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Research task

Accelerator beam physics and beam parameters
measurements.

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Prohlášení:

Prohlašuji, že jsem svůj výzkumný úkol vypracoval samostatně a použil jsem pouze podklady (literaturu, software, atd.) uvedené v příloženém seznamu.

V Praze dne

Title: **Accelerator beam physics and beam parameters measurements**

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Abstract: Particle accelerators are widely used in medical treatment, for industrial purposes, fundamental research and in other fields. The precise knowledge of the beam parameters is important for all mentioned fields but fundamental research requires the most precise measurements. Therefore the practical part of this research task is focused on analyzing and correcting the signal leakage in the bunch by bunch intensity measurements used in CERN on LHC and SPS. Simulations based on real data of LHC-INDIV were done for better understanding the nature of the signal leakage using ROOT framework. The correcting algorithm was created, tested by the simulations and implemented into the FPGA responsible for the data processing of the measurements. Additionally, a study of precision for a proposed single shot bunch by bunch intensity measurements of transfer lines was carried out for different ADC. As another update to the same system, a timing analysis algorithm was developed and implemented to alert operators on incorrect injection to LHC. The whole practical part was carried out in cooperation with the Intensity & Tune section of Beam Instrumentation group of Beams Department of The European Organization for Nuclear Research. In the theoretical part, the foundations of accelerator physics are laid down by introducing different accelerators and the basics of beam physics.

Key words: CERN, BE-BI, Betatron oscillations, Synchrotron oscillations

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Chapter 1

Theoretical part

1.1 Introduction to accelerators

Accelerators are machines increasing kinetic energy of charged particles or nuclei, like . protons, lead nuclei. The final energies can differ vastly in accordance with the usage of accelerated particles or nuclei. Also, there are many purposes for accelerating particle e.g. medical treatment, industrial usage or fundamental research [1].

Accelerator facilities designed for fundamental research do not contain only an accelerator and infrastructure but there is usually a chain of accelerators. This comes from limited energy spread of each accelerator. The chain begins with particle source which often contains the first small electrostatic accelerator in form of a voltage applied between cathode and anode [2]. The following chain segments depend on the facility, linear accelerators are often next accelerating step, circular as well as storage rings, or even some decelerators. The acceleration chain can be complicated and have multiple purposes. For illustration, a scheme of CERN accelerator chain is provided at 1.1.

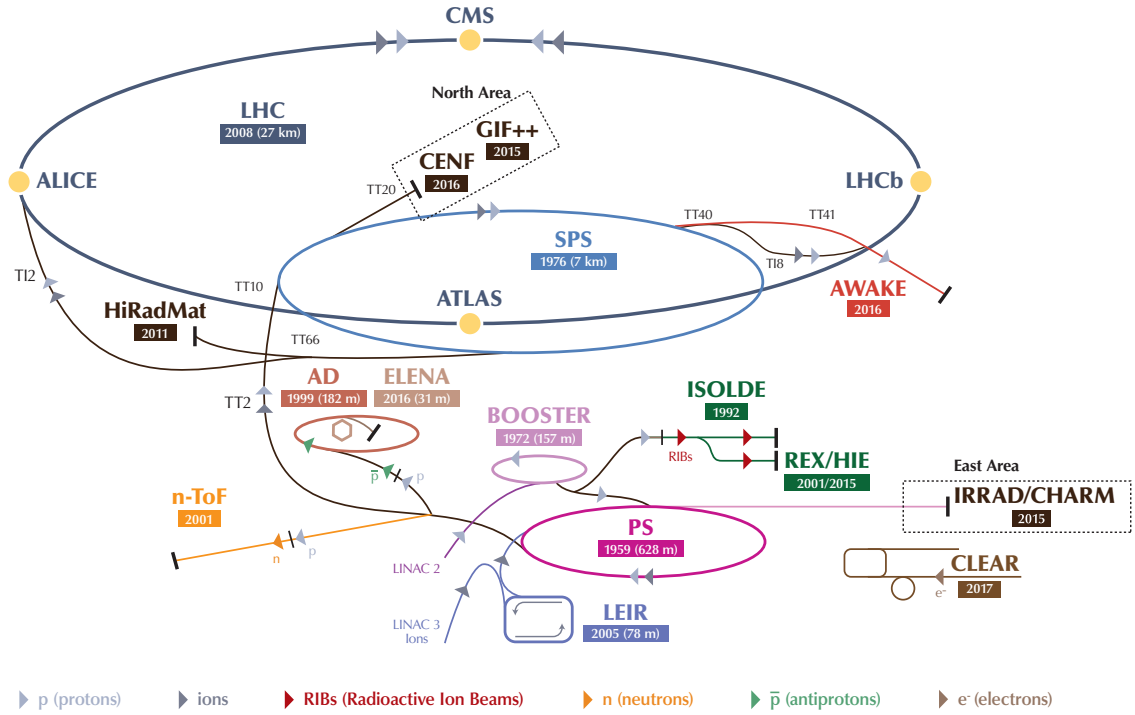
There are several ways of how to classify accelerators. In this text, the accelerators will be characterized as linear or circular and as electrostatic, radio frequency (RF) or induction [1].

1.1.1 Linear accelerators

Linear accelerators are characterized by accelerating the particle in one shot and therefore there is a necessity for a high accelerating gradient. This is achieved by either very high electrostatic differential, i.e. high voltage applied between the start and end of accelerating distance, or using RF field.

There is no need for bending dipole magnets, which makes them more compact in a comparison with circular accelerators. But in the context of achieving very high energies, the main advantage is the lack of synchrotron radiation, especially for light particles. Syn-

The CERN accelerator complex *Complexe des accélérateurs du CERN*



LHC - Large Hadron Collider // SPS - Super Proton Synchrotron // PS - Proton Synchrotron // AD - Antiproton Decelerator // CLEAR - CERN Linear Electron Accelerator for Research // AWAKE - Advanced WAKEfield Experiment // ISOLDE - Isotope Separator OnLine // REX/HIE - Radioactive EXperiment/High Intensity and Energy ISOLDE // LEIR - Low Energy Ion Ring // LINAC - LINear ACcelerator // n-ToF - Neutrons Time Of Flight // HiRadMat - High-Radiation to Materials // CHARM - Cern High energy AcceleraTOR Mixed field facility // IRRAD - proton IRRADiation facility // GIF++ - Gamma Irradiation Facility // CENF - CErn Neutrino platForm

Figure 1.1: Scheme of CERN acceleration chain. From [3].

chrotron radiation is the name given to the radiation which occurs when charged particle's path is being curved. This radiation stems from the fact that any charged particle under acceleration emits electromagnetic radiation, more about synchrotron radiation can be found in [4]. Linear accelerators in the acceleration complex are usually at the start of the accelerating chain because at the low energy domain they are very efficient.

The very high energies are hard to achieve with linear accelerator due to acceleration in one shot. Therefore if such energies are desirable the accelerators have to be substantially long. One of the longest linear accelerator was at SLAC National Accelerator Laboratory 3.2km and accelerating the electrons and positrons up to 50 GeV [5]. The linear accelerator of the European XFEL is the longest superconducting linear accelerator in the world accelerating electrons over a 1.7 km length achieving energy up to 17.5 GeV [6]. The International Linear Collider (ILC) is a project under consideration where two main LINACS of length of $\approx 15 - 25$ km each should accelerate electrons and positrons achieving collisions of 500 GeV [7].

Electrostatic

Electrostatic linear accelerators use applied high-voltage between cathode and anode for accelerating and differ in the source of the high-voltage.

Cockcroft-Walton accelerator was used for first man-made nuclear transmutation: $p + Li \rightarrow 2He$ in 1932. The source is called a Cockcroft-Walton high voltage generator which uses high voltage rectifier units. The maximum energy of this generator was limited to a few MV. The Cockcroft-Walton accelerator has been widely used as the first-stage ion-beam accelerator but now is replaced by more compact and reliable radio frequency quadrupole (RFQ) accelerators [1].

Van de Graff accelerator uses a high voltage source, where the charge is transported by an electrostatic belt and then stored. The whole construction of the generator is placed in a compressed gas for insulation. The maximum energy of the generator is limited by ≈ 10 MV. To this day mostly the tandem variation of Van de Graff is used [8].

The tandem accelerator uses highly negatively charged nuclei. The nuclei are accelerated to the cathode and then stripped off their electrons to make them positively charged. The positively charged, nuclei are accelerated furthermore by repelling away from the cathode. In this way the tandem accelerates the nuclei twice with the same voltage, the accelerating energy maximum is about 25 MV [1].

Radio-Frequency (RF) Accelerators

Radio-Frequency accelerators use RF field for accelerating. The most used example is LINAC. The scheme of this accelerator can be seen in the Figure 1.2.

It uses drift cavities which are charged by the RF field generator. Odd and even cavities have different charge polarity in a way that when a particle is exiting a drift cavity it is

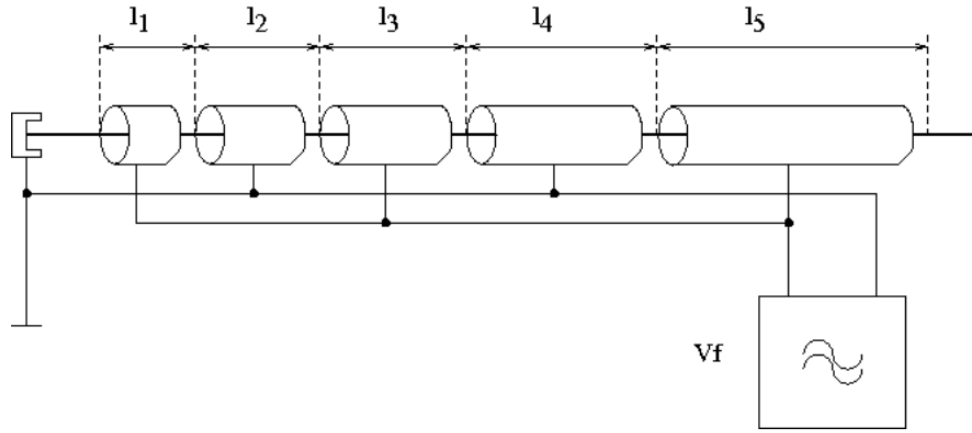


Figure 1.2: Scheme of a LINAC. From [8].

repulsed by the exited cavity and attracted by the next one. Charge polarity on every cavity changes when the particle is traveling through the cavity and is shielded by it. Therefore the particle is accelerated only between the cavities.

The particles are accelerated multiple times and therefore the maximum voltage difference can be lower in comparison to electrostatic accelerators. The accelerating distance is limited to only a portion of the LINAC length, because when the particles are traversing the cavity they are not accelerated. With the rising speed of the particles the cavities have to be longer if constant RF is to be maintained.

Overall the LINAC is widely used for medical, industrial purpose and also in the fundamental research usually as the early stages of the accelerating chain or as the main accelerator e.g. the electron accelerator at SLAC achieving 50 MeV [5].

1.1.2 Induction Accelerators

Induction accelerators use Faraday's law of induction, for accelerating the particles. The law states that when magnetic flux changes, it induces electrical field along the path that encompasses the magnetic flux [1]. The main idea is usually discharging some current source through a circuit to ramp up magnetic flux and the induced electrical field is used for beam acceleration.

The linear induction accelerator (LIA) was invented in the 50's and can be used to accelerate high-intensity short pulse beam, achieving the energy of a few tens of MeV [1].

A Betatron is a circular induction accelerator and was invented also in early 50's. It took a long time to understand the stability of transverse motion in the beam pipe. The

problem was solved in 1941 by using shaped magnet design so the magnetic field is radially modulated with a form [1]:

$$B_z = B_0 \left(\frac{R}{r} \right)^n, \quad n = -\frac{R}{B_0} \left(\frac{dB_z}{dr} \right), \quad (1.1)$$

where R is the ideal orbit radius, r is a radius with small deviation from R and n is a focusing index. The transverse motion is discussed in more detail in the section "Beam physics" 1.2. For now, the important thing is that the stable transverse motion is oscillatory and the stability criteria can be found at 1.7. The Stable motion means that the trajectory is enclosed and this enclosing of transverse motion is called focusing [9]. Because the betatron was the first accelerator that had to solve the problem of the transverse motion the motion is called betatron oscillation even though these oscillations occur in other circular accelerators as well [8]. The focusing described in this subsection is called a weak focusing, the other solution is a strong focusing invented later in 1952 and described in the section "Beam physics" 1.2 [1].

1.1.3 Circular Accelerators

The main characteristic of a circular accelerator is the circular trajectory and therefore the particles can be accelerated multiple times with one accelerating segment of the machine. This means that the particle gains some energy with each turn. With sufficient energy after a certain number of turns the particle is ejected.

The accelerating distance can become long after many turns and therefore low accelerating gradient can be used. The bending of the trajectory into a circle for energetic particles can be challenging. This usually generates one of the constraints on the maximum energy. The light particles suffer from increased synchrotron radiation loss as was mentioned in 1.1.1 [1].

