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Review Work

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The Study of Nuclear Matter in Heavy-Ion Collisions

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Contents

Chapter 1

Introduction

The study of nuclear matter in heavy ion collisions pertain to main topics of nuclear and high energy physics. Energy of collisions varies from several MeV per nucleon to several TeV per nucleon, as planned for LHC. At this high energy scale one can prove recent theory of strong interactions – QCD and theories based on quark-gluon degrees of freedom.

The properties of hot and dense nuclear matter are important for understanding phenomena outside of nuclear physics. Stability of neutron stars, formation of early universe or supernovae explosions depend on characters of nuclear equation of state (EOS). Key role plays compressibility of nuclear matter. Its values differs by a factor of 2 depending on approach used for its derivation [1]. On the contrary, one can check results from heavy ion collisions with above mentioned astrophysics phenomena.

Today, the most important experiments can be divided into three energy regimes: *i*) at about 1 AGeV at BEVALAC in Berkeley or SIS at GSI Darmstadt (at 2009 should be completed new accelerator with energy regime about 40 AGeV); *ii*) at about 2-15 AGeV at AGS in Brookhaven; *iii*) and energy regime about 40-200 AGeV at SPS at CERN and at RHIC (the Relativistic Heavy Ion Collider) in Brookhaven. In the future, one will be able to get much higher energy (of about $\sqrt{s} \approx 6$ ATeV) at the Large Hadron Collider at CERN.

The main aim of this paper is to bring short description of recent situation on the field of theoretical models describing these reactions. Unfortunately, there is not any one model describing whole energy range. Instead of this, there exist few principles used for modeling heavy ion reactions, and concrete models are based on these principles.

In the second chapter, Macroscopic vs. Microscopic, I would like to present the most basic principles used for describing nuclear collisions. Some examples of Macroscopic Models (the only part about macroscopic model in this paper) will be presented as well as brief introduction into Microscopic Models discussed later.

In the third chapter, the microscopic Quantum Molecular Dynamics Models are presented. It begins with Feshbach-Kerman-Koonin Theory, followed by QMD, RQMD and UrQMD.

In the fourth chapter, I will concern on the microscopic Boltzmann-Uehling--Uhlenbeck Models. There are also included modifications of basic BUU, e. g. Vlasov-Uehling-Uhlenbeck Model, Fock-Tani Formalism and ART.

In the last chapter, there are two String Models, the first is JAM and the second one is QGSM.

Chapter 2

Macroscopic vs. Microscopic

2.1 General description

The number of particles participating at nuclear collision is on one hand too small for fully statistical approach, however, on the other hand too big for microscopic description. Microscopic description here mean that the reaction is modeled for every elementary particle separately. On the other hand, the statistical approach, i. e. macroscopic approach, mean complete description of the system with thermodynamic variables. It is instantly clear that both cases could not be possible without any simplifying. For example in heavy ion collisions we suppose assuming complete equilibrium of the system. Degree of this simplification depends on many factors (e. g. energy of participants, their A, centrality of collision etc.).

During a collisions, nuclear matter is compressed and thermalized (thermally excited). Therefore we can describe the state of matter with temperature T and density ρ. In order to disentangle the change of the static energy due to cold compression from the energy due to an increase of the temperature, we define the energy of the equilibrated system [1]:

$$E(\rho, T) = E_T(\rho, T) + E_C(\rho, T = 0) + E_0 \ , \tag{2.1}$$

where E_C is the compressional energy and E_T is the thermal energy. Only a single point of this two-dimensional surface is known experimentally: At normal nuclear matter density $k_{Fermi} \approx 1.36\,\mathrm{fm}^{-1}$, the binding energy per baryon is $E_0 \approx 8\,\mathrm{MeV}$ and the pressure equals zero.

In other words, we defined in the paragraph above the Equation of State (EOS). It seems, that treating with the nuclear collisions via EOS is very useful. In the frame of the thermodynamic approach, we can define many variables, e. g. compressibility $K = \frac{\alpha}{V}\frac{\partial V}{\partial p}$. However, it is not so easy. We have to find out observables, that are connected to variables in EOS. In addition, we have to differentiate moments, they refer to. As an example of observables I would like to mention multifragmentation, i. e. the production of low and medium mass fragments with A ranged from 5 to 30, or the collective flow of nuclear matter.

2.1.1 Spectator-Participant Principle

[2] Before continuing I would like to introduce one useful principle very often used in describing nuclear reactions. This concept is based on fact, that for non central collisions, a certain part of projectile nucleus A touch a certain part of target nucleus B (see Fig. 1). These parts are called participants and in this

concept they lose their velocity completely. The rest of the both nuclei are so called spectators and continue in their original way. If the incident energy is high in comparison with binding energy of nucleon in nucleus (it is about 8 MeV), we can neglect the fact, that both parts (A and A′) were attached. Participants usually disintegrate totally (give to rise many particles p, n, d, π etc.) while the spectators "shadow" the reaction and evaporate a few particles.

Other models using similar idea are firestreak model or row-on-row model. In these models, participants do not stop entirely. Here are interacting horizontal tubes (or rows, respectively) of projectile with horizontal tubes (rows) of target [3].

