREVICH, Alexander V. and Kirill P. ZYBIN. *Runaway Breakdown and the Mysteries of Lightning.* Physics Today. 2005, year 58, issue 5, p. 37-43. ISSN 00319228. DOI: 10.1063/1.1995746. Available at: <`http://www.phy.olemiss.edu/~jgladden/phys510/spring06/Gurevich.pdf`>.

[4] KVASNICA, Jozef. Teorie elektromagnetického pole [Kvasnica, 1985]. First edition, Praha: Academia, 1985. 450 p.

[5] BŘEŇ, David. *Numerické simulace zářivých procesů v plazmatu.* Praha, 2004. Disertation thesis. Czech Technical University in Prague, Faculty of electrotechnics - department of physics.

[6] LUQUE, Alejandro and Ute EBERT. *Lightning above the clouds.* Europhysics News. 2010, vol. 41, issue 5, p. 19-22. DOI: 10.1051/epn/2010501. Available at: <`http://www.europhysicsnews.org/10.1051/epn/2010501`>.

[7] ALDEN, Andrew. *Sprites and Their Siblings.* About.com: Geology [online]. [cit. 2014-03-09]. Available at: <`http://geology.about.com/od/sprites/a/sprites.htm`>.

[8] KASEMIR, Heinz W. *A Contribution to the Electrostatic Theory of a Lightning Discharge.* Journal of Geophysical Research. 1960, year 65, n. 7, p. 1873-1878. ISSN 0148-0227. DOI: 10.1029/JZ065i007p01873. Available at: <`http://www.agu.org/pubs/crossref/1960/JZ065i007p01873.shtml`>.

[9] KULHÁNEK, Petr. *Blýskání, aneb, Třináctero vyprávění o plazmatu*. First Edition, Praha: AGA, 2011, 295 s. ISBN 978-809-0458-239.

[10] ZEMANOVÁ, Hana. *Urychlování elektronů v poli bouřkového oblaku*. Praha, 2012. Available at: <http://physics.fjfi.cvut.cz/publications/mf/2011/bac_thesis_zemanova.pdf>. Bachelor thesis. FNSPE CTU in Prague. Supervisor RNDr. David Břeň, Ph.D.

[11] HUGNES, James. *Velká obrazová všeobecná encyklopedie.* First edition, Praha: Svojtka and Co., 1999, 792 p. ISBN 80-723-7256-4.

[12] PATTERSON, Robert. *Lightning in Super Slow Motion.* In: [online]. [cit. 2012-06-25]. Available at: <http://www.youtube.com/watch?v=RLWIBrweSU8>.

[13] WARNER, Tom A. *LiveLeak.com - Lightning slowed down 300x.flv.* In: [online]. [cit. 2012-06-25]. Available at: <http://www.youtube.com/watch?v=rsokWqplLLE>.

[14] *Electrical charge distribution in a thunderstorm.* In: ENCYCLOPAEDIA BRITANNICA, Inc. Encyclopaedia Britannica: Facts matter [online]. 1999 [cit. 2012-06-30]. Available at: <http://media-1.web.britannica.com/eb-media/78/7578-004-7C794BAD.gif>.

[15] PAPON, Jeremie, Morris COHEN, Benjamin COTTS a Naoshin HAQUE. *Ionospheric Morphology.* In: DEPARTMENT OF ELECTRICAL ENGINEERING, Stanford University. *STAR Lab: Science, Telecommunications, and Radioscience Laboratory* [online]. [cit. 2012-07-02]. Available at: <http://nova.stanford.edu/~vlf/IHY_Test/Tutorials/TheIonosphere/-IonosphericMorphology.pdf>.

[16] Ionosphere: Tutorial. GNSS RESEARCH GROUP, Royal Observatory of Belgium. *GNSS@ROB - Ionosphere* [online]. 2010 [cit. 2012-07-03]. Available at: <http://gnss.be/ionosphere_tutorial.php>.