Circular accelerators use magnets for bending the trajectory, also only a small portion of the trajectory is used for accelerating. Therefore they are often large and the strength of the magnets limits the maximum achievable energy as well.

Cyclotron

The concept of a cyclotron is quite old with first device build in 1932. The scheme is presented in the Figure 1.3. The idea of the cyclotron is to bend the particle trajectory by a uniform magnetic field perpendicular to the plane of particle trajectory. This magnetic field is achieved by a electromagnet with poles above and under the plane of particle trajectory. The accelerating is achieved by plates in the shape of the letter D, sometimes called dees. These plates act as electrode chambers and are powered by RF electric voltage generator, creating the accelerating gradient in the space between them [1].

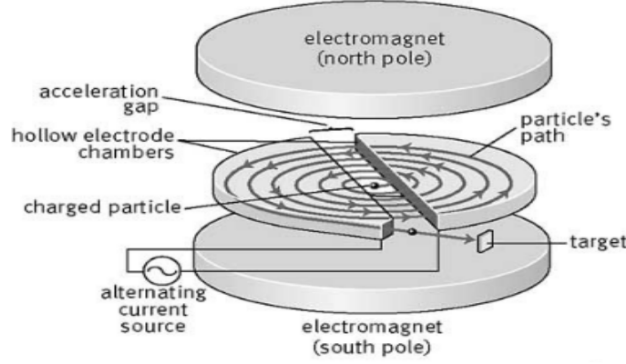


Figure 1.3: A scheme of a classical cyclotron [2].

An important characteristic of a cyclotron is that with the uniform magnetic field the angular velocity of particles is independent of their energy. Therefore the particles oscillate between the electrodes with a constant frequency known as cyclotron frequency. The cyclotron frequency can be written as [2]:

$$f_{cycl} = \frac{hZeB_0}{2\pi mc\gamma}, \quad (1.2)$$

where Z is an atomic number of the accelerated particle, e elementary charge, h Planck constant, c speed of light, m mass of the particle, B_0 the magnetic field and γ the Lorentz factor.

As can be seen in 1.2 the frequency is constant only for non-relativistic particles where $\gamma \rightarrow 1$. This is important because it places a constraint on the maximum energy of the accelerated particle [2].

This constraint is solved for similar accelerators. For example, Synchrocyclotron is using RF frequency modulation so the cyclotron frequency and RF frequency are synchronous. An isochronous cyclotron is using a shaped magnetic field. Microtron is using high accelerating gap so that with each energy gain the particle's frequency changes to an integral multiple of the RF frequency. The solution of Microtron is possible only for light particles, such electrons and is not possible for protons as the energy gain cannot be so high [2].

The protons can be accelerated with cyclotron up to ≈ 25 MeV but with, for example, CERN Synchrocyclotron built in 1957 up to 600 MeV [10]. The Microtron accelerates electron beam and achieves energies of few GeV [11].

Synchrotron

A synchrotron is the next evolution step after cyclotron with confined particle orbit into a closed loop while tuning RF system and magnetic field to synchronize particle revolution frequency [1].

Several advantages arise from the fact, that the particle's path is enclosed into the loop with a fixed radius. Firstly the vacuum chamber can be a torus instead of a disk, this allows more efficient vacuum system and only local and therefore smaller magnets can be used. This allows more cost-effective construction of large-scale accelerator.

Synchrotron can use weak-focusing for example, or strong focusing. The first weak-focusing proton synchrotron was built in 1952 achieving the energy of 3 GeV, but the strong focusing much higher energies can be achieved [1]. For example, CERN PS achieving 28 GeV or BNL AGS achieving 33 GeV built in 1959 and 1960 respectively.

The Future Circular Collider (FCC) is a project of a future accelerator designed to extend the research currently being conducted at the Large Hadron Collider (LHC). The FCC examines scenarios for collisions of hadrons, electron-positron and proton-electron. The emphasis of the project is on a 100 TeV proton collider to be housed in a 80 – 100 km new ring in the Geneva region [12].

1.1.4 Beam types

The beam can be continuous or bunched. The continuous beam consists of a steady stream of particles, the bunched beam consists of bunches of particles entrapped in potential traps. Whenever particles are accelerated by means of RF field the bunched beam is generated, while the continuous beam can be generated only by DC accelerating field. If the bunched beam is in a storage ring and the accelerating RF field is turned off debunched continuous beam is generated. This happens because the potential traps cease to exist and the synchrotron oscillations (described in more detail at 1.2.3) are no longer captured by the separatrix¹. The charge repulsion also plays a role in the debunching of the beam, overall the debunched beam has internal longitudinal oscillations [2]. In this text, the bunched beam will be discussed since it is an object of interest in the practical part.

Bunched beam usually contains sets of bunches with close spacing (e.g. 25ns for LHC type of a beam called LHC25NS) called trains. Between trains, there is a larger space in order to have enough time for activation of the injection, ejection or other utility magnets, which for LHC is in order of μs [13].

¹The separatrix is a boundary trajectory of a particle in the bunch created by the potential trap. Trajectories outside of the separatrix are not enclosed in the trap.

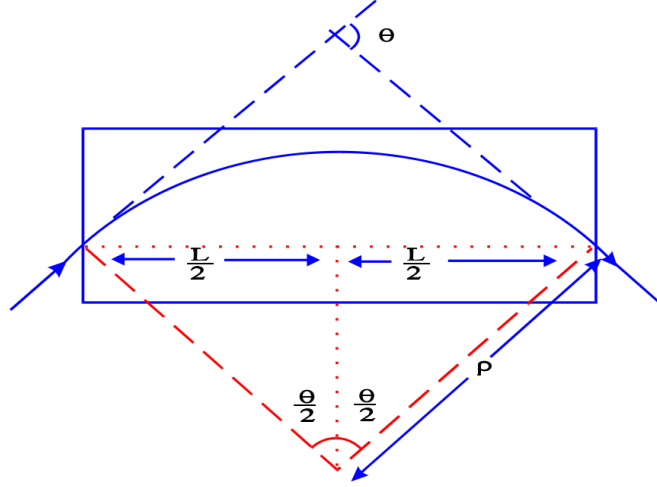


Figure 1.4: Effect of a uniform dipole field of length L and field B on a particle trajectory. The plane of the drawing is perpendicular to the magnetic field generated by the dipole magnet symbolized by the blue rectangle [9].

1.2 Beam physics

Every accelerator has an ideal trajectory which every particle should follow. This trajectory can be for many reasons curved and therefore the bending forces are necessary. As a result of slight deviations in the initial conditions and many other imperfections, most particles will follow a trajectory that is slightly different from the ideal trajectory. The differing trajectories need to be focused by focusing forces to keep the differences minimal [14].

Bending and focusing forces can be achieved by an electromagnetic field, but for high velocities ($v \approx c$) the magnetic field is much more efficient, for example, magnetic field of 1 Tesla corresponds to electric field of $3 \cdot 10^8$ V/m [14]. This is given by the Lorentz force [15]:

$$\mathbf{F} = e(\mathbf{E} + \mathbf{v} \times \mathbf{B}), \quad (1.3)$$

1.2.1 Types of magnets

As was mentioned, it is desirable to use a magnetic field for bending and focusing. In this subsection important types of magnets are described.

Dipole magnet

A dipole magnet creates a dipole field which is used as a bending force. The effect of the dipole field on a particle trajectory is shown in the Figure 1.4.

According to the 1.3 the applied force by the dipole field is perpendicular to the movement of the particle and the magnetic field. Therefore the trajectory of the particle cannot be altered in the direction of the magnetic field.

Quite often the dipole magnets are curved in a way that particles enter and leave the magnet at 90 degree to pole faces, these magnets are called sector magnets.

The trajectory is deviated by an angle θ which can be calculated as:

$$\sin\left(\frac{\theta}{2}\right) = \frac{LB}{B\rho}, \quad (1.4)$$

where L is the length of the magnet, B is the magnetic field and $B\rho$ is called magnetic rigidity which is dependent on the momentum of the particle.

Quadrupole magnet

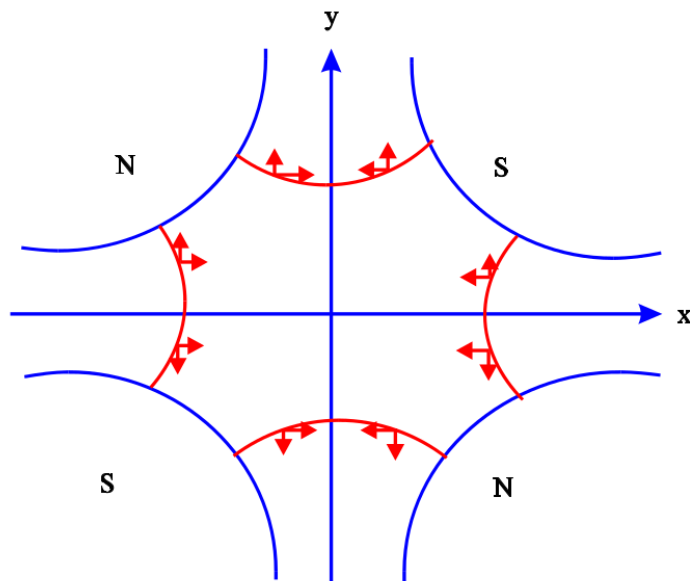


Figure 1.5: The force on the particle moving through a quadrupole. The x and y axis correspond to horizontal and vertical direction. The beam pipeline leads into the plane of the drawing [9].

A quadrupole magnet creates a quadrupole field which is used as a focusing force. The force of the quadrupole magnet on a moving particle is presented in the Figure 1.5 and can be described as:

$$F_x = evB_y(x, y) = -evgx, \quad F_y = -evB_x(x, y) = evgy \quad (1.5)$$

The direction and strength of the force changes with the position of the particle. The strength is rising with the distance from the center, i.e. the particle following the ideal path is not disturbed. The direction of the force follows from the north to south pole the equipotential lines which are hyperbolas $xy = const.$ [9].

Therefore the quadrupole magnet focuses the beam in one direction (e.g. horizontal) and defocuses in the other direction. The change of the focusing and defocussing direction is achieved by a rotation of the quadrupole magnet by 90° around the axis of the beam pipeline. It is also worth noting that the more particle is deviated the more it is focused or defocused.

1.2.2 Transverse motion and Betatron oscillation

The ideal trajectory is bent by the dipole magnets, for the purpose of this section let's assume circular ideal trajectory. Each particle has a slightly different initial condition and therefore there is a deviation between it's and the ideal trajectory.

Let's assume a small difference in an initial horizontal momentum. The trajectory of the deviated particle is still enclosed by the dipole magnets as shown in the Figures 1.6. The deviated particle oscillates around the ideal trajectory, as can be seen in the Figure 1.6 (right). This oscillation is a *Betatron oscillation* and forms the basis of all transverse motion in an accelerator [9].

The trajectory of a particle with different initial vertical momentum is not enclosed and is spiral. This is due to the fact, that the dipole field creates no force in the direction of the magnetic field as was mentioned at 1.2.1. The focusing forces are therefore a necessity.

Weak focusing

The weak focusing was mentioned at 1.1.2. This solution takes advantage of a radially shaped magnetic field in the form of 1.1. The resulting stable transverse motion is described by the formulas [2]:

$$\ddot{x} + \omega_x^2 x = 0, \quad \ddot{y} + \omega_y^2 y = 0, \quad \omega_x^2 = \frac{v}{R} \sqrt{1-n}, \quad \omega_y^2 = \frac{v}{R} \sqrt{n} \quad (1.6)$$

where x is the horizontal axis, y vertical axis, v the speed of a particle, R the ideal radius and n field index defined at 1.1. The equation 1.6 is the equation of harmonic oscillator if The Steenbeck's stability criterion is fulfilled which can be written as:

$$0 < n < 1, \quad (1.7)$$

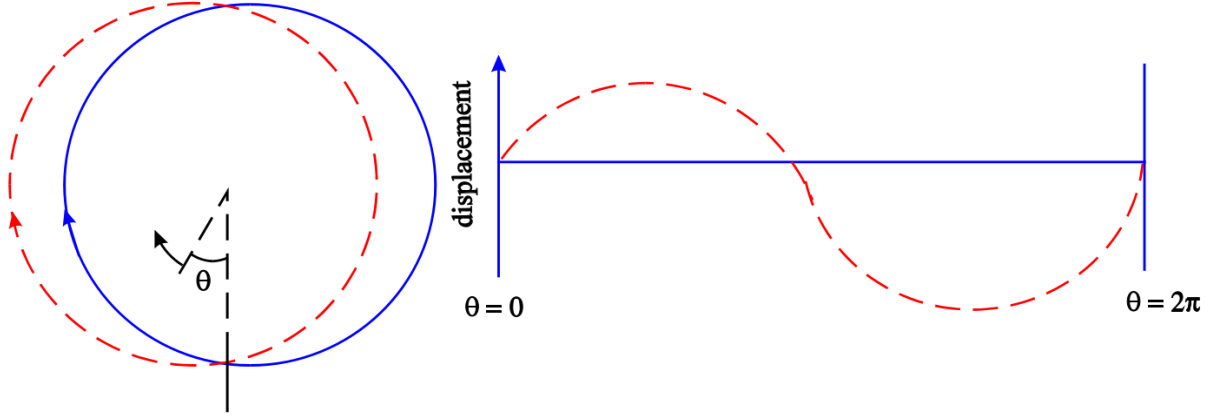


Figure 1.6: The trajectories of ideal (blue) and particle with different initial horizontal momentum (red) enclosed by the dipole field. Drawn in the Cartesian (left) and polar (right) coordinate system. The betatron oscillations of deviated particle around the ideal trajectory can be observed. The magnetic field points into the plane of the drawing [9].