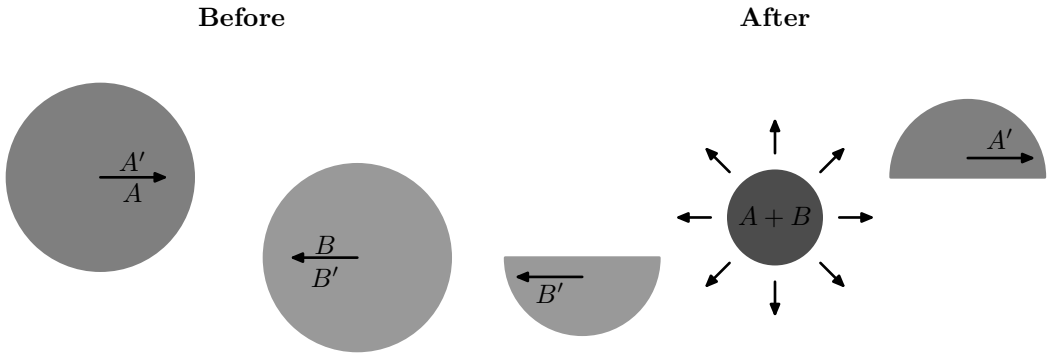


Fig. 1: Participant–spectator concept

Thus the theoretical models used for description of nuclear collisions can be approximately divided into two families [4]: *i*) The macroscopic models, where the complete thermodynamical approach is used (discussed in Chapter 2.2) and *ii*) the microscopic models, where every individual nucleon-nucleon interaction is treated (discussed in Chapter 2.3, 3, 4, 5).

2.2 Statistical Macroscopic Models

[4] Statistical treatment is used. They bring clear transparency to the relevant physics. These models were initially used much more frequently and therefore the concept and the language of heavy ion collisions came from this treatment, e. g. EOS, macroscopic flow, squeeze-out, freeze-out, thermalization, chemical equilibrium, entropy and compressibility. These models are strongly dependent on assumption of local equilibrium. This will not be always valid. More information about equilibration of hadronic matter can be found in [5].

2.2.1 One Fluid Hydrodynamical model

The first representative of statistical models (all necessary calculation are based on statistical mechanics) given here is so called One Fluid Model. This

approach describe nuclear collisions in terms like pressure, energy density, temperature. Particle production is here the way of decreasing free energy. The nuclei are described as spherically symmetric drops of liquid. These drops are divided into grid cells simulated by many test particles because of statistics.

In this model, each cell is in chemical equilibrium, even the suddenly populated cells. Local equilibrium can be expressed by condition:

$$\tau_{rel.} \ll \tau_h, \tag{2.2}$$

where τ_h is the "hydrodynamic" time: the time it takes for two fluids to pass through each other ($\tau_h \approx A^{1/3}/\gamma_{cm}$ [fm /]). The relaxation time τ_{rel} is related to the speed of the sound c_s in hot hadronic matter and can be approximately evaluated as $\tau_{rel} \approx (\rho\sigma_T c_s)^{-1}$, where ρ is the density and σ_T transport cross section. There are at least 2-3 collisions per nucleon needed to system reach thermal equilibrium.

The calculation is made in obvious way, in time steps. In each time step, propagation of each grid cell is evaluated obeying necessary conservation laws as the baryon current conservation, the momentum conservation and the energy conservation. Output of every step is input for the next one. This procedure takes until the freeze-out is reached. Term freeze-out here mean, that density of every cell drop under certain value. Measured observables are calculated from this last time step. Test particles are clustered to real particles (baryons).

2.2.2 Two Fluid Hydrodynamical Model

Model described above is very sensitive to condition (2.2). When this is not valid, the Two Fluid Model takes place. This can occur when the transport cross section is too small, i. e. the system is transparent. This case come at the energy about 1 AGeV and below, when the initial stopping will be lower then assumed in the OFM because average path length required for thermalization equals several mean free paths.

The difference between the One Fluid Model and the Two Fluid Model is in sophistication of thermalazitation when two nuclei collide. The model above assumed instantaneous thermalization while thermalization in the Two Fluid Model is produced by friction between the colliding fluids, which leads on gradual deceleration and heating. On the other hand, definition of this frictional force can be problematic.

The Two Fluid Model is more general in reproducing the experimental data. The differences between these two models increase with energy.

2.2.3 Three Fluid Hydrodynamical Model

For description of ultra-relativistic heavy-ion collisions was developed The Three Fluid Model, that divide particles into three liquids: *i*) the target nucleons; *ii*) the projectile nucleons and *iii*) the particles produced in the reaction. Every liquid is equilibrated (what is necessary for using hydrodynamical model), however, there are not equilibrated to each other. The EOS of projectile and target liquid can be as ideal nucleon gas with appropriate value of compressibility and binding energy, that is implementation dependent. Third liquid is the most interesting here, because in this way the creation of QGP state can be proved (setting this fluid as a pion gas is not appropriate). More about using Three Fluid Model in ultra-relativistic heavy-ion collisions as well as data from S + Au at 200 AGeV (where are some appearance of creation QGP) can be found at [6].

2.3 Microscopic Cascade Models

In this approach, each nucleus consists of point nucleons distributed in the volume of sphere. In the pure cascade model, there is no initial Fermi momenta of nucleons. At the beginning, nuclei start with certain velocities toward each other with certain impact parameter. This is done by Monte-Carlo sampling.

Now, one have to decide when and where nucleons collide. This is done by dividing the time of collision into the time intervals Δt. This interval has to be short enough, the probability for more then one collision of one nucleus is negligible. Its value is obviously $\Delta t \approx 0.5\,\mathrm{fm}/$. Its value depends on ability of computing devices. How to decide when two particles collide? There is simple condition for it:

$$|r_1 - r_2|_{min} \leq \sqrt{\frac{\sigma^t_{NN}\left(\sqrt{s}\right)}{\pi}}, \tag{2.3}$$

where $|r_1 - r_2|_{min}$ means the closest approach of particles and σ^t_{NN} is the total cross section for nucleon-nucleon interaction with c.m. energy $\sqrt{s}$.

When a collision occurs, particles can scatter either elastically or inelastically. We have to decide what channels can be taken into account in our approach. The rule is higher energy imply more channels have to be taken. For example for energy about 1 AGeV, the pion production from direct process is negligible in comparison with indirect via resonance decay.