[17] NOAA/NGDC & CIRES. *Main Field Total Intensity (F).* In: NOAA National Geophysical Data center: The World Magnetic Model [online]. 2010 [cit. 2014-04-06]. Available at: <http://ngdc.noaa.gov/geomag/WMM/data/WMM2010/WMM2010_F_MERC.pdf>.

[18] NOAA/NGDC & CIRES. *Main Field Inclination (I).* In: NOAA National Geophysical Data center: The World Magnetic Model [online]. 2010 [cit. 2014-04-06]. Available at: <http://ngdc.noaa.gov/geomag/WMM/data/WMM2010/WMM2010_I_MERC.pdf>.

Appendices

Appendix A

Coulomb Interaction

A.1 Debye Length

Let us have a plasma consisting of multiple particle types. Particles have a mass m_α, a charge Q_α, where α denotes type of particle. Because a plasma close to the thermodynamic equilibrium is assumed, particles of opposite polarity will move closer to the given particle. They will shield its charge, so its potential will decrease exponentially. On the Debye length λ_D field and potential of given particle decreases to an 1/e of value given by Coulomb law. Therefore, there exist a sphere of radius λ_D centered at given particle named Debye sphere and only particles in it can be taken into account. It holds for Debye length:

$$\lambda_\mathrm{D} \equiv \sqrt{\frac{\varepsilon_0 k_\mathrm{B}}{\sum\limits_\alpha Q_\alpha^2 n_{\alpha 0}/T_\alpha}}\,, \tag{A.1}$$

where $n_{\alpha 0}$ are initial concentrations of particles of type α, T_α their temperature, k_B Boltzmann constant and ε_0 is permittivity of vacuum.

If a Z-ionized plasma is compounded only from electrons and ions with same temperature, then for Debye length simple equation can be written:

$$\lambda_\mathrm{D} = \sqrt{\frac{\varepsilon_0 k_\mathrm{B} T}{(1+Z) n_{\mathrm{e}0} e^2}}\,,$$

where $n_{\mathrm{e}0}$ is initial concentration of electrons, e is elementary charge, T is plasma temperature.

A.2 Chandrasekhar and Error Function

Chandrasekhar function ψ is defined by relation:

$$\psi(x) = \frac{2}{\sqrt{\pi} x^2} \int_0^x \xi^2 \mathrm{e}^{-\xi^2}\,\mathrm{d}\,\xi. \tag{A.2}$$

In limit of large and small arguments Chandrasekhar function has form:

$$x \gg 1 : \psi(x) \approx \frac{1}{2x^2}; \quad x \ll 1 : \psi(x) \approx \frac{2}{3\sqrt{\pi}} x. \tag{A.3}$$

Error function ϕ is defined by relation:

$$\phi(x) = \frac{2}{\sqrt{\pi}} \int_0^x \mathrm{e}^{-\xi^2}\, \mathrm{d}\,\xi. \tag{A.4}$$

In limit of large and small arguments error function has form:

$$x \gg 1 : \phi(x) \approx 1; \quad x \ll 1 : \phi(x) \approx \frac{2}{\sqrt{\pi}} x. \tag{A.5}$$

Chandrasekhar and error function are bound by simple relation:

$$\psi(x) = \frac{\phi(x) - x\phi'(x)}{2x^2}. \tag{A.6}$$

Both functions are drawn on picture A.1.

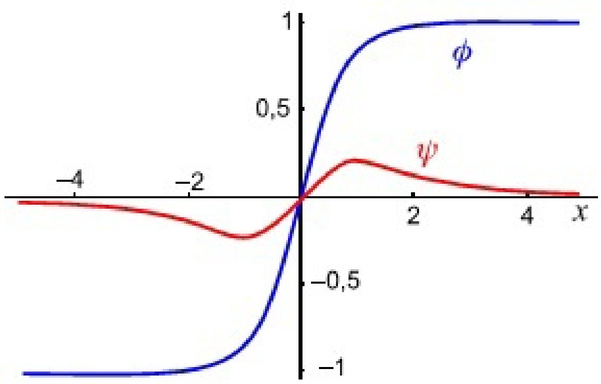

Figure A.1: Graphs of Chandrasekhar and error function [1].