This criteria creates a constraint on the shaping of the magnetic field in order to keep the transverse motion stable.

Strong focusing

The strong focusing stabilizes the motion with quadrupole magnets. The combined magnetic field of dipole and quadrupole magnets has a following form[2]:

$$B_x = -gy, \quad B_y = B_{y0} + gx, \quad (1.8)$$

where B_{y0} is the dipole field and g the gradient of the quadrupole field. The equations of motion in the linear approximation are therefore[2]:

$$x'' + (k_0 + k_{0x}^2)x = 0, \quad y'' + k_0 = 0, \quad (1.9)$$

where k_0 and k_{0x} are coefficients acquired from the dipole and quadrupole field respectively.

The transverse motion can be separated into two independent linear harmonic motions as is indicated by the equations 1.9. Therefore without losing generality only linear equation will be mentioned with a focusing strength defined as [2]:

$$K(z) = k_0(z) + k_{0x}(z), \quad (1.10)$$

where $k_0(z)$ and $k_{0x}(z)$ are dependent on the position in the accelerator as quadrupole and dipole magnets of different sizes and strength can be installed. This leads to the *Hill's equations* which are general equations for transverse motion in a synchrotron [9].

$$u'' + K_u(z)u = 0, \quad (1.11)$$

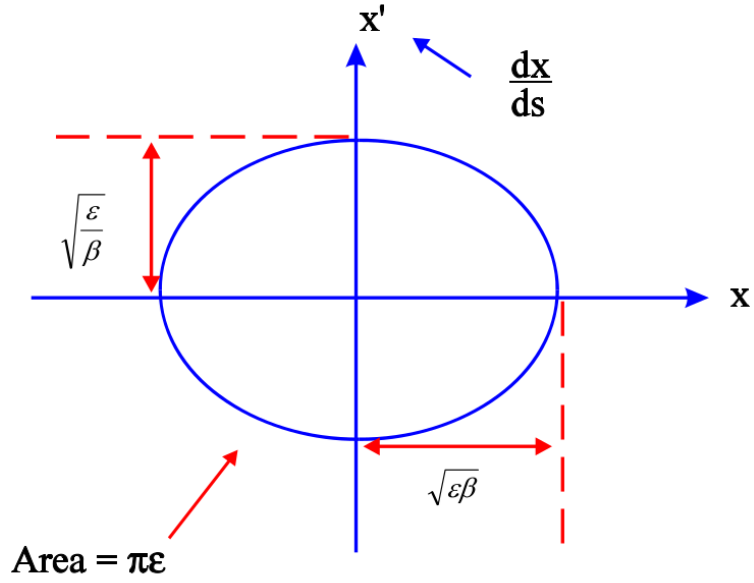


Figure 1.7: A transverse phase space plot of the solution of Hill equation at 1.12 [9].

where u can be either vertical or horizontal as was discussed earlier.

The solution and its derivative for slow varying amplitude and positive focusing strength have the following form [9]:

$$x = \sqrt{\epsilon\beta(z)} \cos(\Psi(z) + \phi), \quad \frac{d\Psi}{dz} \propto \frac{1}{\beta(z)}, \quad x' = \sqrt{\frac{\epsilon}{\beta(z)}} \sin(\Psi(z) + \phi), \quad (1.12)$$

where ϵ and ϕ are constants depending on the initial conditions. The $\beta(z)$ is beta function and characterizes the amplitude modulation due to the changing focusing strength. The $\Psi(z)$ is a phase advance, which also depends on focusing strength.

The ϵ is called the transverse *emittance* and is determined solely by initial conditions. The area of the transverse movement in the phase space is constant and given solely by the transverse emittance as can be seen in the Figure 1.7.

It is possible to define the transverse emittance of a beam with the area of the phase space containing 95%, 98%,... of the particles. Therefore it is important to know the definition of emittance used in a particular context [9].

The Liouville's theorem states that the density of particles in phase space does not change along a beam transport line, when the forces acting on it can be derived from macroscopic electric and magnetic fields [2]. In other words, the beam emittance and therefore the area of the phase space stays constant under the effect of dipoles, quadrupoles and other conservative forces. Therefore the emittance is an important indicator of the quality of a beam.

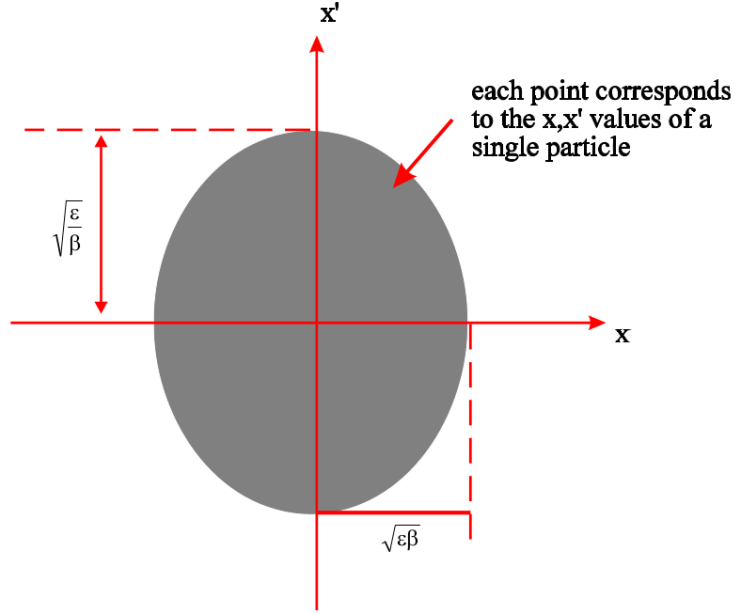


Figure 1.8: A transverse phase space plot of many particles of a beam [9].

The structure called FODO cell consists of two quadrupoles and two drift spaces as is shown in the Figure 1.9. The FODO cell is repeatedly used in the layout of the accelerators such as LHC and SPS for systematic focusing. The strength and positions of the quadrupoles and drift spaces in the FODO cell are designed for overall reducing the transverse size of the bunch.

1.2.3 Longitudinal motion, Transition and Synchrotron oscillation

Successful particle acceleration requires stable and predictable interaction of charged particles and electromagnetic fields. Because oscillating RF-field accelerates the particles special criteria must be fulfilled to ensure systematic particle interaction. For example, the phase of the field is adjusted to be the same at the time of every arrival of the particle. This phase is called the synchronous phase Ψ_s and the particle to which the phase is adjusted is called synchronous [2].

Transition

Every particle differs in the initial momentum, therefore, the time of flight from one accelerating section to the next is not the same for all particles. For higher momentum, the particle travels faster but follows a longer path. The higher momentum particles follow a longer path because with the higher momentum the particles become heavier as is stated

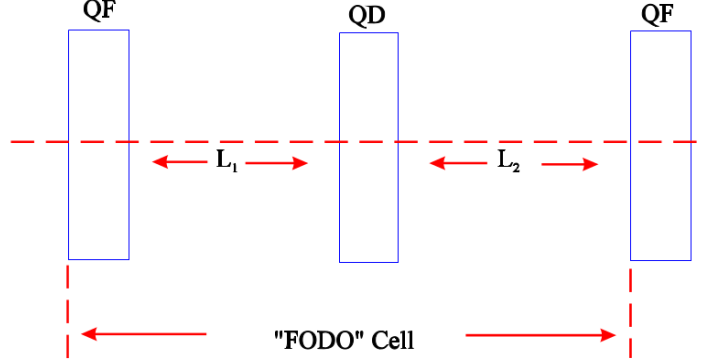


Figure 1.9: The FODO cell consisting of two quadrupoles QF and QD and two drift spaces $L1$ and $L2$. The quadrupole QF and QD are relatively rotated by 90° around the beampipe axis, therefore each one is focusing in the direction the other one is defocusing and vice versa [9].

by the relativity. Therefore in the same magnetic field more energetic particles will be bent less as stated in 1.4.

The change of the revolution frequency of a particle in a constant magnetic field can be described as [9]:

$$\frac{\Delta f}{f} = \frac{\Delta v}{v} - \frac{\Delta r}{r}, \quad (1.13)$$

where $\frac{\Delta v}{v}$ is a change of a velocity and $\frac{\Delta r}{r}$ is a change of a orbit length. This equation can be rewritten with the usage of a change of momentum $\frac{\Delta p}{p}$, Lorenz factor γ and constant α_p as follows [9]:

$$\frac{\Delta f}{f} = \left(\frac{1}{\gamma^2} - \alpha_p \right) \frac{\Delta p}{p} = \eta_c \frac{\Delta p}{p}, \quad (1.14)$$

where η_c is called momentum compaction. The α_p is fixed by the layout and strength of magnetic fields, but γ varies with the momentum [9].

The revolution frequency is therefore rising for momentum where $\gamma^{-2} > \alpha_p$ as is shown in the Figure 1.10 (area I). This is the case of low energy particles where an increase in momentum translates into a relatively high increase of velocity while the increase of the orbit length is negligible.

The revolution frequency is falling for momentum where $\gamma^{-2} < \alpha_p$ as is shown in the Figure 1.10 (area II). This is true for high energy particles where an increase in momentum translates into a relatively high increase of orbit length while the increase of the velocity is negligible. The *Transition* happens for particles when $\gamma^{-2} = \alpha_p$ and all such a particles have the same revolution frequency because $\frac{\Delta f}{f} = 0$. This is important especially for the proton machines as usually, it lies within the range of operating momenta of the machine. For example, the transition momentum is around 6 GeV/c in CERN PS [9].

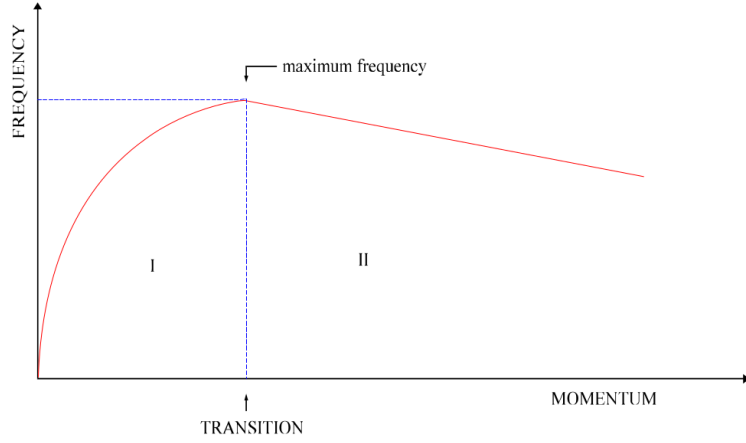


Figure 1.10: Revolution frequency vs particle momentum [9].

Synchrotron oscillation

For small oscillation amplitude and a sinusoidal waveform RF-field the equation of motion can be written as:

$$\ddot{\rho} + \Omega^2 \rho = 0, \quad \Omega^2 = \frac{h2\pi\eta_c}{L_0 p_0 T_0} eV_0 \cos \psi_s \quad (1.15)$$

where $\rho = \psi_s - \psi$ is relative phase, Ω the frequency of the oscillations, $h = \frac{f_{RF}}{f_{rev}}$ is a harmonic number, L_0 the distance between accelerating sections, p_0 the momentum and T_0 the time of flight of synchronous particle, V_0 the amplitude of RF-field and η_c the momentum compaction [2].

The stable solution of the equation of motion 1.15 (where $\Omega^2 > 0$) is oscillatory and achieved by setting correctly the synchronous phase. These oscillations are called *Synchrotron oscillations*. In the Figure 1.11 are presented stable and unstable solutions of the equation 1.15 [2].

The principle of the stable longitudinal movement is that the particle with higher energy than the synchronous particle sees lower accelerating voltage than the synchronous particle and vice versa. Therefore the particles with a different energy than the synchronous particle are oscillating around the synchronous particle.

The Figure 1.12 shows an example of the energy variation of two particles where A is a synchronous particle with lower initial momentum and B is a particle with higher initial momentum, both momenta are above the transition momentum. The Figure 1.13 refers to the same example showing the phase space diagram of the synchrotron oscillations.

The correct setting of synchronous phase depends on the sign of the momentum compaction η_c as can be seen in the equation 1.15. Therefore when a transition energy lies within the energetic range of a circular accelerator the synchronous phase has to change sharply in the acceleration process [2].