Cross sections can be evaluated in several ways. Some of them can be measured in laboratory (e. g.: N + N $\rightarrow$ N + R, where R mean resonance). Other cross sections can be derived via so called detailed balance from inverse process (e. g.: N + R $\rightarrow$ N + N). Some of them are deduced from self-consistent theoretical models.

Many of such simulations at each impact parameter and energy have to be done for true and valuable results. We can take many runs simultaneously. It has

big advantage, we can calculate time evolution of density, number of collisions and entropy (example can be seen in Fig. 2, BUU (see Chapter 4) used [2]).

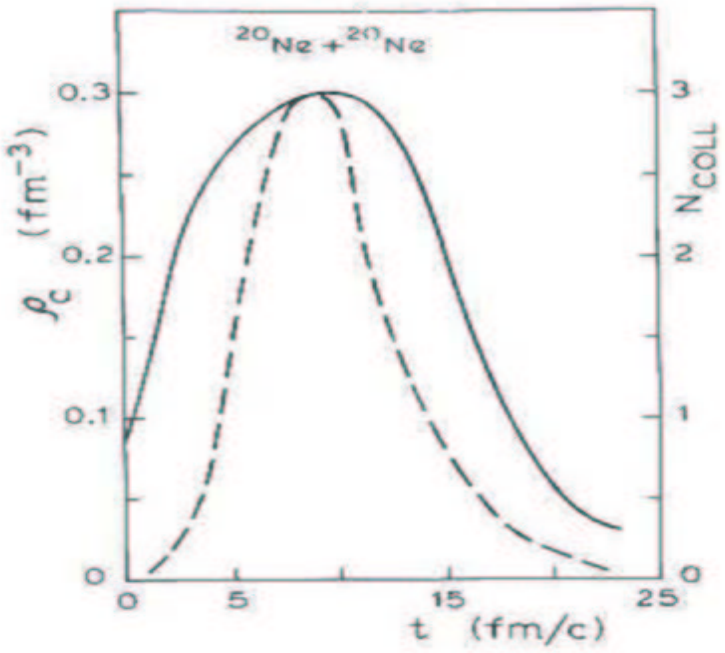


Fig. 2: The central density ρ_c (the solid line), and the number of collisions per 0.5 fm /, N_{coll} (the dashed line), as a function of time for ^{20}Ne on ^{20}Ne at 400 AMeV lab energy. The averages of 60 runs are shown. At the starting time ρ_c is less than ρ_0 because the central region is defined as a (1.5 fm)^3 box and its center is located at the point where the two nuclei touch initially in the c.m.

Entropy is defined:

$$S = -\sum_{cells} \left[n_i \ln(n_i) + (1 - n_i) \ln(1 - n_i) \right], \tag{2.4}$$

where n_i is the occupation probability of cell of size h^3 defined as $n_i = N_i'/\tilde{N}$, N_i' is number of particles in given cell and $\tilde{N}$ is number of simultaneous runs.

Nucleons, which we have discussed here, were without initial momentum, however, it was proved, they have some initial momentum (initial temperature). When we want to add some initial momentum, we have to introduce mean field which nucleons feel. It is because of stability of nucleons. Without mean field, nucleons fly out so there is no nucleus. (Time of this fly out depends on distribution of initial momenta. It will be shortly discussed later.)

Mean-field is usually introduce as a function of the density $U(\rho)$ (so called Skyrme type of potential):

$$U(\rho) = A\left(\frac{\rho}{\rho_0}\right) + B\left(\frac{\rho}{\rho_0}\right)^{\sigma}, \tag{2.5}$$

where $\sigma > 1$, ρ_0 is nuclear density (it is reference density, i. e. value for which we know another parameters, e. g. binding energy of nucleons), A is attractive and B repulsive parts of potential.

That is all for the general description of The Microscopic Cascade Models. In the next parts of this paper, I will discuss some examples, that are based on these basic principles.

Chapter 3

QMD Models

In this chapter will be described microscopic models based on "Quantum Molecular Dynamics". Its name comes from the fact that it can be considered as quantal extension of the classical molecular approach widely used in chemistry and astrophysics [1], [7].

The first attempts to describe heavy-ion collisions build on the participant spectator principle assumed, that the whole reaction machanism is caused by thermalized nuclear gas made of participants. However, this assumption showed to be bad. Therefore there were made some idealization as *i*) the lighter the projectile and the more peripheral collision is, the stronger non-equilibrium appear; *ii*) for lower energies reaction absents collisions and mean-field keep nucleus together; *iii*) for higher energies the frequent nucleon-nucleon collisions cause thermalization and *iv*) Pauli blocking principle is not so severe for higher energies, between low and high energy — Pauli blocking causes the slowing the way toward equilibration. However, these assumptions were valid just at the beginning of the model developement. Sophisticated models are working on the "smaller scale."

3.1 Feshbach-Kerman-Koonin Theory

For the collisions of tens AMeV was the main goal of theoretical models to give the right angular and energy distributions of particles flying out the place of crash. It stands to reason, that for so small energy, it is impossible to get equilibrium. The description of collision was based on particle-hole principle. This principle, in short, say, that in the more energy collision have, the more pairs of particle-hole is created. This principle was incorporated into the Exciton Model, Standard Hybrid Model, Geometry Dependent Hybrid Model and Master Equation, which arose as generalization of Exciton Model. These models are not subject of this paper.

It was natural to incorporate into this model Quantum Mechanics completely. However, it was very computational difficult, therefore Feshbach-Kerman-Koonin (FKK) Theory was created. It is, roughly speaking, quantal particle-hole principle, where the particle production is caused by creation particle-hole toward the more energetic states. More can be found at [8].