Appendix B

Boris Buneman Scheme

Relativistic version of Boris-Buneman scheme was used[1]. This numerical method of the fourth order is well suited for motion of charged particles in both electrical and magnetic fields. Particle acceleration is performed in two parts. The rotation of particle due to magnetic field is executed between these two parts.

For the derivation of values in following time step it holds:

$$
\begin{aligned}
\gamma_n &= 1/\sqrt{1-\mathbf{v}_n^2/c^2}, \\
\widetilde{\mathbf{E}} = \frac{Q\Delta t}{2m_0}\mathbf{E}, &\qquad \widetilde{\mathbf{B}} = \frac{Q\Delta t}{2m_0\gamma_n}\mathbf{B}, \\
\mathbf{u}_n &= \gamma_n \mathbf{v}_n, \\
\widetilde{\mathbf{u}} &= \mathbf{u}_n + \widetilde{\mathbf{E}}, \\
\overline{\mathbf{u}} &= \widetilde{\mathbf{u}} + 2\frac{\left(\widetilde{\mathbf{u}} + \widetilde{\mathbf{u}} \times \widetilde{\mathbf{B}}\right) \times \widetilde{\mathbf{B}}}{1+\widetilde{\mathbf{B}}^2}, \\
\mathbf{u}_{n+1} &= \overline{\mathbf{u}} + \widetilde{\mathbf{E}}, \\
\mathbf{v}_{n+1} &= \frac{\mathbf{u}_{n+1}}{\sqrt{1+\mathbf{u}_{n+1}^2/c^2}}, \\
\mathbf{x}_{n+1} &= \mathbf{x}_n + \mathbf{v}_{n+1}\Delta t.
\end{aligned}
$$

Appendix C

Content of CD

In root folder, there is a file ”DT_Zemanova.pdf” with this diploma thesis. Also, there are two folders: ”computed_solutions” and ”programming”.

In folder ”programming”, there are source codes of my programs. I have two programs, one without Coulomb scattering, see folder ”basic”, and the second with Coulomb scattering, see ”Chandra”.

In folder ”computed_solutions”, there are computed graphs for all physically real situations. The folder ”all_terms” means the situation with all terms, the amplitude of magnetic field of R wave in this case is $B_1 = 10^{-4}$ T. The folder ”maleB” means the situation with all terms, the amplitude of magnetic field of R wave in this case is $B_1 = 10^{-5}$ T. Other folders are clear.

In most of subfolders, there is classification according to angular frequency of R wave:

- nizke – $\omega = \omega_c/100$,
- nizke_mezni – $\omega = \omega_c - 10$,
- velmi_nizke – $\omega = \omega_c/10000$,
- velmi_vysoke – $\omega = \omega_R \times 10000$,
- vysoke – $\omega = \omega_R \times 100$,
- vysoke_mezni – $\omega = \omega_R + 10$.

In these folders, sometimes division to three folders can be found:

- jih – means South Magnetic Pole,
- rovnik – means the equator,
- sever – means North Magnetic Pole.

In these folders, computed solutions can be found:

- intenzita – dipole omponent of radiation power,
- intenzita2 – radiation power,
- kineticka – kinetic energy of electron,
- norma – absoulte value of velocity of electron, bottom image is a beginning segment of top image,
- poloha – projections to axes of position of electron,
- poloha2 – configuration space of electron,
- rychlost – projections to axes of velocity of electron,
- rychlost2 – phase space of electron.

The folder ”delsi_cas”, which is sometimes present, means the computation with larger time interval.