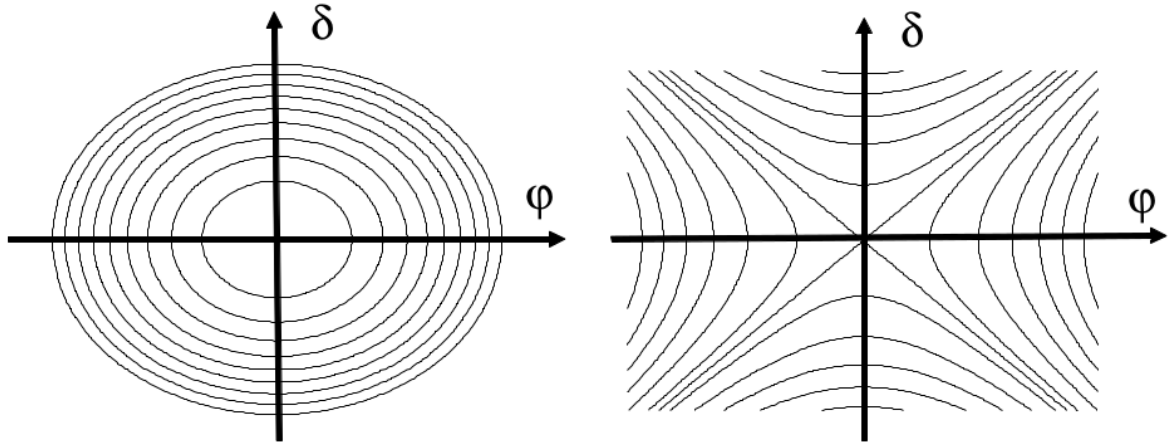


Figure 1.11: The stable solution ($\Omega^2 > 0$) (left) and the unstable solution ($\Omega^2 < 0$) (right) of 1.15 in phase space [2].

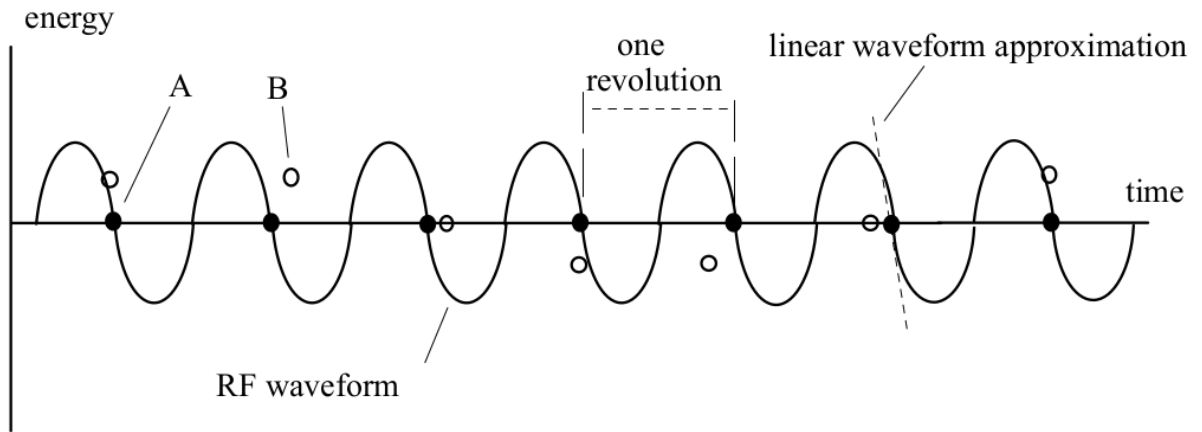


Figure 1.12: Energy variation of synchronous (A) and non-synchronous (B) particle over several turns. Particles above the center line have a higher energy than those below. The Accelerating RF voltage is above the center line and deaccelerating below. [9].

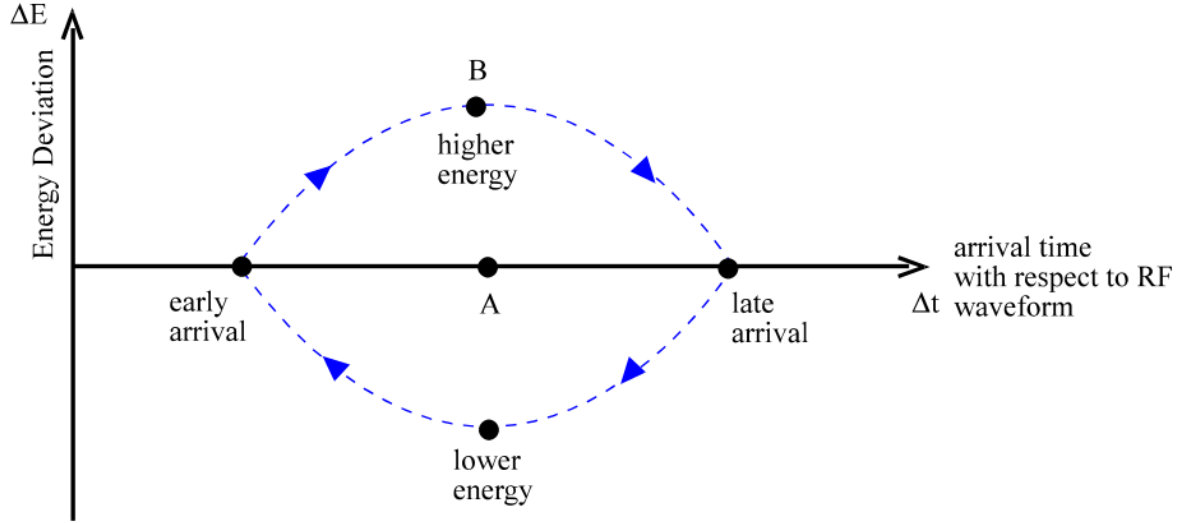


Figure 1.13: The Longitudinal phase space plot for particles A and B from Figure 1.12 showing the synchrotron oscillation. [9].

The equation 1.15 is limited to only a small amplitude of oscillation. The stable motion is also limited by the energy deviation of other particles, a more general solution is presented in the Figure 1.14. The particles are therefore trapped inside the well if the initial conditions are sufficiently good. These potential traps are called buckets and their size and shape is dependent on the synchronous phase. The longitudinal emittance can be defined with the area in the longitudinal phase space which is similar to the definition of transverse emittance[9].

1.3 Bunch by bunch intensity measurements

The LHC accelerates the beam with 400 MHz frequency cavities, creating $\approx 2.5\text{ns}$ wide equidistant buckets. One in ten bucket is filled with a bunch, resulting in the bunch frequency of 40 MHz.

Per bunch intensity measurements use a signal induced in a transformer situated around the beam pipe to give information about charge content of each bunch. The signal is shaped and digitalized for maximum precision, the digitalization sampling is running at 650 MHz which corresponds to 1.54 ns width samples. The charge content is given by digital integration of the signal above baseline. There are 16 samples available per each bunch, which are used by the integrating algorithm. The FPGA chips are used for data processing. The measurements use free-running clock on ADC sampling of the analog

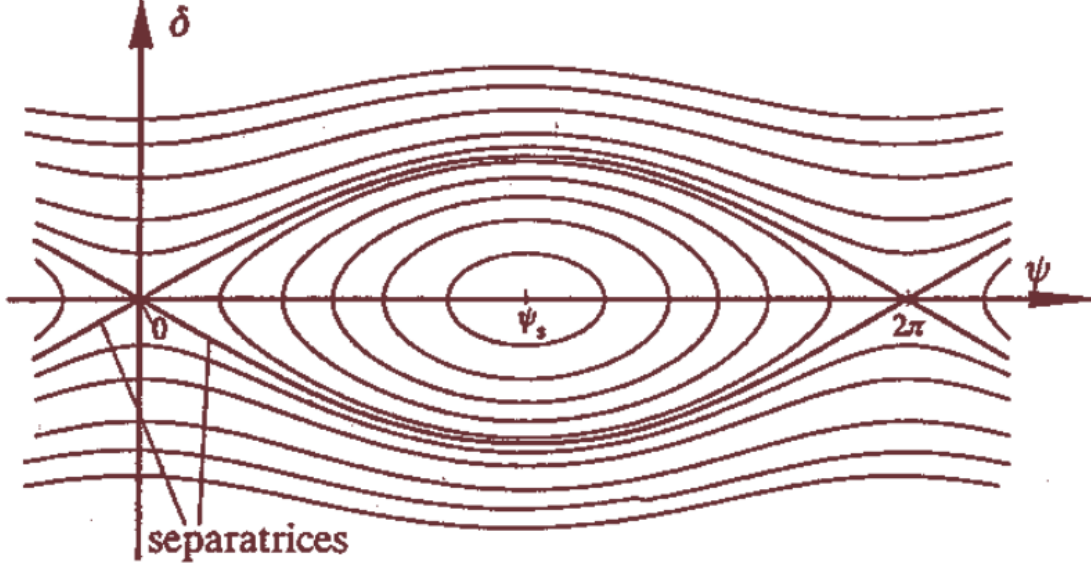


Figure 1.14: The solution of the equation of motion showing the bucket structure created by the RF-field. [2].

signal, therefore the integration is averaged over many turns leading to independence from the sampling phase.

1.3.1 Integration algorithm

The integration algorithm firstly identifies a maximum of each bunch which is defined as two rising and one non-rising sample while the difference between the last rising sample and the fifth sample before is above a threshold. The baseline is estimated as an average of boundary samples which are the 5th sample before and 10th after the maximum, the baseline is therefore linearly approximated. The integration is a sum of a difference between all samples and the interpolated baseline. Therefore the algorithm can be written as:

$$I = \sum_{i=max-5}^{max+10} (c_i - bs), \quad bs = \frac{c_{max-5} + c_{max+10}}{2}, \quad (1.16)$$

where I is the final integral, max is a number of the maximum sample, c_i is a content of sample number i and bs is the baseline. The Figure 1.15 shows the algorithm performance on real LHC data. Data from an LHC Individual beam type were used for this figure data fit is shown as well. The mentioned fit is created by the ROOT framework in later analysis.

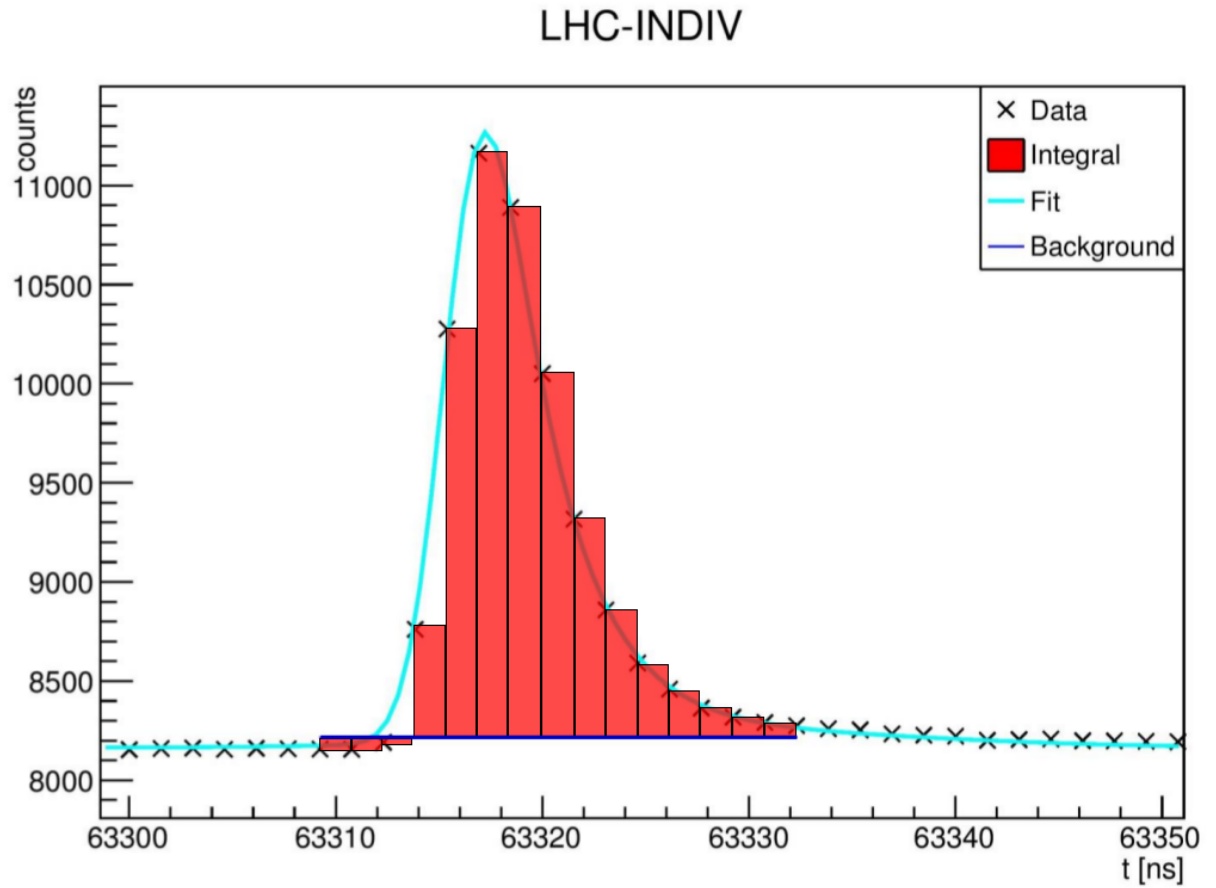


Figure 1.15: An integration algorithm used on real LHC data. baseline (dark blue) is average of first and last used sample. Fit found by ROOT framework is shown.