3.2 QMD

The QMD Model is a N-body theory which simulates heavy-ion reactions at intermediate energies on an event-by-event basis. Taking into account all fluctuations and correlations has two main advantages: *i*) many-body processes, in particular, the formation of complex fragments are explicitly treated and *ii*) the model allows for an event-by-event analysis of heavy ion reactions similar to the methods which are used for the analysis of exclusive high acceptance data. Now, I would like to introduce basic principles.

In QMD each nucleon is represented by a coherent state of the form:

$$\phi(\vec{x};\vec{q}_i,\vec{p}_i,t)=\left(\frac{2}{L\pi}\right)^{\frac{3}{4}}\exp\{-\frac{2}{L}(\vec{x}-\vec{q}_i(t))^2+\frac{1}{\hbar}\mathrm{i}\vec{p}_i(t)\vec{x}\}, \tag{3.1}$$

which is characterized by 6 time-dependent parameters, $\vec{q}_i$ and $\vec{p}_i$, respectively. The parameter L, the extension of packet in phase-space, is fixed. The total n-body wave function is assumed to be the direct product of coherent states:

$$\Phi=\prod_i \phi_i(\vec{x};\vec{q}_i,\vec{p}_i,t) \tag{3.2}$$

Slater determinant is not used here, thus the antisymmetrization is neglected. Successful attempts to simulate heavy-ion collisions with respect to antisymmetrization was performed only for small systems. A consistent derivation of the QMD equations of motion for the wave function under influence of both, the real and the imaginary part of G-matrix, however, has not yet been achieved.

The equations of motion of the many-body system is calculated by means of generalized variational principle:

$$S=\int_{t_1}^{t_2}\mathcal{L}\left[\Phi,\Phi^*\right]\mathrm{d}t \tag{3.3}$$

with the Lagrangian $\mathcal{L}$

$$\mathcal{L}=\left\langle\Phi\left|\mathrm{i}\hbar\frac{\mathrm{d}}{\mathrm{d}t}-H\right|\Phi\right\rangle, \tag{3.4}$$

where the total time derivative includes the derivation with respect to the parameters. Hamiltonian H of the form $H=\Sigma_i T_i+\frac{1}{2}\Sigma_{ij}V_{ij}$ contains kinetic energy T_i and mutual interactions V_{ij}. These interactions are the local hard core repulsion, a Yukawa potential, a Coulomb potential, a momentum dependent potential and a symmetry term homogenizing the proton and the neutron distributions.

The equations of motion get finally the form for parameters $\vec{p}_i$ and $\vec{q}_i$

$$\dot{\vec{p}}_i=-\frac{\partial<H>}{\partial\vec{q}_i}\quad\text{and}\quad\dot{\vec{q}}_i=\frac{\partial<H>}{\partial\vec{p}_i} \tag{3.5}$$

which show the same structure as the classical Hamiltonian equations, and are solved in a same manner as it is done classical molecular dynamics.

In the QMD, there is restriction to binary collisions. The collision occurs if the condition (2.3) is fulfilled. Where the distance of two particles means the relative distance of the centroids of the Gaussians.

Using trial wave functions other then Gaussian in (3.1) yields more complex equations of motion and the analogy to classical physics is lost.

Pauli-blocking principle reduce the cross-section to an effective cross-section. The phase-space is not discretized as in BUU (see Chapter 4.) but the following procedure is applied: The phase space density f_i' at the final states $1'$ and $2'$ is measured and interpreted as a blocking probability. Thus, the reaction is only allowed with a probability of $(1 - f_1')(1 - f_2')$. If the collision is not allowed, particles remain at their original momenta.

3.3 RQMD

The Relativistic Quantum Molecular Dynamics (RQMD) approach has been developed to extend the QMD model up to relativistic energies (AGS and SPS domain). The main improvements compared to QMD are: *i*) covariant dynamics; *ii*) an improved and extended collision term containing heavy baryon-resonances, strange particles and string-excitation for high energy hadron-hadron interactions.

The main drawback of RQMD is connected with computing relations of relative times of the particles, in words: inverting of matrix with number of elements quadratic in the number of particles N.

3.4 UrQMD

UrQMD is the model that incorporates different reaction mechanisms in the energy range from low energies (compound nucleus formation and deep inelastic scattering at the Coulomb barrier) up to ultra-relativistic energies (string-excitation and -fragmentation and parton scattering). Such a microscopic model contains a lot of unknown parameters, which will have to be connected with experimental observables and checked with experimental data. It is important, if one try to find new physical phenomena like phase transition to QGP, to prove that these data cannot be equally described by more then one physical assumption. More information can be found at homepage of UrQMD group [9]. Only hard equation of state has been incorporated into the UrQMD. Now, I would like to described basic principles of UrQMD.

Each nucleon is described by Gaussian (3.1) and nucleus is described as a product of this distributions (3.2). Each initialized nucleus must meet the following constraints: *i*) nucleus is centered in configuration space around $\vec{0}$; *ii*) nucleus is at rest; *iii*) its binding energy should correspond to the value given by the Bethe-Weizsäcker formula; *iv*) the radius should yield the following mass

dependence $R(A) \sim \left(\frac{3}{4\pi\rho_0}\right)^{1/3} \cdot A^{1/3}$ and v) its center has nuclear matter ground-state density. The initial momenta of the nucleons are randomly between 0 and the local Thomas-Fermi momentum $p_F^{max} = \hbar c \left(3\pi^2\rho\right)^1 /3$, where ρ corresponds to local nucleon density.