Chapter 2

Practical part

2.1 Simulation

Simulated data was created by the ROOT framework using a fit of real data of LHC-INDIV beam. An example of the real data and fit can be seen in the Figure 1.15. The fit has the following form:

$$f(x) = A + B \cdot \frac{C}{2} \exp\left(\frac{C}{2} \cdot (2D + C \cdot E^2 - 2x)\right) \cdot \text{Erfc}\left(\frac{D + C \cdot E^2 - x}{\sqrt{2} \cdot E}\right) + F \cdot \exp\left(-\frac{(x - G)^2}{2H^2}\right), \quad (2.1)$$

where A, B, C, D, E, F, G, H are fit parameters with a bit complex interpretation, and *Erfc* is the complementary error function. The whole formula can be boiled down to the baseline, represented by the parameter A , the exponentially modified Gaussian peak (more in [16]) and gaussian peak constrained to be small in the amplitude and placed in the tail area to improve the fitting results.

In total 8 different signals were fitted, the comparison is presented in the Figure 2.1. The fits of signals differ almost exclusively in the area of tails. The analyzed problem is heavily dependent on the tail shapes as is explained in the following section, this difference has an impact on the simulated observables and results. Therefore these fits of multiple signals are used to generate multiple sets of simulated data. These sets of simulated data are treated separately, but all the results are used for tuning the correcting algorithm.

To avoid an overwhelming amount of figures while keeping them simple and readable results of only one or two datasets will be shown in this chapter. The shown results will be a representation of overall results, meaning that results from other fits have similar behavior. All figures can be seen in the appendix ??

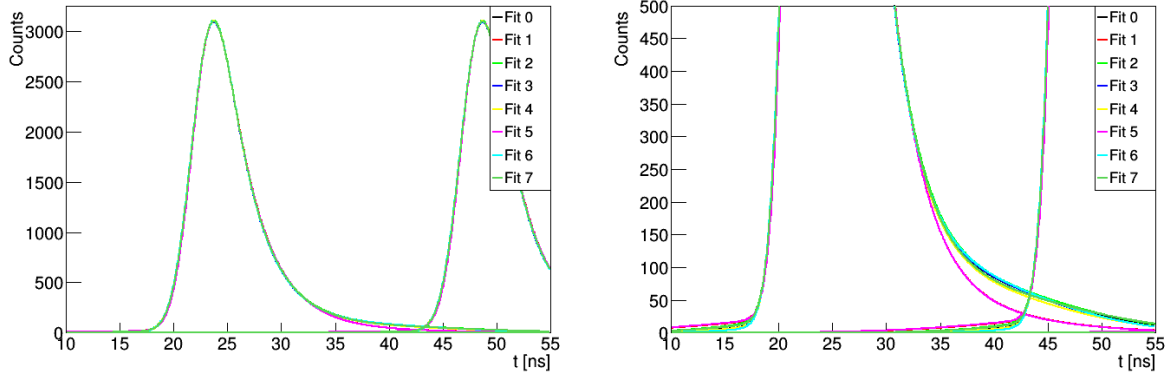


Figure 2.1: Both figures represent fit comparison of 6 different signals, the right figure shows magnified tail region where the differences are important.

2.2 Problem description

The Signal of each bunch from bunch by bunch intensity measurements is approximately 16.2 ADC samples wide. This corresponds to $24.95 \approx 25$ ns spacing in an LHC25NS type of beam. As can be seen in the Figure 2.2 the signal has a longer tail exceeding the 16 samples. This leads to signal leakage to adjacent bunch. In this work, the 25 ns spacing¹ is analyzed.

In the Figure 2.2 (left) two simulated signals with 25 ns spacing are presented and as can be seen the tails overlap. This overlap leads to a different bunch integral and therefore to systematic error in the result.

In the Figure 2.2 (right) the overlapping part is magnified and two contributions to the error can be observed. First is an integral of overlapping part of the tail and the second contribution is a shift of the baseline. The first contribution stems from the fact that the tail of another signal reaches into the 16 samples used in the integration and enhancing them. This can be visualized in the Figure 2.2 (right) as the tail area under the baseline of another signal. This contribution is positive because it enhances the overall signal, but the second contribution is negative since the baseline is shifted by the tail upwards. Also worth noting is that both bunches affect each other, that means that not only first affects the second, but the second bunch alters the first one as well. This can be observed as a slight shift in the baseline of the first signal in the Figure 2.2 (right).

From the simulation, it was estimated that the contribution from the shift of the baseline is larger than from integral of overlapping tail and therefore the overall charge content of each bunch in the train is measured smaller than it should. Approximately the first signal affects the second one by 3% of its own integral and the contribution vice versa is about 0.5%.

¹distance between the maxima of bunches in the train

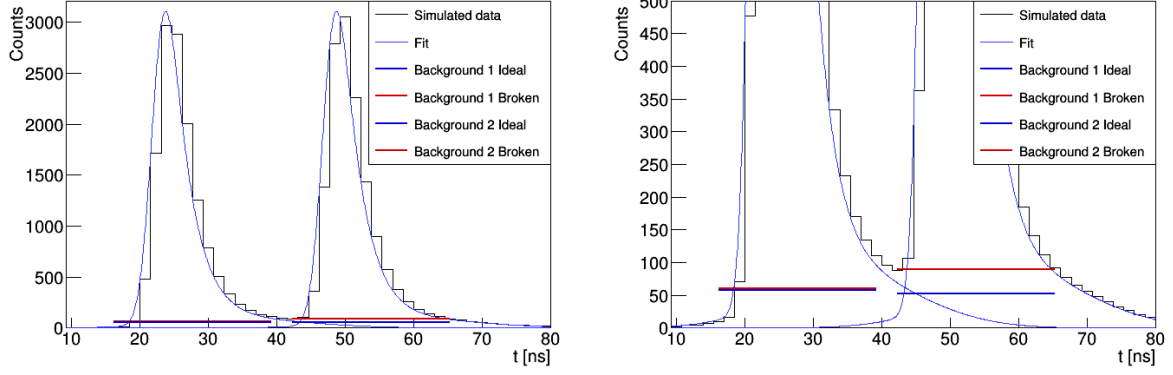


Figure 2.2: Both figures represent two simulated signals with 25 ns spacing, the right figure shows the magnified tail region where the differences are important. Ideal (dark blue) and broken (red) baseline is presented and calculated as the average of the first and the last sample used of ideal and broken signal respectively.

2.3 Analysis

Two kinds of signals were simulated, *Ideal signal* and *Broken signal*. Ideal signal is simulated with no contribution from other signals, therefore it represents the real signal that would be observed if the analyzed problem would disappeared. Broken signal is simulated with contribution from other signals, therefore it shows how the signal would appear in a train with 25 ns spacing. It is also useful to distinguish between three kinds of broken signal, *Starting*, *Middle* and *Ending*. This labeling correspond to a position in a train. Usefulness lies in the realization that the starting signal has no contribution from previous signal and the last signal has no contribution from the following one.

While simulating trains every middle signal behaves the same regardless of its position in the trains, therefore it was concluded that there is contribution to every signal solely from previous and following signal.

2.3.1 Contribution

From now on in this work, the *contribution* will be an observable defined as a difference between an integral of ideal and broken signal. It is worth noting that the contribution is absolute and not relative observable, therefore it is important to keep track of the signal integral or amplitude as well. The contribution can be therefore written as:

$$C = I_s - I_b, \quad (2.2)$$

where C denotes contribution, I_s the integral of a ideal signal and I_b the integral of a broken signal.

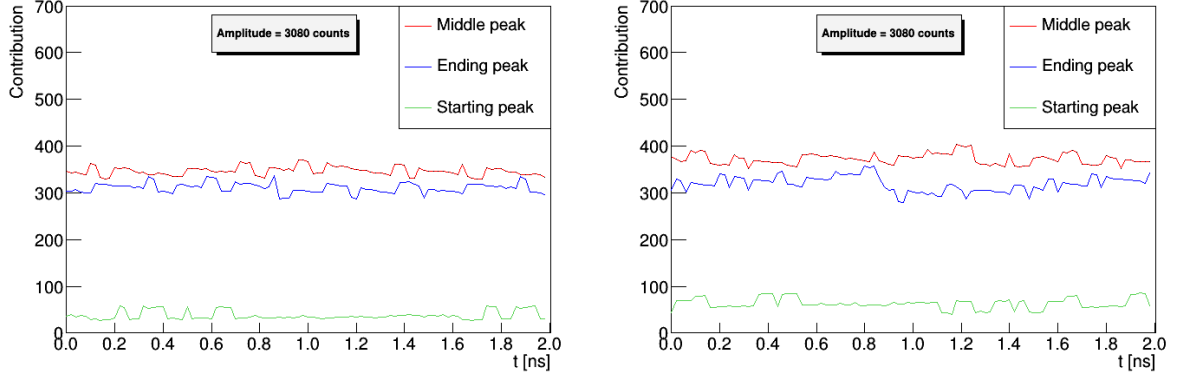


Figure 2.3: Sampling phase dependence of contribution for middle, ending and starting signal. The contribution is a difference between the integral of an ideal and broken signal. Each figure uses a different dataset created from a fit of a different real signal.

Figure 2.3 shows the dependency of a contribution on a sampling phase of a whole train while the amplitude of each signal is 3080 counts. It is observed that the contribution varies slightly, but sharply with different sampling phases. For the purpose of this analysis every following distributions will be average over many sampling phases. The reason for this averaging is that the per bunch intensity measurements uses free running clock, therefore the results are averaged while the signal is processed.

The contribution is studied as well for different signal amplitudes which is presented in the Figure 2.4. It is important to observe that the amplitude dependence is linear. This linearity is important because it shows that the relative contribution may be constant. Therefore a useful observable is *Relative Contribution*, described in the next subsection.

2.3.2 Relative Contribution

The *Relative contribution* is defined as contribution divided by the integral of an ideal signal, which can be written as:

$$RC = 1 - \frac{I_b}{I_s}, \quad (2.3)$$

where RC denotes the relative contribution. As it was mentioned earlier RC is a relative observable and therefore it is meaningful even without the knowledge of the signal amplitude. As can be seen in the Figure 2.5 the relative contribution is constant with varying amplitude. This behavior is very useful because it allows to use constant parameters in the correcting algorithm.

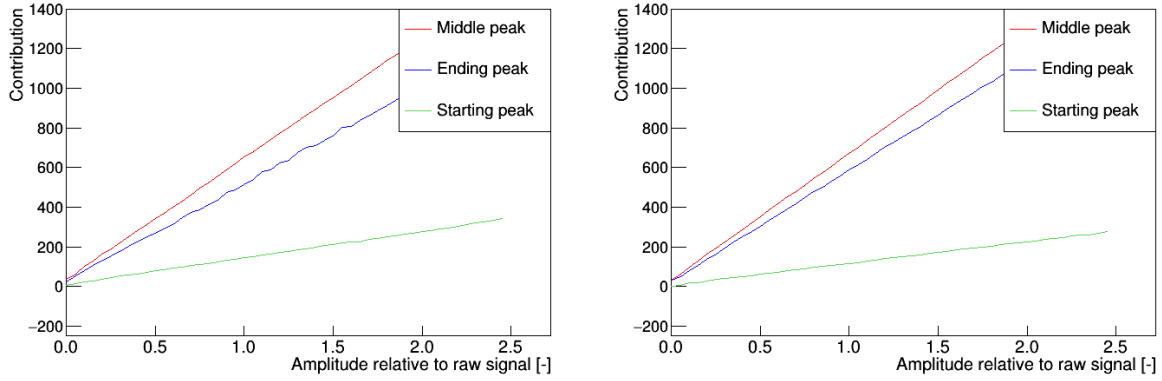


Figure 2.4: Amplitude dependence of contribution for middle, ending and starting signal. The contribution is a difference between the integral of an ideal and broken signal. Each figure uses a different dataset created from a fit of a different real signal. All datasets used in the analysis are shown in the appendix 2.10

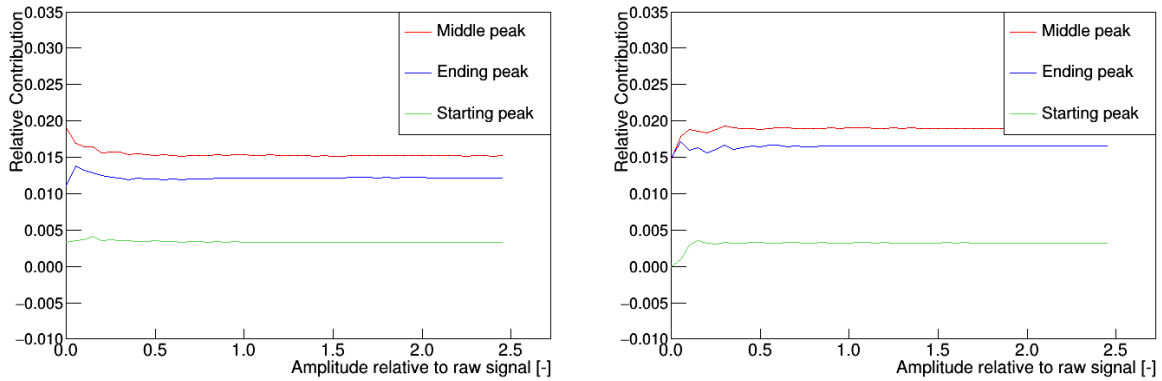


Figure 2.5: Amplitude dependence of relative contribution for middle, ending and starting signal. The relative contribution is an integral of a broken signal divided by ideal signal minus one. The relative contribution was found to be constant.