The classical UrQMD Hamiltonian of the parameters $\vec{r}_j$ and $\vec{p}_j$ is given by:

$$H = \sum_{j=1}^{N} E_j^{\text{kin}} + \frac{1}{2}\sum_{j=1}^{N}\sum_{j=1}^{N}\left(E_{JFK}^{\text{SK2}} + E_{JFK}^{\text{Yuks}} + E_{JFK}^{\text{Coulomb}} + E_{JFK}^{\text{Pauli}}\right) + \frac{1}{6}\sum_{j=1}^{N}\sum_{k=1}^{N}\sum_{l=1}^{N} E_{kl}^{\text{SK3}}, \tag{3.6}$$

where E_j^{kin} is the kinetic energy, E_{JFK}^{SK2} and E_{kl}^{SK3} are the two-body and the three-body Skyrme-Potentials, respectively, and the last three are Yukawa, Coulomb and optional Pauli potentials. Their form can be found also at [7].

The collision criterion has the same form as (2.3).

The UrQMD collision term contains 55 different baryon species and 32 different meson species, which are supplemented by their corresponding anti-particles and all isospin-projected states. For excitations with higher masses than 2 GeV/c^2 a string picture is used. Generally, for ultra-relativistic energies, the effects of Quantum Chromodynamics are incorporated, i. e. string-excitations and fragmentations and color fluctuations, opacity and transparency. Figure 3. shows hadrochemistry results of recent SPS data.

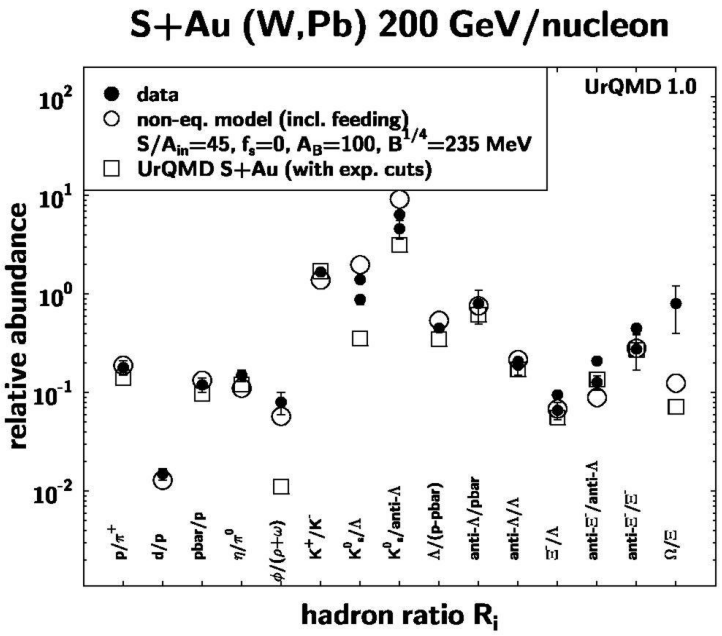


Fig. 3: Comparison between the UrQMD model and data for the system S+Au (W, Pb) at 200 AGeV.

Chapter 4

BUU Models

4.1 BUU – Boltzmann-Uehling-Uhlenbeck

This method is based on Boltzmann-Uehling-Uhlenbeck equation:

$$\frac{\partial f}{\partial t} + \vec{v}\cdot\vec{\nabla}_r f - \vec{\nabla}_r U\cdot\vec{\nabla}_p f = -\frac{1}{(p\pi)^6}\int \mathrm{d}^3p_2\mathrm{d}^3p_{2'}\mathrm{d}\Omega\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}v_{12} \times\{[ff_2(1-f_{1'})(1-f_{2'}) - f_{1'}f_{2'}(1-f)(1-f_2)] \times(2\pi^3\delta^3(\vec{p}+\vec{p}_2-\vec{p}_{1'}-\vec{p}_{2'}))\}, \tag{3.1}$$

where U is the mean-field potential, f distribution function, left hand side of equation is derived from quantum mean field theory and right hand side of equation is the collision integral. Terms description will be explained in detail in the next paragraphs describing this method in detail or in [2].

At the end of the last chapter, we introduced mean field. It improves the cascade model in two ways. Let we have isolated nucleus of N nucleons. We will describe it via $N\tilde{N}$ test particles which are in the sphere of radius R. In cascade model we have distributed just N nucleons. Now, we can introduce Fermi motion of nucleons. It is initial motion of nucleons that they have before collision. We assign momenta to nucleons up to p_F. Some more details about distributions of initial momenta will be displayed later or in [10]. Nucleus will not fly off because of mean field. The radii R and momentum p_F are not independent, they are conjuncted via $4(4\pi/3)^2R^3p_F^3 = h^3N$, where isospin degeneracy have been taken into account.

Let assume, that two test particles collide. They change from $(\vec{r}_1, \vec{p}_1)(\vec{r}_2, \vec{p}_2)$ to $(\vec{r}_1, \vec{p}_1')(\vec{r}_2, \vec{p}_2')$. If the phase space around $(\vec{r}_1, \vec{p}_1')$ and $(\vec{r}_2, \vec{p}_2')$ are essentially empty then scattering is allowed. If on the other hand they are essentially filled, the scattering should be suppressed. The word "essentially" vary from one implementation to another, but this is a basic procedure. The phase space around test particle can be represented by sphere with radius r around r' and p around p'. Values of these radii are specified by relation $r/p = R/p_F$, where p_F is the Fermi momentum and R is the radius of static nucleus. The number n of test particles, that fill phase-space volume cannot be taken too small as statistical fluctuations, inherent in any numerical calculation, become important and on the other hand not too large since we need to sample phase-space close to $(\vec{r}, \vec{p}')$. Now, we can define $f_a = n_a/(n-1)$ where $a = 1,\ 2,\ 1',\ 2'$ and n_a is the number of test particles in the phase space volume not including the test particle at $(\vec{r}_1, \vec{p}_1)$.

The BUU equation is a differential equation for the classical one-body phase-space distribution function $f(\vec{r}, \vec{p}, t)$ corresponding to the classical limit of the Wigner function.

An example of usage BUU can be found at [11], where the proton energy spectra were compared with simulations. The parameters of collision was ^{36}Ar + ^{45}Sc at 80, 120 and 160 AMeV and the collision was central. On the Figure 4. is shown good agreement of the model and the experiment. However, in the same reference are shown disagreements for ^{40}Ar + ^{197}Au at 200 AMeV where the QMD was needed.

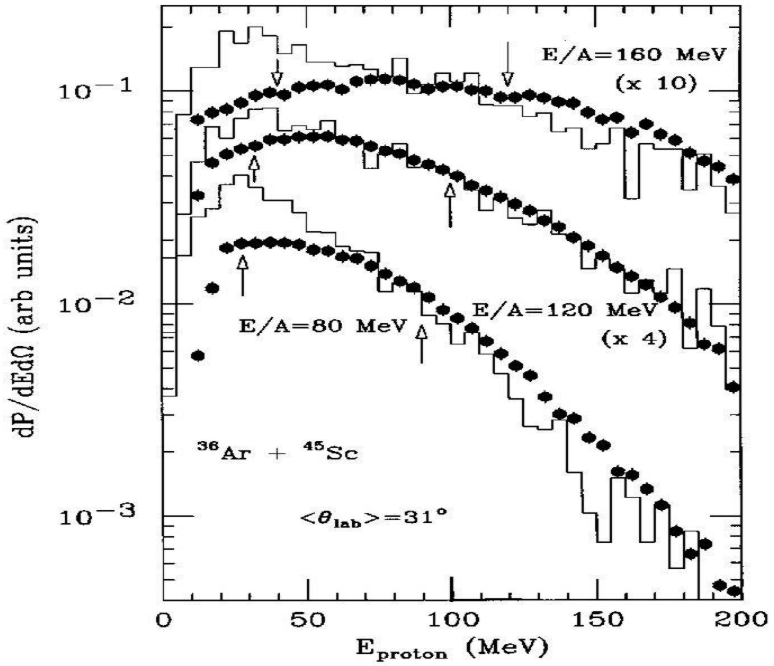


Fig. 4: Laboratory frame proton energy spectra measured at $\Theta_{\rm lab} = 31^\circ$ for central ($b/b_{\rm max} \leq 0.3$) collisions of^{36}Ar + ^{45}Sc at 80, 120 and 160 AMeV (solid points) are compared with predictions of BUU (histograms). Relative normalization gives equal areas for measured and predicted spectra for $E_{\rm proton} \geq 50\,{\rm MeV}$.

The BUU is made, broadly speaking, for energy range from 30 AMeV to 2 AGeV. There were also made many improvements for relativistic energies, like RBUU, CBUU (usage of this model can be found at [12]) or more complex improvements like Hadron String Dynamic (HSD) or A/Another Relativistic Transport (ART) described later. Here we mention also VUU and Fock-Tani formalism.

4.2 VUU – Vlasov-Uehling-Uhlenbeck

Equation (3.1) without right hand side is so called Vlasov equation. This can be derived as an approximation to time dependent Hartree-Fock theory, which is well known and used in the theory of heavy ion collisions. I will not show derivation here, it can be found at [2]. The example of using VUU can be found at [13] (shortly described below) or in the previous chapter.

In the Ref. [13], the collisions of ^{40}Ca + ^{40}Ca have been studied at an incident energy of 1.8 AGeV and an impact parameter of 2 fm, that corresponds to a nearby central collisions. In the numerical simulations each nucleon was replaced by 80 test particles. Time steps were 0,5 fm/c. Smaller time intervals did not show any significant difference. Calculated transverse momentum is displayed in Fig. 5 as long as experimental data ^{40}Ca + ^{40}Ca at 1.8 AGeV by Ströbele. We can see, this model depends on given compressibility K and effective mass M^* ($M^* = M - g_S\Phi$, where M is nucleon mass, g_S coupling constant of scalar mesons to the nucleon and Φ scalar meson). In some articles, the VUU and the BUU models were considered as the same or at least very similar.

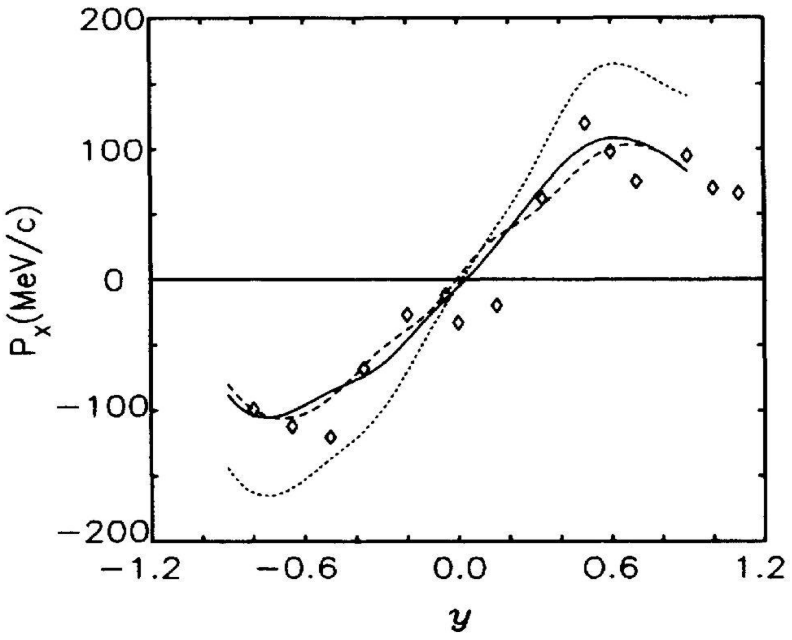


Fig. 5: The transverse momentum projected onto the reaction plane and averaged over all perpendicular momentum P_x, as a function of the rapidity y for the reaction ^{40}Ca + ^{40}Ca at 1.8 AGeV. The open diamonds are experimental data from Ströbele. The solid curve and the dashed curve are from the theoretical calculations with the same effective mass $M^* = 0.83M$ but different values of the compressibility $K = 380$ MeV and $K = 200$ MeV, respectively. The dotted curve corresponds to $M^* = 0.7M$ and $K = = 380$ MeV.