2.3.3 Correcting algorithm

To tune the correcting algorithm two main proprieties are tested, a number and a form of the signals used for correcting.

Two different algorithms were tested, the direct and sequential algorithm. The direct algorithm (DA) can be written as:

$$I_c[i] = I_b[i] + \sum_{j=-n}^n A_j \cdot I_b[i + j], \quad (2.4)$$

where $I_b[i]$ denotes an integral of a broken signal number i^2 , $I_c[i]$ the calculated corrected integral, A_j the parameters of the algorithm and $2n$ is the number of signals used for correcting each integral. As can be seen, the DA calculates the corrected integral directly from the integrals of broken signals.

The sequential algorithm (SA) can be written in form of the following recursive formula:

$$I_{pc}^k[i] = I_{pc}^{k-1}[i] + A_k \cdot I_{pc}^k[i + (-1)^k \cdot \left\lceil \frac{k}{2} \right\rceil], \quad I_c[i] = I_{pc}^m[i], \quad I_b[i] = I_{pc}^0[i], \quad (2.5)$$

where $I_{pc}^k[i]$ denotes a calculated partially corrected integral of order k , the order k expresses the number of correcting sequences and m is the maximum order calculated. For a better understanding of the sequence algorithm the example is presented in the Figure 2.6.

The sequential algorithm is correcting each signal for one contribution per sequence and then uses the integral of this partially corrected signal for calculating other contribution in the next sequence. The final corrected integral is given by the maximum order that is chosen. The calculation of the corrected integral can be traced down to the usage of only broken signals, but the relationship is more complicated than for the direct algorithm.

The algorithms were tested for up to 4 signals used for correcting each signal on the trains of 8 bunches. This corresponds to $n = 2$ and $k = 4$ in the equation 2.4 and 2.5 respectively. Only A_0 and A_1 were found nonzero which is in the accordance with the conclusion in 2.3.1. Furthermore, the sequential algorithm was very slightly modified for a bit better results. The modification lies in the usage of $I_{pc}^1[i + 1]$ instead of $I_{pc}^2[i + 1]$ in the second order, the final form can be seen at 2.6.

The comparison of the direct (left) and sequential algorithm (right) is presented in the Figures 2.7. Overall it was established that the modified sequential algorithm has slightly better results, in spite of the fact that DA has better performance for some datasets, an example is shown in the Figure 2.7 (up). The final correcting algorithm uses two parameters, A and B , which are constant as discussed in 2.3.2. Since the second order of

² i corresponds to a position in the train

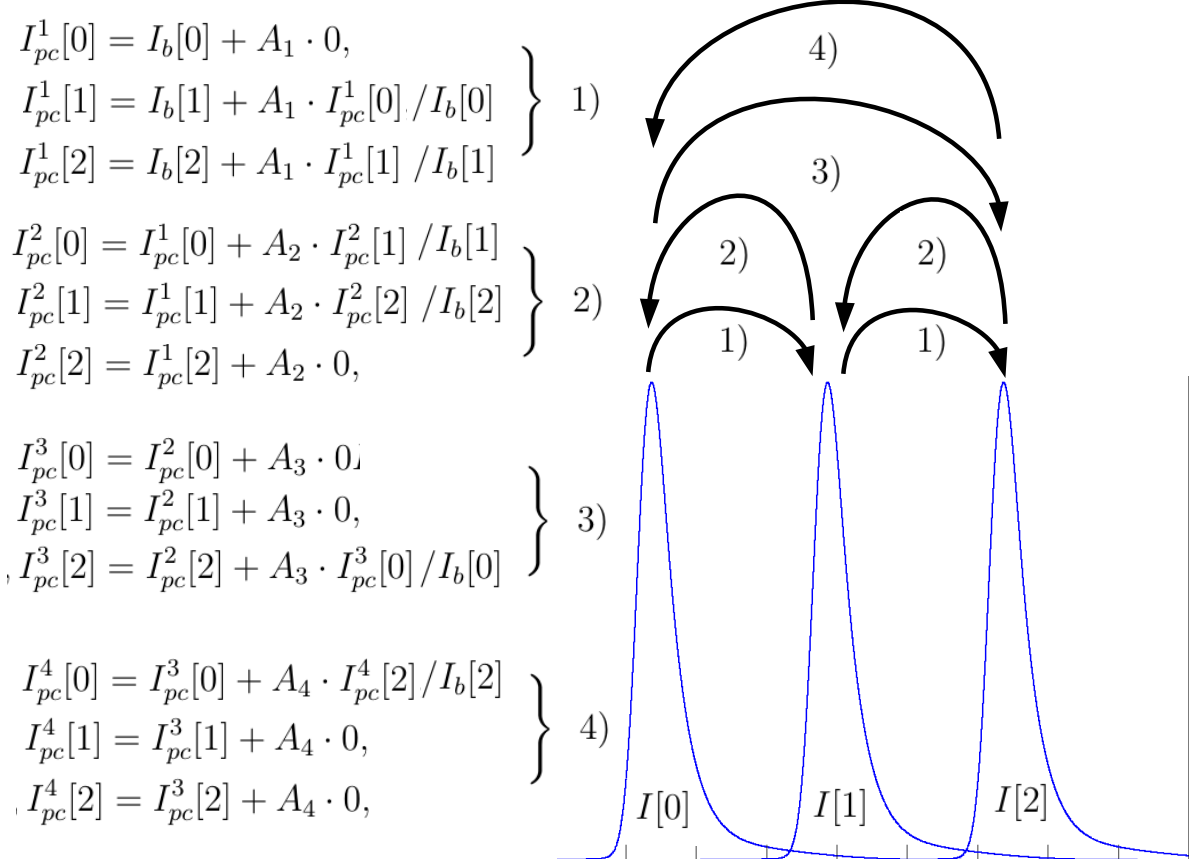


Figure 2.6: The example of the sequence/direct algorithm of 4th. order on 3 signals with visualization of each order. Arrows indicate from which signal (tail of the arrow) the correction is calculated and to which signal (the head of the arrow) this correction is added. The number under the arrows indicate in which order of the sequence algorithm the corresponding corrections are calculated. The zeros in the equations come from signals that are not present, i.e. $I_{pc}^i[-1] = I_{pc}^i[-2] = I_{pc}^i[3] = I_{pc}^i[4] = 0$

modified sequential algorithm was chosen the final algorithm corrects each integral in two steps and has a form:

$$I_{pc}[i] = A \cdot I_{pc}[i - 1] + I_b[i], \quad I_c[i] = B \cdot I_{pc}[i + 1] + I_{pc}[i], \quad (2.6)$$

$$I_c[i] = B \cdot I_{pc}[i + 1] + /I_b[0] \quad /I_b[1] \quad /I_b[2] \quad (2.7)$$

where $I_{pc}[i]$ denotes an integral of a partially corrected signal number i^3 corresponding to a broken signal corrected for the previous peak, $I_b[i]$ a broken signal number i and $I_c[i]$ a fully corrected signal.

The parameters of the algorithm were found by numerical minimization using Minuit framework. All datasets were used in the process of the minimization for more universal results. The resulting in: $A = 1.21 \cdot 10^{-2}$ and $B = 3.43 \cdot 10^{-3}$.

2.3.4 Simulation with correcting algorithm

The comparison between simulation with and without usage of the correcting algorithm is presented in this subsection. Relative contribution is shown in the Figures 2.8. It can be seen that some datasets (upper row) are corrected almost perfectly by the algorithm and some are still nonzero. This can be expected since as was discussed at the beginning of chapter 2.1 the fits that were used to generate the datasets differ especially in the tail area. The maximum observed difference from zero is about 0.7% for ending peak, 0.5% for the middle and 0.2% for the starting signal. Overall standard deviation of corrected relative contribution is: $\sigma = 0.19\%$ which is in accordance with the 1% required for current measurements. The corrected contribution is presented in the appendix 2.11.

2.4 Implementation and measurements with algorithm

The intensity measurements use FPGA for data processing, therefore the algorithm was implemented into FPGA. This implementation was done by rewriting part of the code responsible for the signal deconvolution before integration. The code is written in System Verilog language and afterwards is compiled and loaded into FPGA.

2.5 Precision of a single shot measurements

The bunch by bunch intensity measurements are considered to be used for a single shot bunch by bunch intensity measurements for transfer lines. These measurements are dependent on the sampling phase. This is in the contrast with the measurements of SPS and

³ i corresponds to position in the train

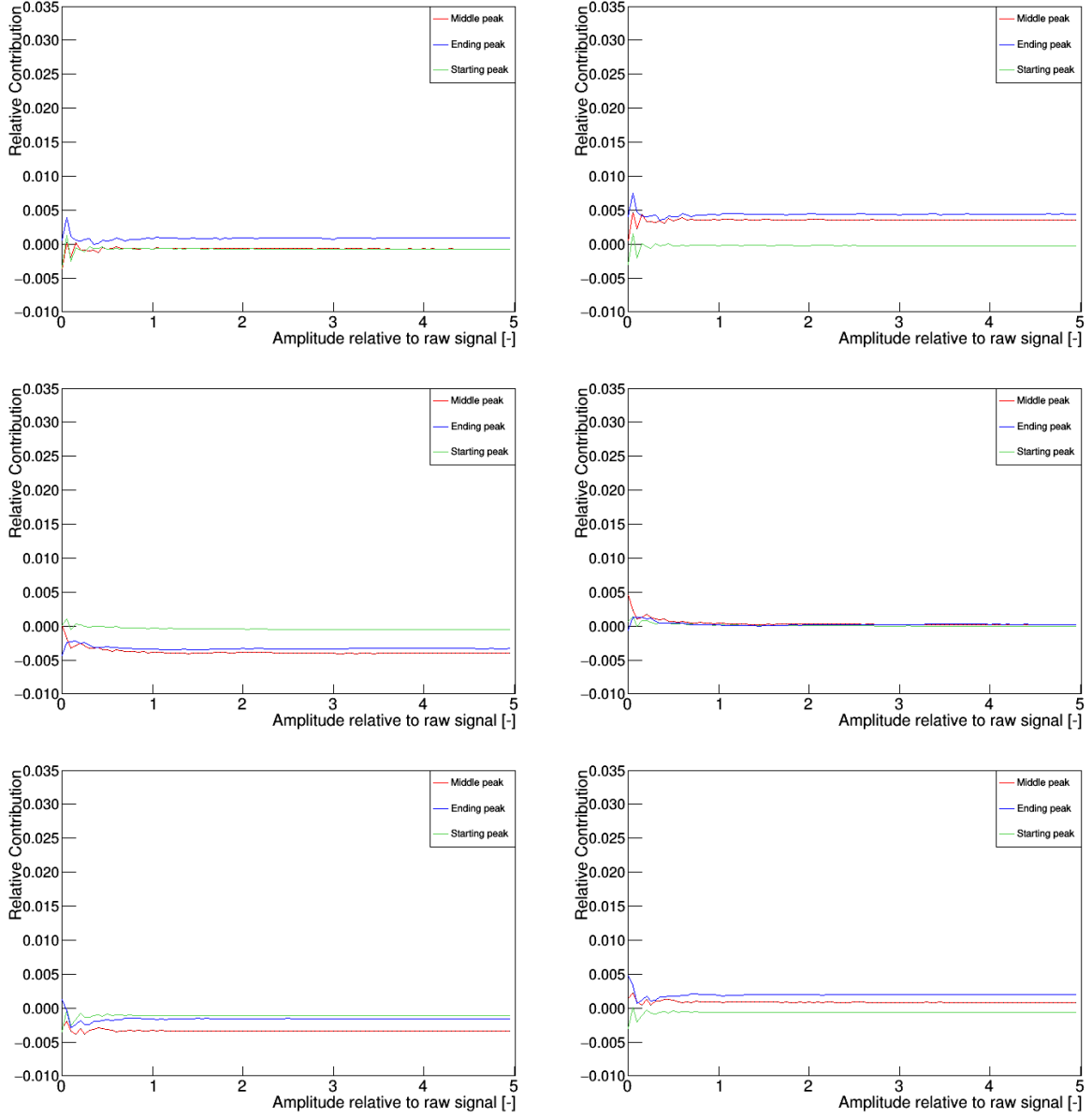


Figure 2.7: Comparison of direct (left) and modified sequential (right) algorithm. The relative contribution is an integral of a broken signal divided by ideal signal minus one. Each figure in a column uses a different dataset created from a fit of a different real signal, while each row compares the algorithms used on the same dataset.