4.3 Fock-Tani formalism

Some improvements of BUU can be done at (3.1) in cross-section. In the present case, the cross-section is obtained from Fock-Tani Formalism [14]. This approach, in short, shows, that even in the intermediate energies (below RHIC), the collision of heavy ions depends on exchange of quarks, i. e. we should use quark formalism. Fock-Tani cross section is defined as

$$\sigma = \frac{4\pi^5 s}{s - 4m^2} \int\limits_{-(s-4m^2)}^{0} dt |h_{f_i}|^2, \tag{3.2}$$

where h_{f_i} is the scattering amplitude, s and t are the Mandelstam variables, m is the nucleon mass.

4.4 ART

Relativistic transport model for heavy-ion collisions called ART [15] (with version extension) was developed for AGS energies. Its main purpose was to study formation hot and dense hadronic matter for beams of Au with $p_{beam}/A = = 11.6\,\mathrm{Ge\,V}\,/$ reached at AGS.

ART is based on the same principles as BUU. It has the same "philosophy," but with some new physics and better numerical algorithm. More baryons and mesons and reactions among them are included, as well as their explicit isospin degrees of freedom. Antiparticles and heavier mesons are not included, they can be included into this model via perturbance approach. Also nearly all cross sections and angular distributions recently used in BUU model are replaced by empirical expressions based on double-logarithmic interpolations of experimental data.

The advantage of this approximation is that the finite lifetime of these resonances takes into account partially the effects of the finite formation time for produced secondaries.

The most inconsistent, but also probably the most interesting, feature shared by this model and others is the prediction, that the conditions for forming the QGP have already been reached in heavy-ion collisions at AGS energies.

The example of usage of ART can be found at [16], where they simulated collisions of uranium, whose nuclei are eccentric, and observe the dependency of collisions (i. e. creating QGP) on the angles of main axes of nuclei-nuclei – elipsoids. On the Fig. 6, we can see very similar picture as Fig. 2, however, using ART.

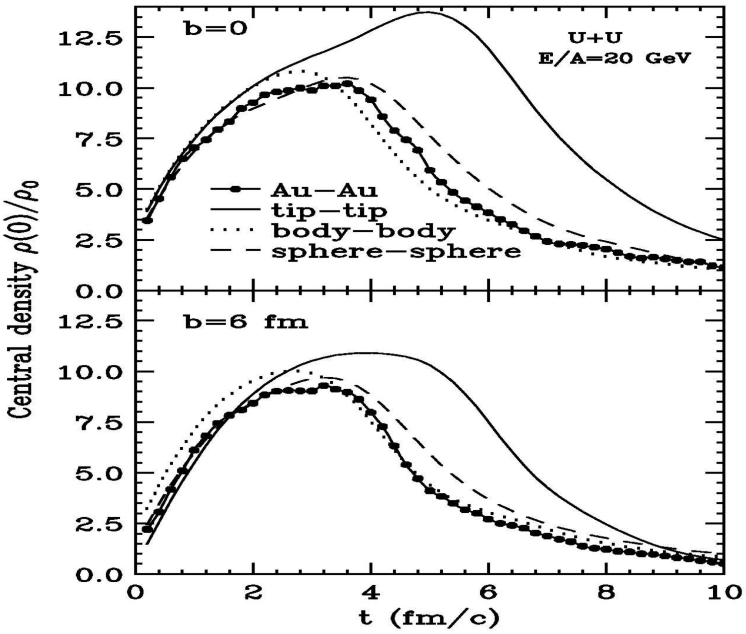


Fig. 6: The evolution of central baryon density in Au-Au (filled circles), tip-tip (solid line), body-body (dotted line) and sphere-sphere (dashed line) U-U collisions at a beam energy of 20 AGeV and an impact parameter of 0 fm (upper panel) and 6 fm (lower panel), respectively.

Chapter 5

String models

In this chapter, I would like to introduce two "string" models. Quotes are here because of today we cannot say, we are using pure string theories for describing collision processes. We are using here some pieces of whole theory. As was mentioned above, some string approach is used in RQMD. Strings are, in short, transformation from wordsheet to n-dimensional Minkovski spacetime, which should be Lorentz invariable (restriction to n). String theory can be found at [17].

5.1 JAM

Jet AA Microscopic Transport Model (JAM) was introduced by Yasushi Nara [18]. JAM is designed to simulate (ultra-) relativistic nuclear collisions from low incident energies $1 \approx 10$ AGeV up to collider energies.

JAM has been developed on the concept that should reduce hadronic transport models like BUU or QMD at low energies, in order to describe consistently nuclear collision from low to high energy, in addition to be able to treat the final state interaction among produced hadrons, while at high energies, multiple minijets production is included.

In this model the trajectories of all hadrons, resonances and produced particles are followed explicitly as a function of space and time. Nuclear collisions are modeled as a sum of independent binary hadron-hadron or parton-hadron collisions. The closest distance approach is used (see Microscopic cascade models). Excitations of hadrons are realized via resonances or strings. The initial positions govern by the parametrized distribution of nuclear density. Fermi motion is assigned to every nucleon. Pauli blocking principle is also included.

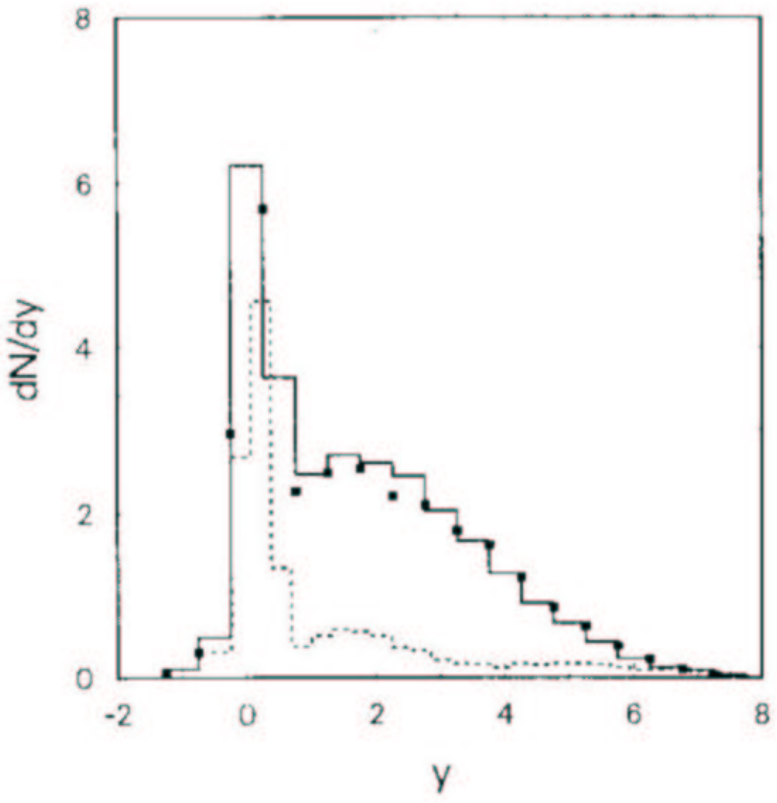
Next features are that all established hadronic states are explicitly included with explicit isospin-spin states as well as their anti-particles. The inelastic hh collisions produce at low energies resonances, whereas at high energies color strings are created. For its decay into hadron Lund string model is used. It gives roughly formation time of 1 fm/c.

Hadrons which have original constituent quarks can scatter with hadrons assuming the additive quark cross section within a formation time. This point is important for SPS energies.

Low energy baryon-baryon, baryon-meson and meson-meson rescatterings are also included assuming resonance/string excitation picture in order to treat final state interaction of hadronic gas.

5.2 QGSM

The Quark Gluon String Model is a realistic microscopic model based on a string phenomenology. QGSM is a model, which does not assume QGP formation. Hadron-hadron collisions are described by well-established phenomenological string model approach. In collisions one or more strings (or flux tubes) are created, which later decay via secondary hadron formation. In this model, the secondaries can rescatter moving the system toward equilibrium. This possibility is not included in similar Fritiof model [19]. Parameters of this model are $h + h$ and $h + A$ data. An example of comparison of QGSM results and experiments is shown at Figure 7.



Picture 7: Rapidity distributions of protons (dashed line) and all positively charged particles (full line) calculated in the QGSM for 200 GeV p+^{130}Xe reaction compared to experimental data (full squares). The sharp peak at y≈0 is the result of rescattering; spectator nucleons are not included in the plot.

The most important new physical features of this model are the rescattering of secondarily produced particles and the interactions of strings (treated in an approximate way by including the scattering of valence diquarks in a string which

had not yet hadronized.) There exist two other models, which also include these approach: VENUS [20] and RQMD discussed above.

Energy ranges for this model are about SPS, i. e. about 200 AGeV, medium and heavy projectiles. It was shown with QGSM, that for later stages of a collision it is reasonable to use the fluid dynamic model to estimate quantities we cannot evaluate in other models, like sensitivity to the EOS and QGP formating [21].

Conclusion

In this work, I poked out only some of used models for description of relativistic heavy ion collisions. I tried to descrie their basic principles and main agreements with experimental data or at least bring in some references. Many more models can be achieved from [22].

Hydrodynamics models are used as a first step into the new, deeper level of structure of matter. Today, we are at the beginning of the experimental quark-quark-gluon area. Many hydrodynamics models were introduced into describing data from RHIC. They depend on many different factors, e. g. stiffness of nuclear matter or initial phase-space distributions. Next model, not introduced in the second chapter but no less important is A Transversally Thermalized Model [23].

It seems that in the second step of this procedure one try to use more or less sophisticated cascade models. On the hadron level of matter, it showed itself as good way into the understanding of nature laws. I introduced in this paper three groups of microscopic models.

In the QMD group are Feshbach-Kerman-Koonin theory (only units or tens of AMeV), QMD (hundreds of AMeV), RQMD (up to AGeV's with first string approximations) and UrQMD (hundreds of AGeV up to TeV's, with incorporated QCD, i. e. strings and color charge).

In the BUU group are models based on Boltzmann-Uehling-Uhlenbeck equation. These models can be used, with some sophistications, up to tens AGeV. Sophistications mean involving quark exchange between nucleons (Fock-Tani formalism), extension of possible reaction channels for higher energies and improvement of their values (ART).

Finally string models try to incorporate QCD effects and they are obviusly realized on the resonance and string levels. Example of string models are JAM ($1 \approx 10$ AGeV) and QGSM (completely based on string phenomenology). The most ideal model would probably be based on "pure" string theory, but this is impossible today.

What's the difference between these models? These models have own supporter and opponents. With some improvements can be used in nearly every energy. Their disadvantages (neglection of any parameter) can be on the other hand their advantages becuase one of the criterion of the value is computational simplicity.

The models for heavy-ion collisions are just an approximation of our theories on the grade that is computable by recent machines. And, of course, nobody knows the basis on which they are built on are stable.

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