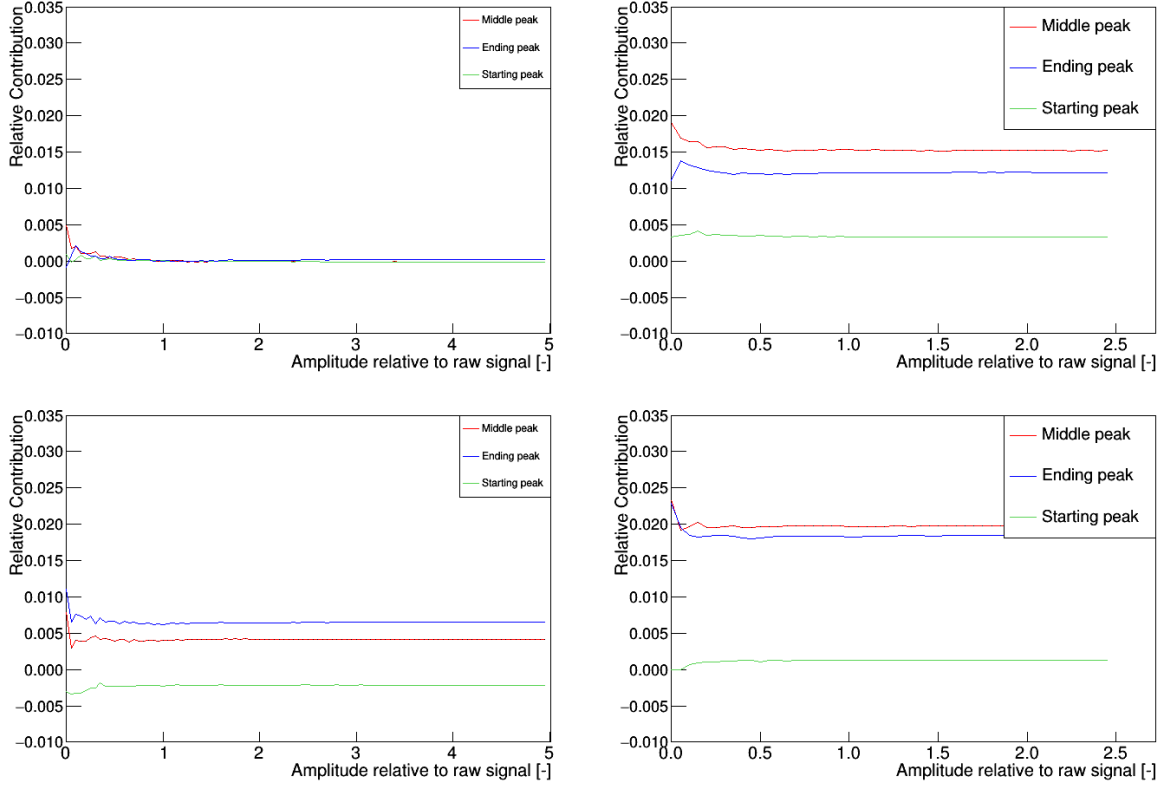


Figure 2.8: Comparison of amplitude dependence of relative contribution with (left) and without (right) correcting algorithm use. The best (up) and the worst (down) case are shown. The relative contribution is an integral of a broken signal divided by ideal signal minus one. Each figure in a column uses different dataset created from a fit of a different real signal, while each row compares the algorithms used on the same dataset. All datasets used in the analysis are shown in the appendix 2.6

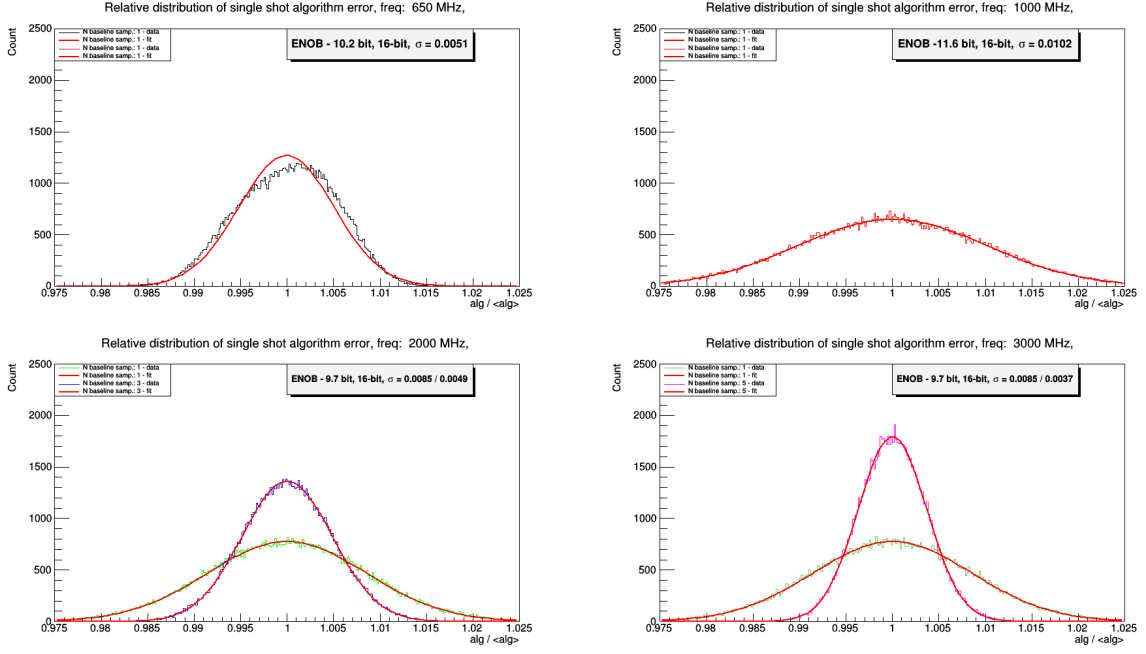


Figure 2.9: The comparison of the errors of combined ADC noise and a single shot measurement for different ADCs. The distribution calculated as the value of the algorithm (same algorithm as at 1.3.1) for varying sampling phase and divided by the average value. The algorithm uses 1 and 3 or 5 samples at each side for the estimation of the baseline for the ADCs with higher sampling rate of 2 and 3 GHz (down).

LHC beam which are averaged and therefore independent of the sampling phase as was mentioned in 2.3.

Several analog to digital converters (ADCs) were considered for this single shot measurements. The properties of each ADC were compared. In particular, the ADC noise was calculated and a single shot error was simulated. The ADC noise was calculated for each ADC from the ENOB stated in the datasheets. The single shot error was estimated from a simulated distribution of the algorithm⁴ value for varying sampling phase and divided by the average value .

The final errors of combined ADC noise and a single shot measurement are calculated and shown for considered ADC in the Figure 2.9.

⁴The same algorithm as described at 1.3.1

2.6 Wrong bucket injection

The bunch scheme of the LHC was mentioned at 1.3 where one in every 10 buckets is filled with the bunch. This scheme is important to maintain since a lot of the acquisition works with it as a fact and would be erroneous otherwise. In the past, the bunch was injected into the wrong bucket twice and it cost 6 hours of operational time at a minimum since no alerting mechanism existed for this case in the time.

The alerting algorithm was therefore developed utilizing some signals used in the bunch by bunch intensity measurements. This algorithm was implemented into the FPGA responsible for bunch by bunch intensity measurements' data processing.

Summary

The topic of this research task is *Accelerator beam physics and beam parameters measurements*. Therefore the different types of accelerators and beams are described at the start of this work. The introduction into a beam physics is presented as a main topic of the theoretical part. The transverse and longitudinal motion are discussed as the key component of the beam physics. Important concepts such as Betatron oscillations, Weak and Strong focusing, Transition or Synchrotron oscillation were introduced in these sections. The theoretical part of this work is enclosed with the description of a bunch by bunch intensity measurement which is the main object of interest in the practical part.

The bunch intensity measurements for LHC and SPS was designed to fulfill 1% and 5% precision, respectively. To achieve better measurements precision the analog signal that overlaps bunch boundaries must be deconvoluted. This is related to the signal leakage of the bunch by bunch intensity measurements. The correcting algorithm was created based on the analysis and simulation of the signal leakage and implemented into the data processing FPGA. The results from measurements using the algorithm will be available after the technical stop 2. Thanks to this algorithm the errors should be well below the specification.

The generalization of the bunch by bunch intensity measurements for the single shot bunch by bunch intensity measurements for the transfer lines is considered. The analysis of the precision of such measurements for different ADCs was carried out, to study feasibility and sustainable precision.

The injection of a bunch into a wrong bucket happened for the LHC beam twice in the past and cost at least 6 hours of operational time. Algorithm to notify operators of such wrong injection was developed and implemented. This algorithm reports the wrong injection and therefore the cost of such a error should be minimized.

The whole practical part was carried out in cooperation with the Intensity & Tune section of Beam Instrumentation group of Beams Department of The European Organization for Nuclear Research.

Appendix

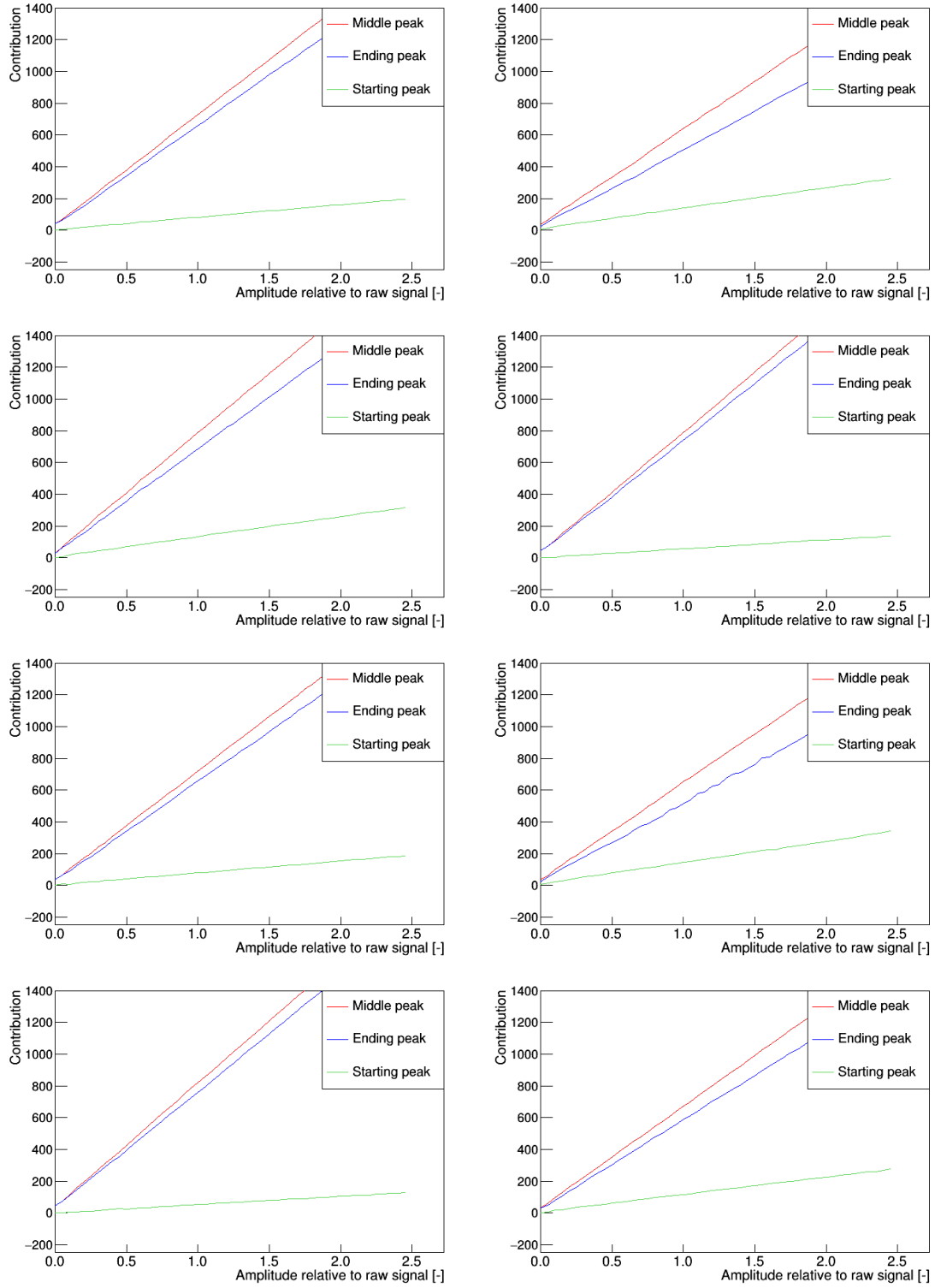


Figure 2.10: Amplitude dependence of contribution for middle, ending and starting signal. Without correcting algorithm use. The contribution is a difference between the integral of an ideal and broken signal. All datasets used in analysis are shown..

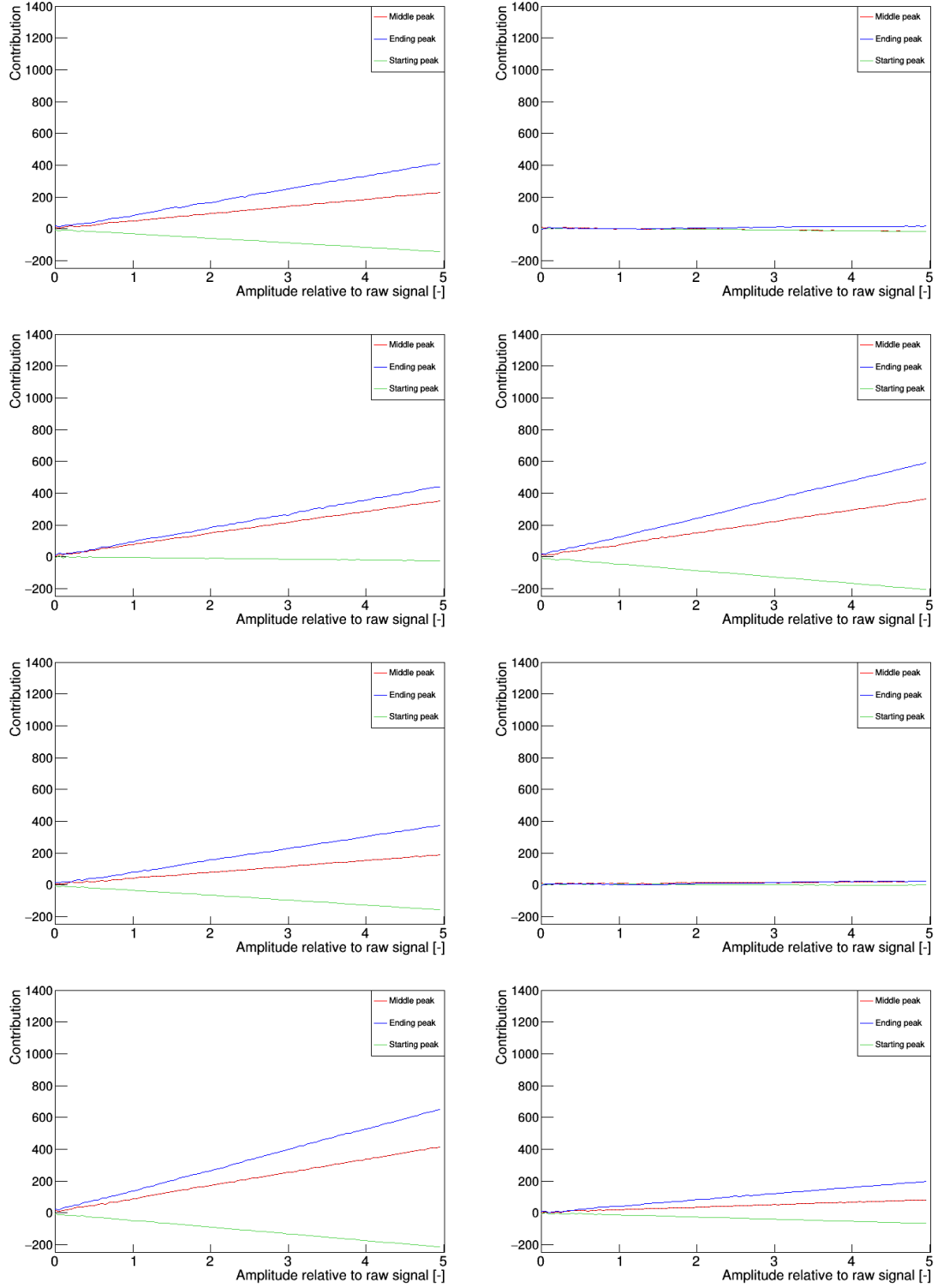


Figure 2.11: Amplitude dependence of contribution for middle, ending and starting signal. With correcting algorithm use. The contribution is a difference between the integral of an ideal and broken signal. All datasets used in analysis are shown.

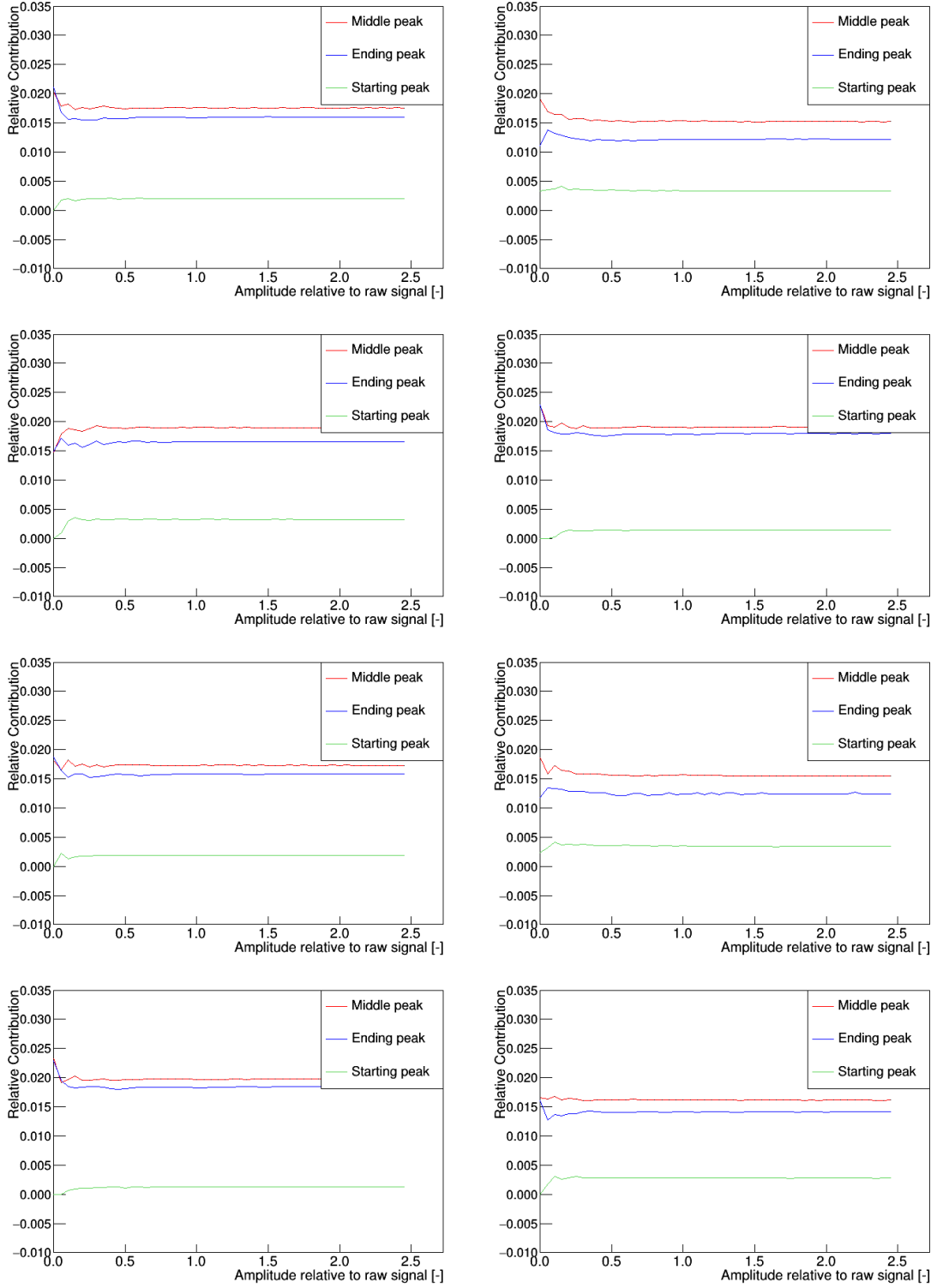


Figure 2.12: Amplitude dependence of relative contribution for middle, ending and starting signal. Without correcting algorithm use. The relative contribution is an integral of a broken signal divided by ideal signal minus one. All datasets used in analysis are shown.

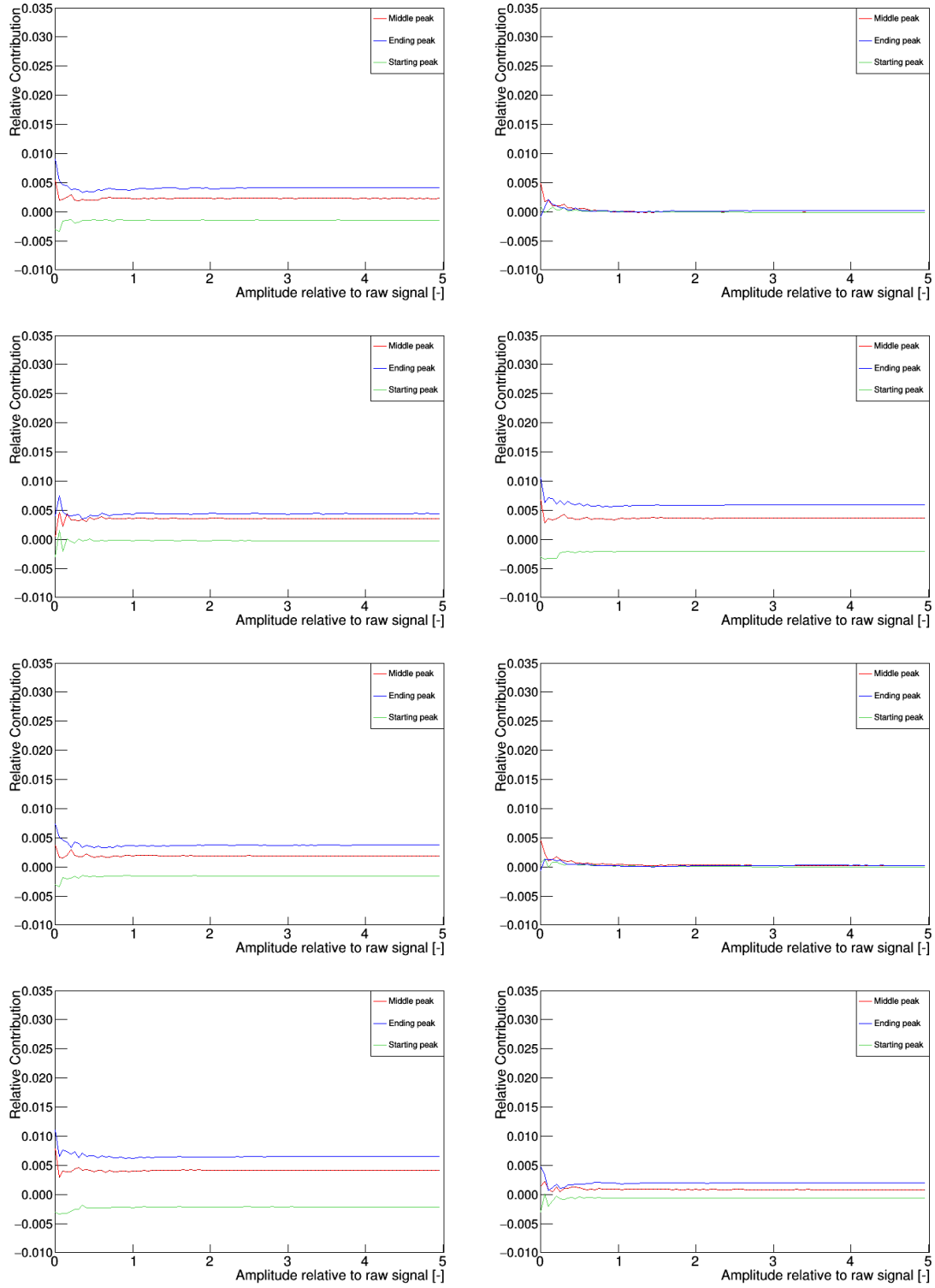


Figure 2.13: Amplitude dependence of relative contribution for middle, ending and start-signal. With correcting algorithm use. The relative contribution is an integral of a broken signal divided by ideal signal minus one. All datasets used in analysis are shown.

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